Optimizing Non-MSE Distortion for Data Compression in Emitter Location Systems

Mo Chen and Mark L. Fowler
Department of Electrical and Computer Engineering
State University of New York at Binghamton
Binghamton, NY 13902
{mchen0,mfowler}@binghamton.edu

***Abstract* – We present a new distortion measure for compression in systems that share data in order to locate RF signal emitters and discuss its optimal use. These results are relevant in wireless location systems, electronic warfare systems, and sensor networks. The optimal bit allocation with respect to this distortion measure is accomplished through use of a genetic algorithm with a penalty function. We present the general principles on which the distortion measure is based and then derive its specific form for the problem at hand. Then we describe a penalty function used with a genetic algorithm to optimize the bit allocation with respect to the new distortion measure. Finally, presented simulation results demonstrate the effectiveness of the method: (i) a 60% improvement in compression ratio for the same level of TDOA accuracy and (ii) a 30% reduction in TDOA error at a compression ratio of 8.**

I. INTRODUCTION

A common way to locate electromagnetic emitters is to measure the time-difference-of-arrival (TDOA) between pairs of signals received at geographically separated sensors [1]. The measurement of TDOA between these signals is done by coherently cross-correlating the signal pairs [2] and requires that the signal samples of the two signals are available at a common site, which is generally accomplished by transferring the signal samples over a data link from one sensor to the other. Often the available data link rate is insufficient to accomplish the transfer within the time requirement unless some form of lossy data compression is employed in the emitter location system. Emitter location is a typical task in wireless systems, electronic warfare systems, and sensor network scenarios.

We begin by summarizing previous compression-for-location results. For the case of white Gaussian signals and noises, bounds on the rate-distortion performance for the TDOA problem were established and compared to the performance achievable using non-adaptive scalar quantizers, where distortion is measured in terms of lost SNR due to the mean-square error (MSE) of lossy compression [3]. Adaptive scalar quantizers developed specifically for RF signals were considered in [4]. Wavelet-based methods have been proposed and demonstrated [5]. All these approaches use mean-square error (MSE) distortion measures and therefore measure the effect of the lossy compression on the SNR.

More recently, we have developed a non-MSE distortion measure has been developed and have shown it improves compression in TDOA-based geolocation systems [6], [7]. The measure is based on the insight on TDOA estimation accuracy provided by the Cramer-Rao bound and provides a mathematically-based trade-off between the impact of the compression on the MSE as well as the impact of the compression on signal characteristics that determine accuracy (i.e., the signal's Gabor bandwidth in the case of TDOA). Although this distortion measure has been used to advantage [6], [7], no means for using it to *optimally* allocate bits in transform-based compression has been previously available, although we have developed three sub-optimal algorithms [8].

Optimizing the non-MSE distortion measure is a bit-allocation problem that requires combinatorial optimization of an objective function that is very non-linear. This makes conventional optimization methods incapable of coping well with this problem; however, the genetic algorithm is a very strong approach for this setting. This paper applies the genetic algorithm to find the optimal bit allocation according to this new distortion measure.

II. NON-MSE DISTORTION MEASURE FOR TDOA

In TDOA applications it is crucial that the compression methods minimize the impact on the TDOA estimation performance rather than simply stressing minimization of MSE, as is common in many compression techniques. The accuracy of the TDOA estimates is governed by the Cramer-Rao bound (CRB) for TDOA given by

$$\sigma_{TDOA} \geq \frac{1}{2\pi B_{rms}\sqrt{2SNR_o}} \tag{1}$$

where B_{rms} is the signal's RMS (or Gabor) bandwidth in Hz given by

$$B_{rms}^2 = \frac{\int f^2 |S(f)|^2 df}{\int |S(f)|^2 df}, \qquad (2)$$

with $S(f)$ being the Fourier transform of the signal, and

$$SNR_o = \frac{WT}{\frac{1}{SNR_1} + \frac{1}{SNR_2} + \frac{1}{SNR_1 SNR_2}},$$

where SNR_1 and SNR_2 are the SNRs of the two signals to be cross-correlated.

If we wish to choose a distortion measure that reflects the TDOA accuracy we could choose to minimize the right-hand side of (1) for a given desired rate R. It is clear that this is equivalent to maximizing

$$SNR_{TDOA} = B_{rms}^2 SNR_o \qquad (3)$$

for a given desired rate R [7]. From (2) we see that allocating bits to transform coefficients using the distortion measure (3) tends to exclude those coefficients that have very little impact on the RMS bandwidth.

It is convenient to have a more usable form for (3). Let SNR_1 be the SNR of the signal to be compressed and let SNR_2 be the SNR at the other sensor (i.e., for the signal that is *not* compressed). We consider here only the case where $SNR_1 << SNR_2$ for which we have shown [7] that

$$SNR_{TDOA} = \frac{\int f^2 |S(f)|^2 df}{N_o W}, \qquad (4)$$

where N_o is the noise power spectral density level of the white noise and $S(f)$ is the Fourier transform of the signal.

So the general goal of our compression method is the following, expressed as transform coding with a non-MSE distortion. Given some orthogonal signal decomposition (e.g., wavelet transform, DFT, etc.)

$$s(k) = \sum_{n=1}^{N} c_n \psi_n(k)$$

of the signal to be compressed, we wish to coded the coefficients c_n to achieve a desired rate-distortion goal where distortion is measured using (4). That is, we wish to selected a subset $\tilde{\Omega}$ of coefficient indices and an allocation of bits $B = \{b_i \mid i \in \tilde{\Omega}\}$ to the selected coefficients such that the signal given by

$$\tilde{s}(k) = \sum_{n \in \tilde{\Omega}} \tilde{c}_n \psi_n(k),$$

where $\{\tilde{c}_n \mid n \in \tilde{\Omega}\}$ are the quantized versions of the selected coefficients. Obviously, the goal of the selection/allocation is to maximize (4) while meeting the rate constraint given by

$$\sum_{i \in \tilde{\Omega}} b_i \le R \quad , b_i \text{ is nonnegative integer}.$$

In general, *jointly* determining the optimal selection and allocation is quite difficult because of (i) the nonlinear, non-monotonic dependence of (4) on the coefficients, and (ii) the fact that removing a coefficient from $\tilde{\Omega}$ effects the numerator and denominator of (4). Previously, we have developed three suboptimal algorithms [8]: (i) using a frequency-quadratic weighting on the coefficients before quantizing with a quantizer having a dead zone – this increases the number of low frequency coefficients that get quantized using zero bits (i.e. not selected); (ii) using integer programming to first select coefficients and then using dynamic programming to allocate bits to the remaining coefficients; (iii) first allocating bits to achieve a specified MSE distortion (in excess of the rate constraint) and then using the knapsack algorithm to select the coefficients that maximize (4) while not exceeding the required rate constraint.

Our goal in this paper is to use the genetic algorithm to *jointly* optimize the selection/allocation. We consider here the use of the DFT as the transform; it is chosen not because it is necessarily the best transform to use (it isn't!), but because it is simple to apply and our focus here is on the optimization methods used. In the absence of compression, for the N-point DFT $S[k]$, $k = -N/2, -N/2+1, \ldots, N/2-1$ having frequency bin size Δf, the measure in (4) becomes

$$SNR_{TDOA} = \frac{\sum_{k=-N/2}^{N/2-1} (k\Delta f)^2 |S[k]|^2}{N_0 \, \Delta f \, N} \qquad (5)$$

Let the set of indices of the DFT coefficients be the set $\Omega = \{-N/2, -N/2+1, \ldots, N/2-1\}$. Let the selection set $\tilde{\Omega} \subseteq \Omega$ be the subset of indices of the DFT coefficients that are kept during compression, let $B = \{b_i \mid i \in \tilde{\Omega}\}$ be the set of bit allocations for the selection set (i.e., the i^{th} DFT coefficient in $\tilde{\Omega}$ is allocated b_i bits for quantization), and let $P_q(B)$ be the quantization noise power due to this allocation. Then (5) becomes

$$SNR_{TDOA}(\tilde{\Omega}, B) = \frac{\sum_{k \in \tilde{\Omega}} (k\Delta f)^2 \left| S[k + N/2] \right|^2}{N_0 \Delta f \left| \tilde{\Omega} \right| + P_q(B)} \qquad (6)$$

where $\left|\tilde{\Omega}\right|$ denotes the number of elements in the selection set $\tilde{\Omega}$. Hence, the selection/allocation problem reduces to the maximization of (6) under the constraint of

$$\sum_{i \in \tilde{\Omega}} b_i \le R, \quad b_i \text{ is nonnegative integer} .$$

III. Non-MSE Allocation Using Genetic Agorithm

The genetic algorithm has shown great promise for solving challenging optimization problems that are plagued by multimodal and nonlinear objective functions [9]. That capability combined with relatively low complexity compared to other methods (e.g., dynamic programming) has lead to the GA's increasing use in signal processing applications (for example, complexity is low enough that GA-based adaptive IIR filters are being proposed [9]) . In addition, the GA is well suited to constrained optimization problems [10]. These characteristics make the GA particularly well suited to the selection/allocation problem described above in Section II.

The basic idea of the GA is to start with a set of possible solutions and then create new possible solutions through a process that mimics the genetics of human reproduction. Because of this mimicking, possible solutions are called "chromosomes". The GA consists of a cycle of three steps – selection, genetic operation, and replacement – that are repeated until some specified termination criteria is met. There are also three sets of possible solutions on which these steps operate: the population, the mating pool, and the subpopulation. The GA starts with a randomly selected set of chromosomes. The first step is to select a subset of chromosomes from the population to create the mating pool. Members in the mating pool (called parents) are mated in pairs by applying a genetic operation that creates the new offspring chromosomes that make up the subpopulation. These offspring are inserted back into the population, replacing some of the original population chromosomes to create a new population.

There are many ways to implement each of these steps [9],[10]. Many methods of selection exist but they all strive to mimic the "survival of the fittest" paradigm: chromosomes that yield higher values of the objective function (assuming maximization) are more likely to be selected to create offspring. Crossover is one particular genetic operation that creates two new chromosomes from two parents by interchanging parts of the two mating chromosomes. After crossover, mutation then can be applied to each new chromosome by randomly flipping bits in the chromosome. The newly created offspring chromosomes then replace poorer quality chromosomes in the population and the cycle continues. The selection and replacement steps evaluate the goodness of chromosomes based on the objective function; choosing a good form of the objective function is an important aspect of designing a GA for a given use [9], [10].

When the optimization problem is a constrained problem there are various methods that can be used to enforce the constraints [10]. One method is to force the offspring chromosomes to satisfy the constraint – sometimes by simply modifying the offspring chromosome to meet the constraint. A more common approach, though, is to apply a penalty function to the objective function. The penalty function forces the objective function to have small values for chromosomes that do not satisfy the constraint. There are many ways to construct and apply penalty functions [10].

The penalty function method transforms a constraint optimization problem into an unconstrained optimization problem where infeasible solutions are penalized using penalty coefficients. The general constrained maximization problem has the following form: maximize the function $f(\mathbf{x})$ with respected to solution vector $\mathbf{x}$ subject to K inequality constraints

$$g_k(\mathbf{x}) \ge 0 \quad \text{for} \quad k = 1, 2, \cdots, K$$

and M equality constraints

$$h_m(\mathbf{x}) = 0 \quad \text{for} \quad m = 1, 2, \cdots, M \; .$$

The conventional method for penalty function selection uses

$$\phi(\mathbf{x}, r) = f(\mathbf{x}) - r \sum_{k=1}^{K} G_k \left[g_k(\mathbf{x}) \right]^2 - r \sum_{m=1}^{M} \left[h_m(\mathbf{x}) \right]^2$$

where G_k has $G_k = 0$ for $g_k(\mathbf{x}) \ge 0$ and $G_k = 1$ for $g_k(\mathbf{x}) < 0$, and r is a positive multiplier that controls the magnitude of the penalty terms, the speed of convergence and the quality of the optimal value. Note that this uses an subtractive form of applying the penalty function – the penalty function subtracts off value when the constraint is not met. There are several other penalty function strategies for single criteria genetic algorithm optimization available in the literature (see [10] for an overview). Typical ones are (i) Homaifar-Lai-Qi (HLQ) method, (ii) Yokota-Gen Ida-Taguchi (YGT) method and (iii) Osyczka's tournament selection method. For our bit allocation problem, we have only one constraint (the rate inequality constraint) and under this condition the HLQ method collapses into the conventional penalty function method described above. Thus we consider in more detail the YGT method, which is stated as follows.

Yokota-Gen Ida-Taguchi (YGT) Method: For the following nonlinear programming problem:

$$\max f(\mathbf{x})$$

such that

$$g_m(\mathbf{x}) \le b_m, \quad m = 1,2,\cdots,M$$

take the multiplicative form of the penalty function:

$$\phi(\mathbf{x}) = f(\mathbf{x})\,p(\mathbf{x}) .$$

The penalty term $p(\mathbf{x})$ is constructed as follows:

$$p(\mathbf{x}) = 1 - \frac{1}{M}\sum_{m=1}^{M}\left[\frac{\Delta b_m(\mathbf{x})}{b_m}\right]^{\alpha}$$
$$\Delta b_m(\mathbf{x}) = \max\{0, \quad g_m(\mathbf{x}) - b_m\} \tag{7}$$

where $\Delta b_m(\mathbf{x})$ is the value of violation of the m^{th} constraint. In this method the penalty function is designed with the non-parameterized approach and is problem independent [10].

When we applied this method to our problem our simulation results showed that the infeasible points were penalized very little, which caused the final solution to lie in the infeasible region.

Our Modified YGT Method: To overcome this problem we modified the YGT method's penalty function given in (7) by adding coefficients r_m to make sure the constraint would be enforced. Our modification replaces the $p(\mathbf{x})$ (7) by

$$p(\mathbf{x}) = 1 - \frac{1}{M}\sum_{m=1}^{M} r_m\left[\frac{\Delta b_m(\mathbf{x})}{b_m}\right]$$

where $\Delta b_m(\mathbf{x})$ is the value of violation of the m^{th} constraint and r_m is a variable penalty parameter for the m^{th} constraint.

In the context of the above discussion, our problem is stated as

$$\max_{\tilde{\Omega},B}\left\{SNR_{TDOA}(\tilde{\Omega},B)\right\}$$

subject to

$$\sum_{i\in\tilde{\Omega}} b_i \le R$$

where $\tilde{\Omega}$ is the selection set for transform coefficients and $B = \{b_i \mid i \in \tilde{\Omega}\}$ is the allocation set. The modified penalty strategy can be simplified because there is only one constraint:

$$p(B,\tilde{\Omega}) = 1 - r\frac{\Delta V}{R}$$
$$\Delta V = \max\left\{0, \quad \left[\sum_{i\in\tilde{\Omega}} b_i\right] - R\right\} .$$

Constraint Tournament Selection Method. In [10], the author states that this method is superior to the other penalty function strategies. We implemented this method to compare the result from our penalty method (see [10] for details). Surprisingly, it did not perform well here.

IV. Simulation Results

We tested the GA-based allocation methods using simulated linear FM (LFM) radar signals. An LFM signal was generated and a delayed version was also created. White Gaussian noise was added to each to yield desired SNR_1 and SNR_2 on the two signals. To provide a reference case, these two noisy signals were cross-correlated without compression to estimate the TDOA value of delay between the two signals. To test the GA-based selection/allocation methods, one of the two signals was compressed as follows: compute the DFT and then determine the selection/allocation parameters $B,\tilde{\Omega}$ using one of the GA methods discussed above, then compress the signal by quantizing the selected DFT coefficients according to the allocation B; form the decompressed version of the signal and cross-correlate it with the non-compressed signal to estimate the TDOA. As another reference result for comparison, we compressed one of the signals using an MSE-optimized bit allocation for the DFT coefficients.

The genetic algorithm was implemented using the follows parameters:

- Chromosome String Length: 8 bits
- Crossover Rate (Simple Crossover): 0.8
- Mutation Rate (Binary Mutation): 0.005
- Penalty Parameter:
    - r = 0.5 for conventional method
    - r = 2.5 for our method
- Population Size: 100

- Number of Generations: 200

We studied the ability of the various GA methods to maximize the objective function $SNR_{TDOA}(\tilde{\Omega}, B)$. For each GA method we ran ten simulations (same signal on each run, different noises on each run) and checked the achieved value of $SNR_{TDOA}(\tilde{\Omega}, B)$. The results are shown in Figure 1, were it is seen that our modification of the YGT performs the best of the three penalty function methods considered.

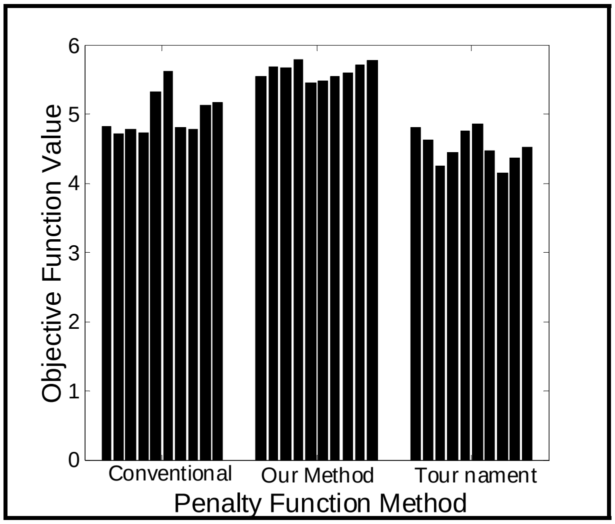

Figure 1: Performance of penalty function methods

Having established that our modified penalty function is preferred, we studied the TDOA accuracy performance achieved through its use. The signal to be compressed had its SNR varied over the 5 – 35 dB range while the signal not compressed had an SNR of 40 dB. At each SNR value we ran 200 Monte Carlo simulation runs where the lower SNR signal is compressed (once using the DFT/GA method described above; once using MSE-based allocation) and was then cross-correlated with the other signal to estimate TDOA. The RMS value of the TDOA error was computed by averaging over the 200 Monte Carlo runs. This was repeated for two different rate constraints: compression ratio of CR = 5 and CR = 8. The simulation results are shown in Figure 2 for the CR = 5 case and in Figure 3 for the CR = 8 case. In each case we see that the TDOA RMS error using the GA-optimized non-MSE distortion criteria is lower than using the more-standard MSE criteria for allocating bits to the DFT coefficients. Note that at higher SNR values the MSE and non-MSE error approaches are equivalent – this is because the form of the non-MSE distortion measure given in (6) was developed for the case where SNR of the signal being compressed was much less than the SNR of the signal not being compressed (which was set to 40 dB for these results). The results at low SNR values show:

- The TDOA error at CR = 8 using the non-MSE method is about the same as the error at CR = 5 using the MSE method – that is, a 60% improvement in CR using the non-MSE method.
- For CR = 8, about a 30% reduction in TDOA error when using the non-MSE method vs. using the MSE method.

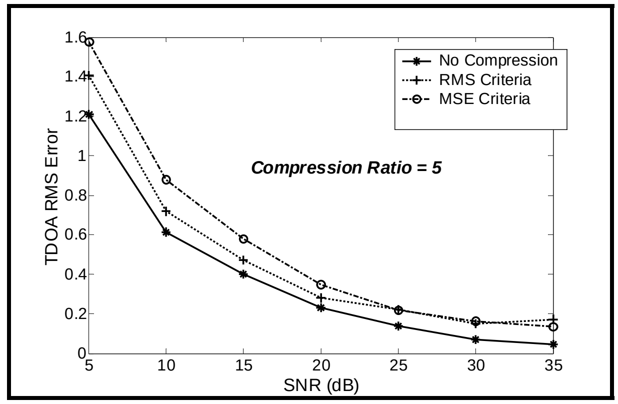

Figure 2: TDOA accuracy performance using proposed genetic allocation method when CR = 5.

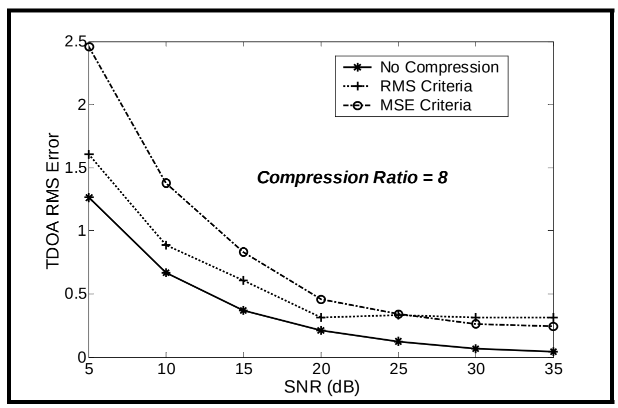

Figure 3: TDOA accuracy performance using proposed allocation method when CR = 8.

V. SUMMARY

We have considered the case of data compression for TDOA-based emitter location and have demonstrated the importance of using the CRB-based non-MSE distortion criteria. In a transform coding environment this criteria should be optimized by jointly selecting transform coefficients and allocating bits to the selected coefficients. Previously only sub-optimal methods have been proposed to perform the selection/allocation; these sub-optimal methods do not *jointly* perform the selection and allocation. Here for the first time we have proposed a method for the joint selection/allocation problem – a GA-based method. We have specified a penalty function approach to solve this constrained optimization and have demonstrated that it outperforms other existing penalty function approaches.

The TDOA-based location problem is important in electronic warfare systems to allow location and targeting of hostile emitters. It is also applicable in wireless systems to locate emergency calls from wireless phones. Another application lies in locating targets in a sensor network application – acoustic sources could be located using similar processing. Furthermore, similar ideas could be used for other estimates required in a sensor network: derive the CRB for the estimate and use it as a distortion measure that then gets optimized via use of a GA-based allocation scheme. We are currently investigating such extensions.

References

[1] P. C. Chestnut, "Emitter location accuracy using TDOA and differential doppler ," *IEEE Trans. Aero. and Electronic Systems*, vol. AES-18, March 1982, pp. 214-218.

[2] S. Stein, "Differential delay/doppler ML estimation with unknown signals," *IEEE Trans. Sig. Proc.*, vol. 41, August 1993, pp. 2717 - 2719.

[3] D. J. Matthiesen and G. D. Miller, "Data transfer minimization for coherent passive location systems," Report No. ESD-TR-81-129, Air Force Project No. 4110, June 1981.

[4] M. L. Fowler, "Coarse quantization for data compression in coherent location systems," *IEEE Trans. Aerospace and Electronic Systems*, vol. 36, no. 4, Oct. 2000, pp. 1269 – 1278.

[5] M. L. Fowler, "Data compression for emitter location systems," Conference on Information Sciences and Systems, Princeton University, March 15-17, 2000, pp. WA7b-14 – WA7b-19.

[6] M. L. Fowler**,** "New distortion measures for data compression for emitter location," Thirty-Fifth Asilomar Conference on Signals, Systems, and Computers, November 5 – 7, 2001, pp. 658 – 662.

[7] M. L. Fowler, "Non-MSE wavelet-based data compression for emitter location," in *Mathematics and Applications of Data/Image Coding, Compression, and Encryption IV*, Mark S. Schmalz, Editor, Proceedings of SPIE, Vol. 4475, San Diego, CA, July 29 – August 3, 2001, pp. 13 – 22.

[8] M. L. Fowler and M. Chen, "Integer optimization methods for non-MSE data compression for emitter location," in *Mathematics and Applications of Data/Image Coding, Compression, and Encryption V*, Mark S. Schmalz, Editor, Proceedings of SPIE, Seattle, WA, July 8 – 11, 2002, pp. 188 – 195.

[9] K. S. Tang et al., "Genetic algorithms and their applications," *IEEE Signal Processing Magazine*, vol. 13, no. 6, Nov. 1996, pp. 22 – 37.

[10] A. Osyczka, *Evolutionary Algorithms for Single and Multicriteria Design Optimization*, Physica Verlag, 2001.