36.5: Application of MEMS for Improved Emitter Location Accuracy

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Abstract

This paper considers the problem of locating a stationary radar emitter from a single airborne platform making frequency measurements in the presence of aperture state uncertainty. It is shown that the location accuracy is most sensitive to aperture *velocity* uncertainty and that location accuracy can be improved by using a MEMS accelerometer at the aperture A decentralized method is developed for estimating the aperture velocity that integrates the on-board navigation data, the MEMS data, and Doppler shifts of any aperture-acquired GPS signals. An upper bound is given that shows the potential for significant location accuracy enhancement.

Keywords

Emitter location, MEMS, antenna vibration, accelerometer.

I. INTRODUCTION

Passive location of a stationary radar emitter with unknown frequency from a single aircraft is a problem of recent interest [1],[2], which has been shown in field tests to be an effective method (see references in [2]). The approach uses a signal model that describes the relationship between the frequency measured at a receiving aperture and the emitter location, together with least-squares to estimate the emitter's location. The signal model depends on knowledge of the position and the velocity of the aperture and any uncertainty in them ("aperture error") leads to uncertainty in the knowledge of the signal model, and hence to increased error in the location estimate.

Previous analyses of the performance of frequency-based location have considered the impact of the frequency measurement error but not the impact of aperture errors. The contribution of this paper is two-fold: (i) we present an analysis of aperture error that leads to an assessment of their relative impact, and (ii) we propose and analyze methods for reducing the aperture error, namely use of aperture-local microelectromechanical systems (MEMS) accelerometers and aperture-acquired GPS signals.

As a means to exploit these aperture-local measurements we develop a decentralized, multiple-sensor processing scheme to fuse together measurements from the on-board navigation system, an aperture-local accelerometer, and aperture-acquired GPS measurements. The estimator is structured as a federated estimation scheme and is interpreted in terms of least-squares and minimum variance optimality criteria. The performance of this scheme is assessed by developing an upper bound on the resulting aperture velocity error covariance.

II. EMITTER LOCATION ESTIMATION ERRORS

Consider a stationary coherent emitter with an unknown center frequency f_o located at an unknown position (p_x^e, p_y^e, p_z^e). We are interested here in the impact of uncertainties in the aperture position and velocity on the estimation accuracy of the parameter vector $\mathbf{p}^e = [p_x^e \quad p_y^e \quad p_z^e \quad f_o]^T$. The signal model for the Doppler-shifted center frequency $f(k)$ at the aperture at times $\{t_k\}, k = 1, 2, \ldots, M$, can be written as

$$f(k) = f_o + \frac{f_o}{c}\mathbf{v}^T(k)\mathbf{u}_R(k) \stackrel{\Delta}{=} f(\mathbf{p}^e, \boldsymbol{\theta}(k)) . \tag{1}$$

where c is the speed of light, $\mathbf{v}^T(k)$ is the aperture velocity vector, $\mathbf{u}_R(k)$ is the unit vector pointing from the aperture to the emitter location, and $\boldsymbol{\theta}(k)$ is the vector of aperture positions and velocities. Measurements of these frequencies can be put into vector form

$$\mathbf{m} = \mathbf{f}(\mathbf{p}^e, \boldsymbol{\Theta}) + \mathbf{e} , \tag{2}$$

where $\mathbf{e}$ is a vector of additive measurement noise. For use in processing it is not possible to perfectly know $\boldsymbol{\Theta}$ (hereafter called the local states); thus, the algorithm has access only to an estimate $\hat{\boldsymbol{\Theta}}$ that differs from $\boldsymbol{\Theta}$ due to errors in the on-board navigation system as well as uncertainty due to flexure and vibration of the airframe holding the aperture.

The emitter location problem then becomes: given frequency measurement vector $\mathbf{m}$ and local state estimate vector $\hat{\boldsymbol{\Theta}}$, compute an estimate $\hat{\mathbf{p}}^e$ of the emitter parameter vector $\mathbf{p}^e$. The goal of this section is to characterize the impact of using $\hat{\boldsymbol{\Theta}}$ in this processing rather than the unavailable error-free $\boldsymbol{\Theta}$. Because the statistics of $\mathbf{e}$ and $\hat{\boldsymbol{\Theta}}$ are not known in practice and may not be well-modeled as Gaussian, the estimation approach used is often least-squares, which, because of the nonlinear measurement model, is generally solved by iterating a linearized least-squares model in terms of the residual [1]. Here we focus on characterizing the direct impact of the aperture uncertainty on the location estimation error.

For a given $\hat{\boldsymbol{\Theta}}$, least-squares processing seeks a location estimate $\hat{\mathbf{p}}^e$ based on measurement vector $\mathbf{m}$ and estimated signal model $\mathbf{f}(\mathbf{p}^e, \hat{\mathbf{\Theta}})$ that minimizes a weighted least-squares cost with positive definite weighting matrix $\mathbf{W}$. The condition for a least-squares solution is

$$\mathbf{H}^T \mathbf{W}[\mathbf{m} - \mathbf{f}(\hat{\mathbf{p}}^e, \hat{\mathbf{\Theta}})] = 0, \tag{3}$$

where

$$\mathbf{H} = \left. \frac{\partial \mathbf{f}(\mathbf{p}^e, \hat{\mathbf{\Theta}})}{\partial \mathbf{p}^e} \right|_{\mathbf{p}^e = \hat{\mathbf{p}}^e} \tag{4}$$

Using the definition of $\mathbf{H}$ in (4) as a derivative and assuming that the location estimate is close to the true value gives

$$\mathbf{H}(\mathbf{p}^e - \hat{\mathbf{p}}^e) = \mathbf{f}(\mathbf{p}^e, \hat{\mathbf{\Theta}}) - \mathbf{f}(\hat{\mathbf{p}}^e, \hat{\mathbf{\Theta}}). \tag{5}$$

Multiplying both sides of (5) by $\mathbf{H}'\mathbf{W}$, solving for $\mathbf{p}^e - \hat{\mathbf{p}}^e$, and then using (3) and (2) gives

$$\hat{\mathbf{p}}^e - \mathbf{p}^e = (\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1} \mathbf{H}' \mathbf{W} [\mathbf{e} + \tilde{\mathbf{f}}_\theta], \tag{6}$$

where $\tilde{\mathbf{f}}_\theta \equiv \mathbf{f}(\mathbf{p}^e, \mathbf{\Theta}) - \mathbf{f}(\mathbf{p}^e, \hat{\mathbf{\Theta}})$ is the signal model error. Therefore, (6) shows that the location error is affected as much by the signal model error $\tilde{\mathbf{f}}_\theta$ as it is by the measurement error $\mathbf{e}$.

III. CRUCIAL ROLE OF APERTURE VELOCITY

In this section, we analyze the sensitivity of the signal model with respect to the local state estimates. The contribution from all error sources is then compared to the currently achievable frequency measurement accuracy. This allows us to determine where to focus our improvement in local state estimation accuracy.

The physical separation between the aperture and the navigation system reduces the local state estimate accuracy at the aperture from that afforded by an integrated on-board navigation system. Consider an aperture at a wingtip. It is known that the deflection of wingtips of large aircraft can be several feet, and even fighter aircraft exhibit sizable flexibility effects. The position perturbation caused by such wingtip deflections, however, is at most *0.01%* relative to the typical range under consideration, and at most *10%* relative to the best position accuracy provided by an INS/GPS system [3]. On the other hand, the frequency of the first flexible mode is in the range of several Hertz, which can result in an adverse velocity component of several meters/second. The velocity perturbation is approximately *1%* relative to the nominal velocity under consideration, and is 10^4*%* at the worst relative to the best velocity accuracy provided by an INS/GPS [3]. Therefore, more accurate aperture velocity measurement is needed. We next analyze how these perturbations propagate through the signal model to the calculated frequency.

More precisely, the extent the signal model is influenced by aperture velocity vector $\mathbf{v}$ vs. aperture position vector $\mathbf{p}$ can be seen from

$$df = \frac{\partial f}{\partial \mathbf{v}} d\mathbf{v} + \frac{\partial f}{\partial \mathbf{p}} d\mathbf{p}, \tag{7}$$

where $\partial f/\partial \mathbf{p}$, $\partial f/\partial \mathbf{v}$ are the gradients of f with respect to $\mathbf{p}$, $\mathbf{v}$, respectively. The uncertainty in $\mathbf{v}$ is reflected in f through

$$\left\| \frac{\partial f}{\partial \mathbf{v}} \right\| = \left\| \frac{f_0}{c} \mathbf{u}_R \right\| = \frac{f_0}{c}, \tag{8}$$

while the uncertainty in $\mathbf{p}$ is reflected in f through

$$\left\| \frac{\partial f}{\partial \mathbf{p}} \right\| \le \frac{f_0}{c} \frac{2\|\mathbf{v}\|}{R}, \tag{9}$$

where R is the aperture-to-emitter range, and $R_x = p_x^e - p_x$, $R_y = p_y^e - p_y$, $R_z = p_z^e - p_z$.

A comparison between (8) and (9) reveals that the sensitivity to velocity errors is $R/2\|\mathbf{v}\|$ times the sensitivity to position errors, and for the typical values of R and $\|\mathbf{v}\|$ under consideration, $R/2\|\mathbf{v}\|$ is on the order of *100*. Thus, reducing the uncertainty in $\mathbf{v}$ is at least *100* times more effective than reducing the uncertainty in $\mathbf{p}$ in order to achieve the same relative accuracy improvement.

IV. DECENTRALIZED PROCESSING

A. Aperture-Local Sensor Approaches

Local measurements made at the aperture and subsequent processing of them are needed to reduce the aperture velocity uncertainty. Bearing in mind that the best effort should be made to minimize the need for added hardware, two possible arrangements of sensors for local velocity measurements are proposed.

1. Doppler-shifted GPS frequencies acquired at the aperture can be used for determining the local states.
2. Aperture-mounted accelerometers can be used to correct the on-board navigation state estimates.

These two options together provide opportunity for development of fault tolerant velocity estimation.

We consider that three sets of measurements are individually processed to estimate the aperture velocity: one set from the on-board navigation system, one set from an aperture-local MEMS accelerometer, and one from the aperture-received GPS signal carrier frequency measurement. We have chosen to separately process these three sets of measurements and then combine the estimates in a sensible way. The reasons are simple: (i) the navigation system should be kept intact for its original function, (ii) it allows progressive system upgrade, (iii) synchronization among sensors is not needed, and (iv) it is tolerant to sensor outages. We first describe the three individual local velocity estimates and then develop a federated local state estimator that combines them.

B. Individual Local Velocity Estimates

1. Using On-Board INS/GPS Measurements

Suppose the on-board navigation system has a dedicated filter that also takes into account the on-board GPS measurement integration (i.e., not the aperture-acquired GPS of Option 1). Let $\hat{\mathbf{x}}_N(k/k-1)$ and $\hat{\mathbf{x}}_N(k/k)$ be the time and the measurement updates of the navigation state estimates, and let $\mathbf{P}_N(k/k-1)$ and $\mathbf{P}_N(k/k)$ be the corresponding estimation error covariances. Define T as the (time-varying) function used by the processing to map from the navigation states referenced at the inertial navigation system to the aperture velocity; for example, T may be a lever-arm adjustment. The resulting estimate of the aperture state is $\hat{\mathbf{x}}_{DN}(k/*) = T[\hat{\mathbf{x}}_N(k/*)]$, where subscript '$DN$' stands for "Doppler (local) state from Navigation data;" it suffers from errors in the navigation state estimate $\hat{\mathbf{x}}_N(k/*)$ as well as uncertainty in the mapping function T. Denote by $\mathbf{x}_D(k)$ the local state, where subscript 'D' stands for 'Doppler'. Then

$$\begin{aligned}\mathbf{x}_D(k) &\approx \hat{\mathbf{x}}_{DN}(k/*)\\ &\quad + \boldsymbol{\tau}(k/*)[\mathbf{x}_N(k) - \hat{\mathbf{x}}_N(k/*)] + \mathbf{w}_T(k),\end{aligned} \tag{10}$$

where $\boldsymbol{\tau}(k/*)$ is a derivative of T w.r.t. $\mathbf{x}_N(k)$ and $\mathbf{w}_T(k)$ is the additive local state uncertainty associated with the transformation T. Its covariance is denoted by $\mathbf{Q}_T(k)$. It follows from (10) that local estimation error covariances obey

$$\mathbf{P}_{DN}(k/*) = \boldsymbol{\tau}(k/*)\mathbf{P}_N(k/*)\boldsymbol{\tau}^T(k/*) + \mathbf{Q}_T(k)\,. \tag{11}$$

It can be seen that if the uncertainty introduced by transformation T is large ($\mathbf{Q}_T$ large), the local state estimate error $\mathbf{P}_{DN}$ will be large, regardless of the accuracy of the on-board navigation system ($\mathbf{P}_N$ small or large). Since this is likely the case in practice due to vibration and (unmodeled) flexure, this points out the need for using local measurements.

2. Using Aperture-Local Accelerometers

Suppose acceleration vector $\mathbf{y}_a$ is measured by a tri-axis accelerometer at the aperture, and the additive measurement error $\mathbf{w}_a$ has a covariance $\mathbf{Q}_a$ that captures the accuracy of the accelerometer. The measurement is then integrated using Euler integration with update rate $1/T_s$ to obtain the local velocity. Then local state $\mathbf{x}_D = \mathbf{v}$ is governed by

$$\begin{aligned}&\mathbf{x}_D(k+1) = \mathbf{x}_D(k) + \mathbf{y}_a(k)T_s + \mathbf{w}_a(k)T_s,\\ &\mathbf{y}_L(k) = \mathbf{x}_D(k) + \mathbf{v}_L(k),\end{aligned} \tag{12}$$

where $\mathbf{v}_L$, with a covariance $\mathbf{R}_L$, is the sensor noise associated with the velocity measurement. Subscript 'L' stands for 'Local'. Note that since $\mathbf{y}_a$ is accessible, it is considered as an input to process equation (12). The model described in (12) alone leads to a Kalman filter local estimate given by

$$\begin{aligned}&\hat{\mathbf{x}}_{DL}(k/k-1) = \hat{\mathbf{x}}_{DL}(k-1/k-1) + \mathbf{y}_a(k-1)T_s,\\ &\hat{\mathbf{x}}_{DL}(k/k) = \hat{\mathbf{x}}_{DL}(k/k-1) + \mathbf{K}_{DL}(k)[\mathbf{y}_a(k) - \hat{\mathbf{x}}_{DL}(k/k-1)],\end{aligned} \tag{13}$$

where $\mathbf{K}_{DL}(k)$ is the gain matrix. The velocity estimation error covariances are given by

$$\begin{aligned}&\mathbf{P}_{DL}(k/k-1) = \mathbf{P}_{DL}(k-1/k-1) + \mathbf{Q}_a(k-1)T_s^2,\\ &\mathbf{P}_{DL}^{-1}(k/k) = \mathbf{P}_{DL}^{-1}(k/k-1) + \mathbf{R}_L^{-1}(k).\end{aligned} \tag{14}$$

Subscript 'DL' above stands for 'Doppler (local) state from Local accelerometer measurements'. It has become very clear now from the above equations that three factors contribute to the accuracy of this local velocity estimate: the accuracy of the local acceleration measurement ($\mathbf{Q}_a$); the accuracy of the local velocity measurement ($\mathbf{R}_L$); and the update rate ($1/T_s$).

3. Using Aperture-Acquired GPS Signals

In addition to the emitter signal picked up by the aperture, it is also possible that the aperture picks up the Doppler-shifted carrier frequencies of visible GPS signals, assuming receiver resources are available. Note that this is in addition to the GPS signals picked up by the navigation system's integrated GPS system. Also note that if the aperture is mounted on the bottom surface of the wing then visibility of GPS signals is highly limited; in that case virtually similar results could be obtained by mounting a dedicated GPS antenna on top of the wing, directly above the aperture – this would minimize the mapping error from this GPS antenna to the aperture location. The GPS frequency measurements can be stacked into a vector equation given by

$$\mathbf{y}_G(k) = \mathbf{H}_G(k)\mathbf{x}_D(k) + \mathbf{v}_G(k)\,. \tag{15}$$

The measurement in (15) can be used in a stand-alone static processor, or a single step dynamic processor when available *a priori* local information is used. We explore only the second one here, where the estimate is given by

$$\begin{aligned}\hat{\mathbf{x}}_{DG}(k/k) &= \hat{\mathbf{x}}_{DL}(k/k-1)\\ &\quad + \mathbf{P}_{DL}(k/k-1)\mathbf{H}_G^T(k)\times\\ &\quad [\mathbf{H}_G(k)\mathbf{P}_{DL}(k/k-1)\mathbf{H}_G^T(k) + \mathbf{R}_G(k)]^{-1}\times\\ &\quad [\mathbf{y}_G(k) - \mathbf{H}_G(k)\hat{\mathbf{x}}_{DL}(k/k-1)]\end{aligned} \tag{16}$$

where $\hat{\mathbf{x}}_{DL}(k/k-1)$ is the most recent output of the local estimator associated with the accelerometer given in (13) and $\mathbf{P}_{DL}(k/k-1)$ is given in (14). The Kalman filter estimation error covariance is

$$\mathbf{P}_{DG}(k/k) = [\mathbf{P}_{DL}^{-1}(k/k-1) + \mathbf{H}_G^T(k)\mathbf{R}_G^{-1}(k)\mathbf{H}_G(k)]^{-1}. \tag{17}$$

C. Federated Local State Estimator

We now combine these three estimates using a federated estimator [6] given by

$$\hat{\mathbf{x}}_D(k/k) \equiv \mathbf{P}_D(k/k)[\mathbf{P}_{DN}^{-1}(k/k)\hat{\mathbf{x}}_{DN}(k/k) + \mathbf{P}_{DL}^{-1}(k/k)\hat{\mathbf{x}}_{DL}(k/k) + \mathbf{P}_{DG}^{-1}(k/k)\hat{\mathbf{x}}_{DG}(k/k)], \tag{18}$$

with

$$\mathbf{P}_D^{-1}(k/k) \equiv \mathbf{P}_{DN}^{-1}(k/k) + \mathbf{P}_{DL}^{-1}(k/k) + \mathbf{P}_{DG}^{-1}(k/k) \tag{19}$$

Note that $\mathbf{P}_{DX}(k/k)$, (X=N, L, or G) is set to infinity whenever the X-th sensor outage occurs. The optimal solution [6] is not pursued here because of the difficulty in applying Carlson's procedure here. Nevertheless, the estimate in (18) can be shown [7] to be the solution derived under a weighted least squares cost and can also be given a minimum variance interpretation.

More essential than optimality is the accuracy of the solution at hand. For simplicity arguments k and k/k will be suppressed in the following. The estimation error covariance can be shown to be

$$Cov\{\mathbf{x}_D - \hat{\mathbf{x}}_D\} = \mathbf{P}_D\begin{bmatrix}\mathbf{P}_{DN}^{-1} & \mathbf{P}_{DL}^{-1} & \mathbf{P}_{DG}^{-1}\end{bmatrix}\mathbf{P}\begin{bmatrix}\mathbf{P}_{DN}^{-1}\\ \mathbf{P}_{DL}^{-1}\\ \mathbf{P}_{DG}^{-1}\end{bmatrix}\mathbf{P}_D,$$
$$\mathbf{P} = \begin{bmatrix}\mathbf{P}_{DN} & \mathbf{P}_{NL} & \mathbf{P}_{NG}\\ \mathbf{P}_{LN} & \mathbf{P}_{DL} & \mathbf{P}_{NG}\\ \mathbf{P}_{GN} & \mathbf{P}_{GL} & \mathbf{P}_{DG}\end{bmatrix}. \tag{20}$$

All off-diagonal cross-covariance terms in $\mathbf{P}$ can be expressed via recursive manipulation, but we focus on deriving an upper bound for the estimation error covariance that uses only the diagonal covariance terms. These are the terms readily provided by the decentralized processors. To further simplify the notations, let $\mathbf{P}_1 = \mathbf{P}_{DN}$, $\mathbf{P}_2 = \mathbf{P}_{DL}$, and $\mathbf{P}_3 = \mathbf{P}_{DG}$. Then $\mathbf{P}_D = \left[\mathbf{P}_1^{-1} + \mathbf{P}_2^{-1} + \mathbf{P}_3^{-1}\right]^{-1}$.

<u>Theorem</u>: An upper bound on $Cov\{\mathbf{x}_D - \hat{\mathbf{x}}_D\}$ is given by

$$Cov\{\mathbf{x}_D - \hat{\mathbf{x}}_D\} \le \mathbf{P}_D[k_1\mathbf{P}_1^{-1} + k_2\mathbf{P}_2^{-1} + k_3\mathbf{P}_3^{-1}]\mathbf{P}_D, \tag{21}$$

where

$$k_i = \frac{\sum_{j=1}^{3}\sqrt{\mathbf{z}_j'\mathbf{P}_j\mathbf{z}_j}}{\sqrt{\mathbf{z}_i'\mathbf{P}_i\mathbf{z}_i}}, \quad \mathbf{z}_i \ne \mathbf{0}, \quad i = 1,2,3. \tag{22}$$

In addition,

$$E\{\|\mathbf{x}_D - \hat{\mathbf{x}}_D\|^2\} = Trace\{Cov\{\mathbf{x}_D - \hat{\mathbf{x}}_D\}\} \le \left(\sum_{i=1}^{3}\sqrt{Trace\{\mathbf{P}_D\mathbf{P}_i^{-1}\mathbf{P}_D\}}\right)^2 \le Trace\{\mathbf{P}_D[k_1\mathbf{P}_1^{-1} + k_2\mathbf{P}_2^{-1} + k_3\mathbf{P}_3^{-1}]\mathbf{P}_D\}. \tag{23}$$

Proof: (see [7])

Remarks: (i) A particular set of feasible k_i's in (22) can be, for example,

$$k_i = \frac{\sum_{j=1}^{3}\sqrt{\sigma_j}}{\sqrt{\sigma_i}}, \quad \sigma_{\min}(\mathbf{P}_j) \le \sigma_j \le \sigma_{\max}(\mathbf{P}_j), \quad i,j = 1,2,3.$$

(ii) The upper bound tends to be exact when one sensor error is much smaller than the others or when the estimation errors of all processors are completely correlated. The upper bound becomes more conservative when the correlation becomes weaker.

D. Assessment of Benefits

The following analysis provides a crude numerical estimation on the potential for accuracy improvement of local velocity estimates. We consider (11), (14), and (17) in steady state form and to simplify further, assume that

$$\mathbf{P}_{DN} = \sigma_{DN}^2\mathbf{I},\ \boldsymbol{\tau}\mathbf{P}_N\boldsymbol{\tau}' = \sigma_N^2\mathbf{I},\ \mathbf{P}_{DL} = \sigma_{DL}^2\mathbf{I},\ \mathbf{P}_{DG} = \sigma_{DG}^2\mathbf{I},$$
$$\mathbf{Q}_T = \sigma_T^2\mathbf{I},\ \mathbf{Q}_a = \sigma_a^2\mathbf{I},\ \mathbf{R}_L = \sigma_L^2\mathbf{I},\ \mathbf{R}_G = \sigma_G^2\mathbf{I}. \tag{24}$$

All identity matrices are of size 3×3. Two different situations for $\mathbf{H}_G$ are considered:

$$\mathbf{H}_G \approx \frac{f_o^i}{c}\mathbf{I}, \quad \text{or} \quad \mathbf{H}_G \approx \frac{f_o^i}{c}\begin{bmatrix}1 & 0 & 0\end{bmatrix}. \tag{25}$$

These represent two extreme situations of GPS satellite visibility: three satellites and one satellite, respectively. The ranges of the parameter values to be used in the above equations are now given. The references from which the numbers come, whenever available, are cited. $\sigma_N = 0.02$ m/s (see [3], Table 1), $\sigma_T = 0, 0.2, 2$ m/s (small, medium, and large lever-arm-adjustment uncertainty), $\sigma_a = 10\ \mu\text{m/s}^2 \sim 10\ \text{mm/s}^2$ (high to medium resolution of capacitive MEMS accelerometers [4]), $T_s = 0.02$ s (medium update rate), $\sigma_L = 0.002$ m/s~ 2 m/s (high to medium velocity accuracy (see [5], Table 1)), $\sigma_G = 1$ Hz (consistent with the GPS velocity accuracy from Doppler shift measurement), $f_o^{L1}/c = 5.25\ \text{m}^{-1}$. The velocity estimation error bound given in (23) is calculated as a function of σ_a and T_s with σ_T as a parameter, and $\sigma_L = 200\sigma_a$ [3],[5].

The results of accuracy improvement computation for local velocity estimate are shown in Figure 1 and Figure 2. The horizontal axes for these two figures are labeled as local accelerometer resolution at a *50* Hz measurement update rate. The vertical axes in these figures are labeled

$20\log_{10}(\sigma_D/\sigma_{DN})$, where σ_D represents the *1*-σ local velocity estimate accuracy upper bound for the federated estimator, and σ_{DN} represents the *1*-σ local velocity estimate accuracy inferred from the on-board navigation state estimates without additional measurements. Therefore, the vertical axis indicates the amount of local velocity accuracy improvement in dB. For each plot, the uncertainty σ_T associated with the navigation state to local velocity transformation is set at *0*, *0.2*m/s, and *2*m/s. Figure 1 shows the velocity accuracy improvement bounds for the case of using on-board navigation data together with local accelerometer measurements but no aperture-local GPS measurements. Figure 2 shows the velocity accuracy improvement bounds for the case of using on-board navigation data together with local accelerometer measurements and GPS carrier frequency measurements from one or three satellites – line plots show the result for 1 GPS satellite while the circle marker plots show the result for 3 GPS satellite. Note that the 3-satellite result is not shown for the $\sigma_T = 0$ case because it was virtually identical to the 1-satellite result.

It can be concluded from these numerical results that significant local velocity accuracy improvement can be expected by adding a high-resolution local accelerometer. For example, it can be seen from Figure 1 that a *100*μg local accelerometer would provide at least a *60*dB reduction of local velocity estimation error assuming a *2*m/s *1*-σ local velocity uncertainty in the absence of the accelerometer. This is a factor of 10^{-3} accuracy improvement, giving an improved local velocity accuracy of at least $2\times10^{-3} = 0.002$m/s. Note that the current MEMS technology can already produce devices an order of magnitude more sensitive than *100*μg [4]. On the other hand, without filtering the local measurements, the accelerometer resolution translates into a local velocity accuracy at around *0.2*m/s [3],[5].

Comparing Figure 1 to Figure 2 shows that the benefits gained by acquiring local GPS measurements on top of the accelerometer measurements are quite limited. The benefits become noticeable only when the uncertainty of the local velocity estimate based on the lever-arm-adjusted navigation-state estimate is very severe, the local accelerometer accuracy is very poor, and three satellites are visible.

V. CONCLUSIONS

Signal model error adds directly to the measurement error to impact the location accuracy. Velocity error has 100 times the impact of position error. Expected levels of aperture velocity errors can have a significant impact on location accuracy unless they are reduced. Aperture-local MEMS accelerometers and aperture-acquired GPS measurements together with the on-board navigation system's measurements can reduce the impact of the aperture errors to a negligible level. Our derived bounds on the resulting aperture velocity error covariance for the proposed processing show that significant benefit can be gained. We also showed that the addition of the aperture-local GPS measurements, even when three satellites are available, provides little further improvement.

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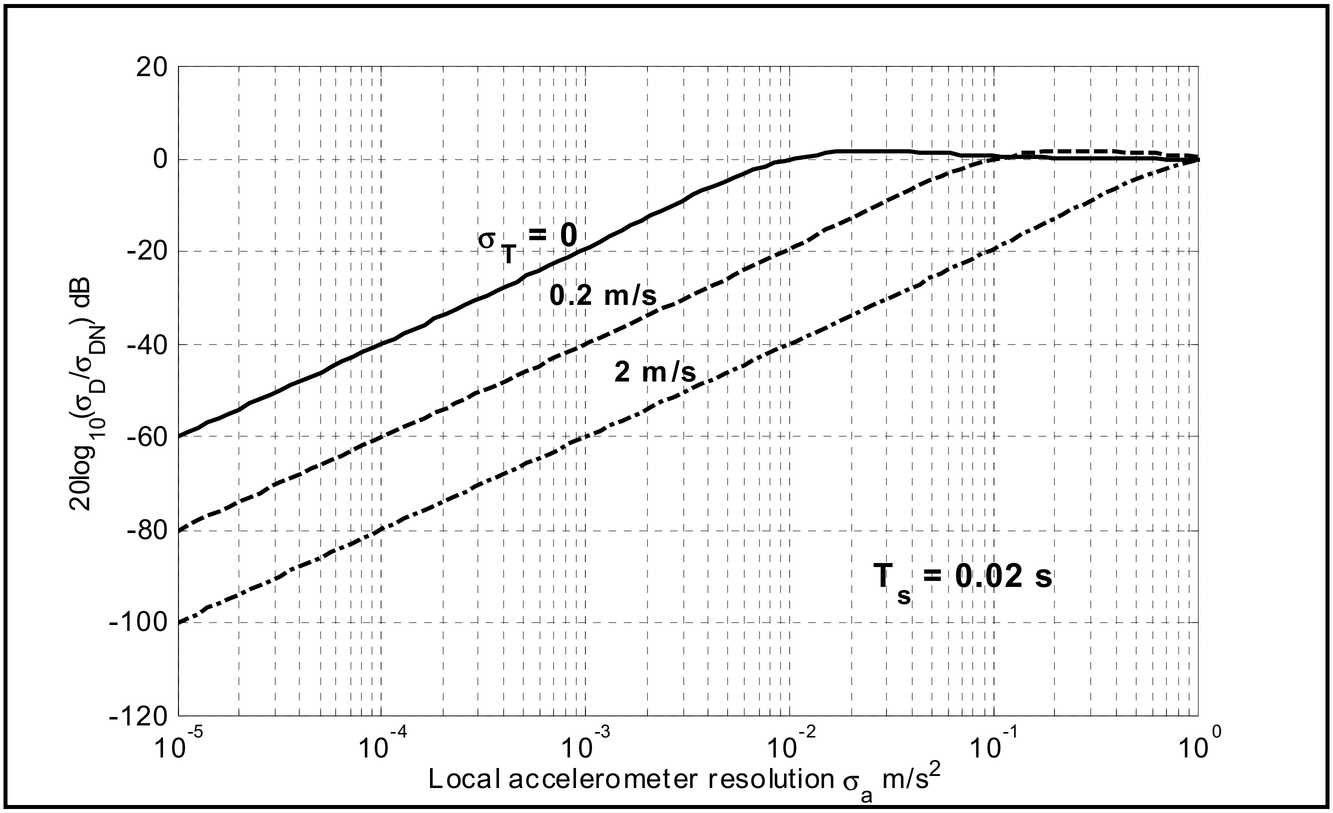

Figure 1: Improvement using local accelerometer and 0 GPS signals

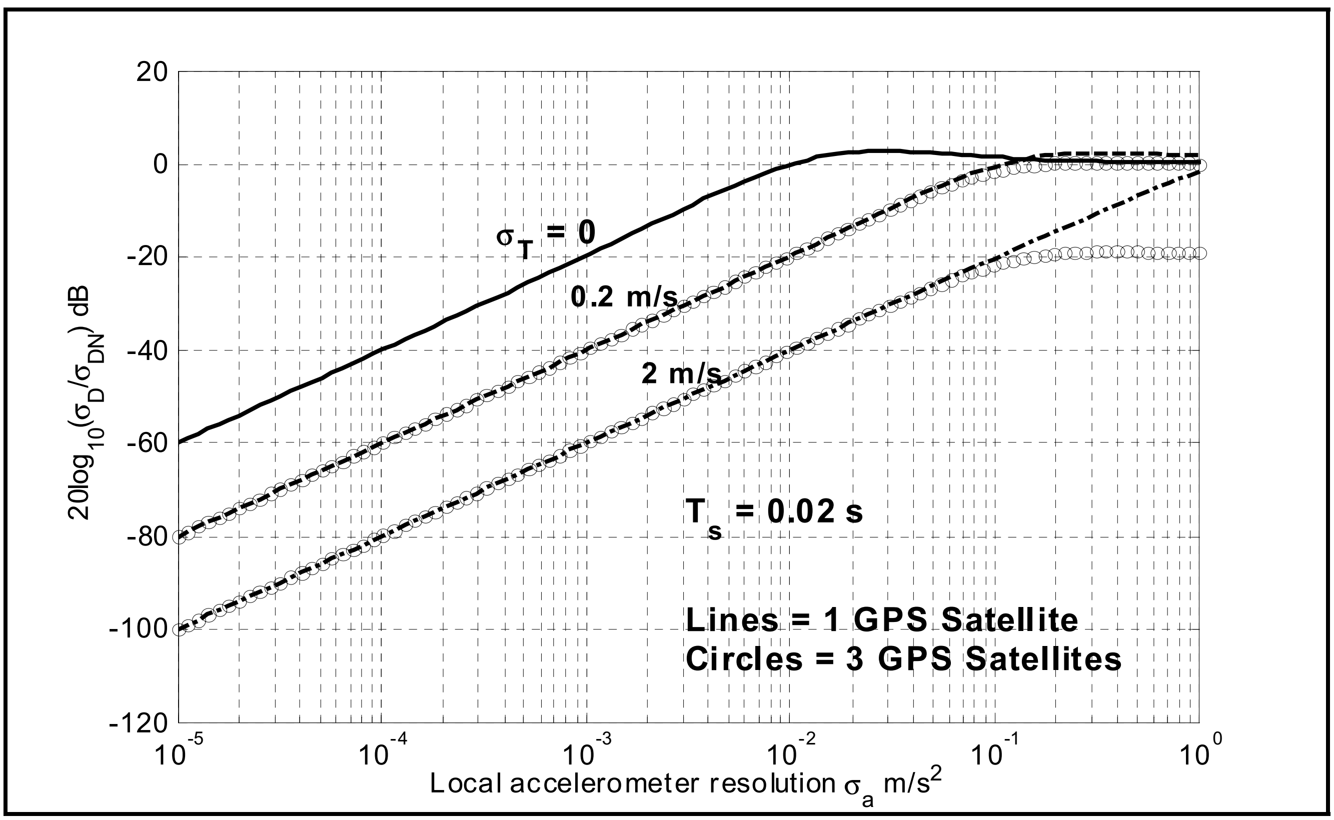

Figure 2: Improvement using local accelerometer and 1 or 3 GPS signal