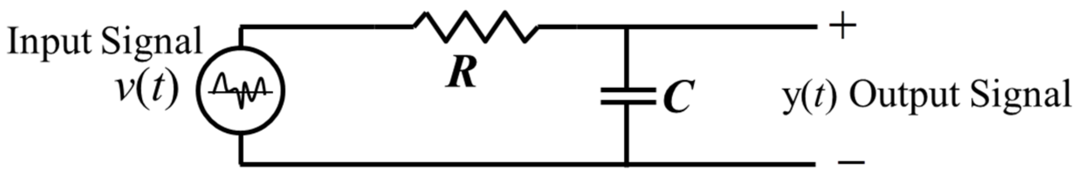
EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #37

- C-T Systems: Using Bode Plots

Bode Plot Ideas Can Help Visualize What Circuits Do…

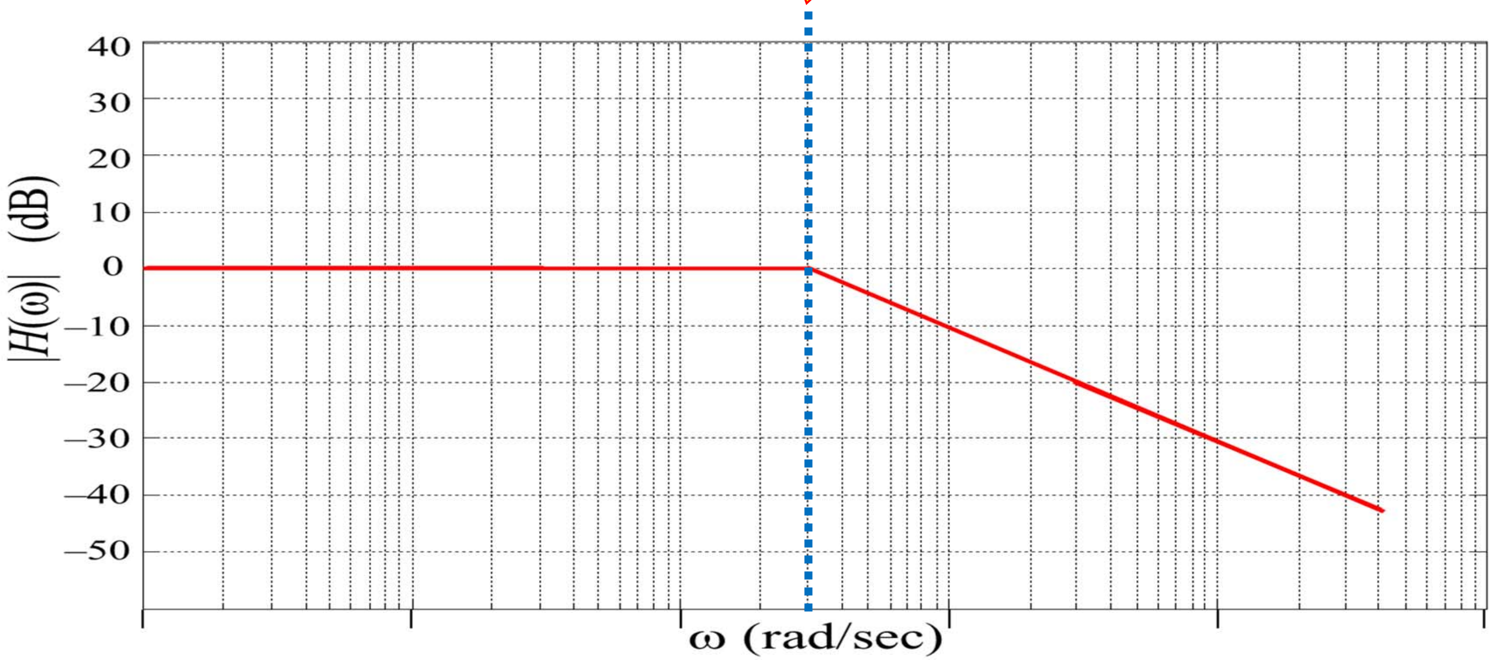
RC Lowpass Filter



$$H(s)=\frac{1}{1+RCs}=\frac{1/RC}{s+1/RC}$$

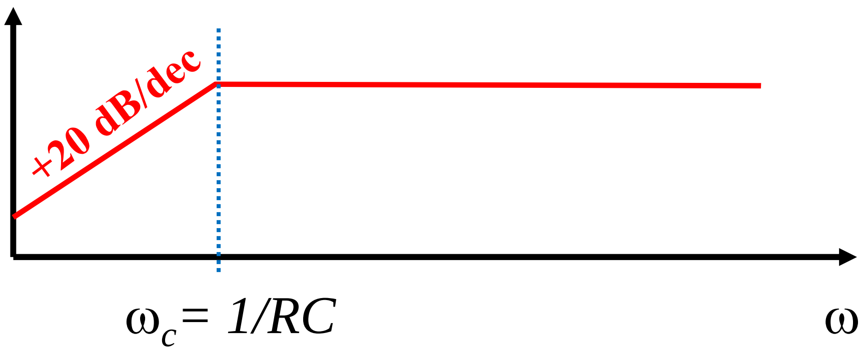
$$H(\omega)=\frac{1}{1+jRC\omega}$$

Break Point = 1/*RC*



RC Highpass Filter

$$H(s)=\frac{RCs}{1+RCs}=\frac{s}{s+1/RC}$$

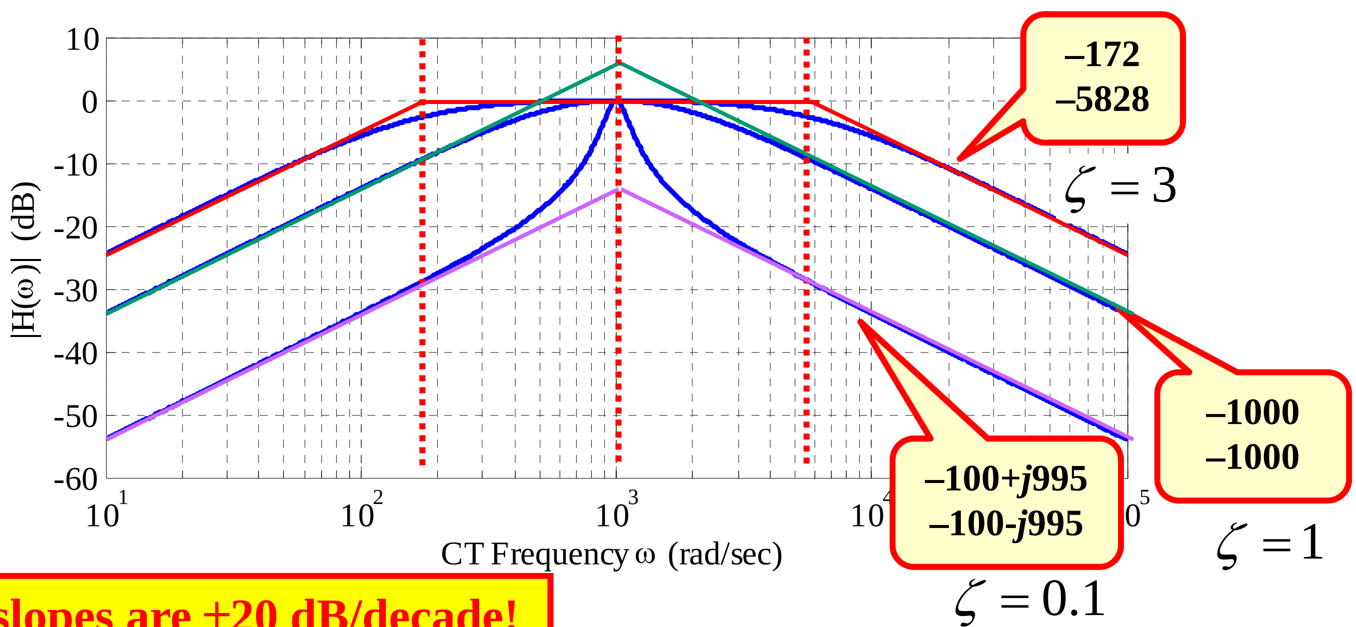


RLC Bandpass Filter

$$H(s)=\frac{2\zeta\omega_n s}{s^2+2\zeta\omega_n s+\omega_n^2}$$

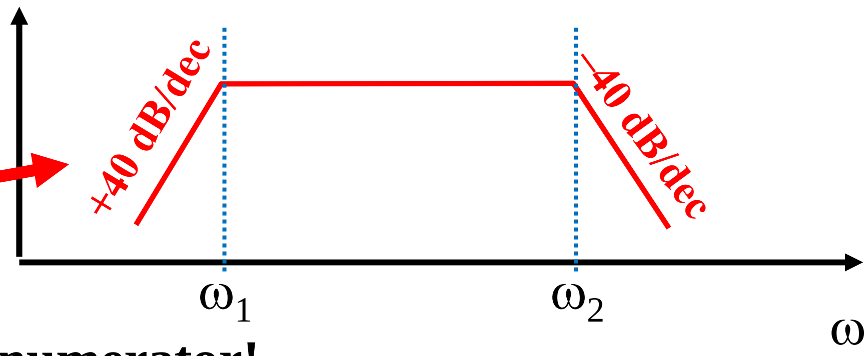
$$H(s)=\frac{2\zeta\omega_n s}{(s+a)^2}$$

$$H(s)=\frac{2\zeta\omega_n s}{(s+a)(s+b)}$$



All slopes are ±20 dB/decade!

A Better Bandpass Filter? Suppose you want a BPF…. with faster "rolloff"!



Need an s^2 term in the numerator!

At ω_1 we need to change slope by –40 dB/dec… So need double pole @ ω_1!

At ω_2 we need to change slope by –40 dB/dec… So need double pole @ ω_2!

$$H(s) = \frac{Ks^2}{(s+\omega_1)^2(s+\omega_2)^2}$$

There are (at least) three ways to get this!

$$H(s) = \left[\frac{K_1 s}{(s+\omega_1)(s+\omega_2)}\right]\left[\frac{K_2 s}{(s+\omega_1)(s+\omega_2)}\right]$$

BPF RLC w/ distinct poles

$$H(s) = \left[\frac{K_1 s}{(s+\omega_1)^2}\right]\left[\frac{K_2 s}{(s+\omega_2)^2}\right]$$

BPF RLC w/ repeated poles

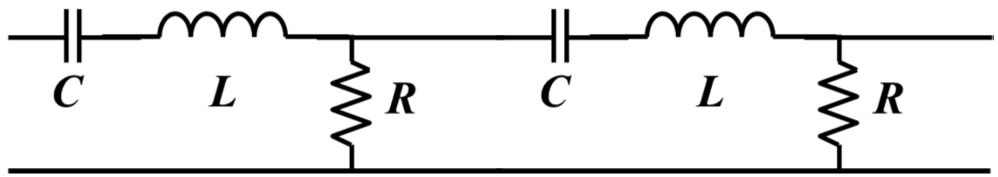
$$H(s) = \left[\frac{K_1 s^2}{(s+\omega_1)^2}\right]\left[\frac{K_2}{(s+\omega_2)^2}\right]$$

HPF RLC w/ Rep. poles

LPF RLC w/ Rep. poles

Same exact circuits… just different choices of RLC!!!

Looks like we could just cascade two of our RLC circuits… Here we cascade BPFs.

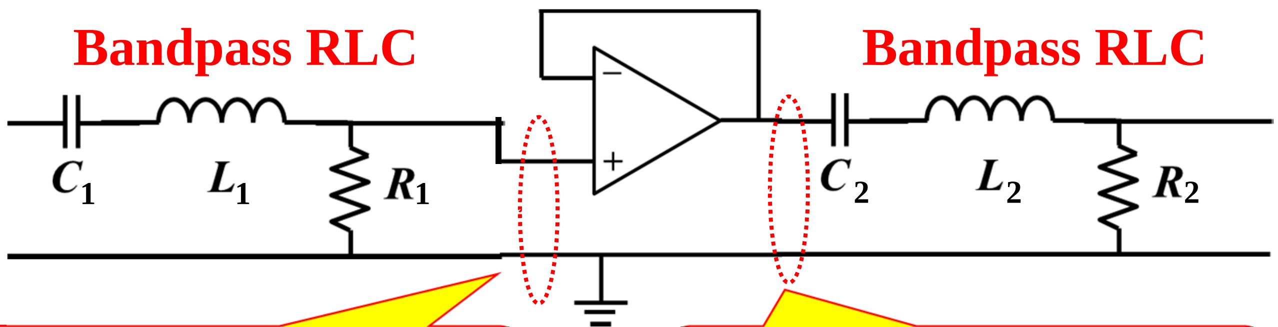


Our "cascade theory" only holds when attaching the 2nd system does not change how the first one behaves!

Although TF Theory says this will work… the problem is that the second circuit "Loads" the first one!!!

So… one approach would be to re-analyze this cascade and see if it will still work but with some "tweeks" on the component choices.

Another approach is to use an op amp as a "buffer" between the stages!



VERY large input resistance of op amp prevents "loading" of first stage!

VERY small output resistance of op amp minimizes impact on 2nd stage!

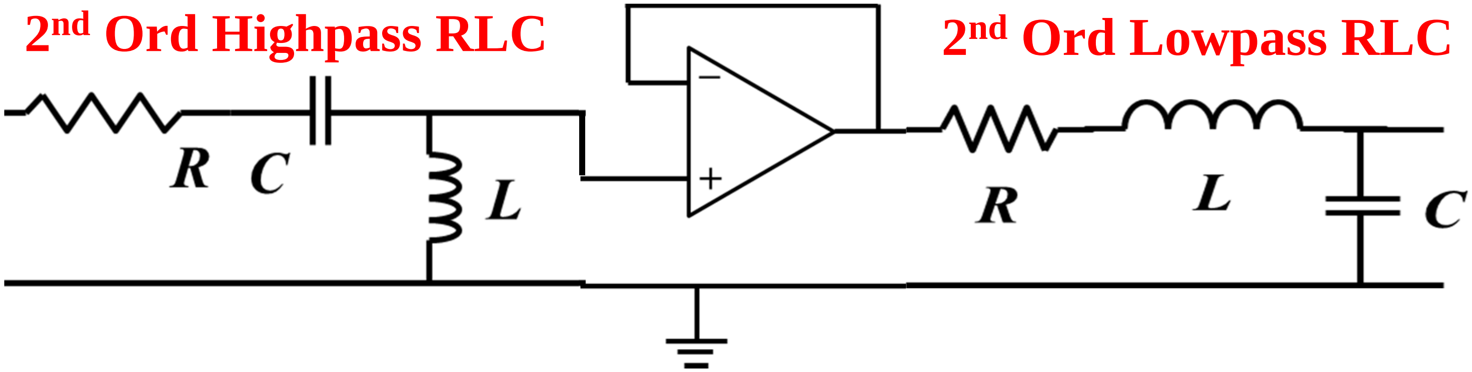
It may be desirable to add another buffer here…

Remember… there are two ways to choose the components here:

1. Each stage has repeated poles
2. Each stage has distinct poles

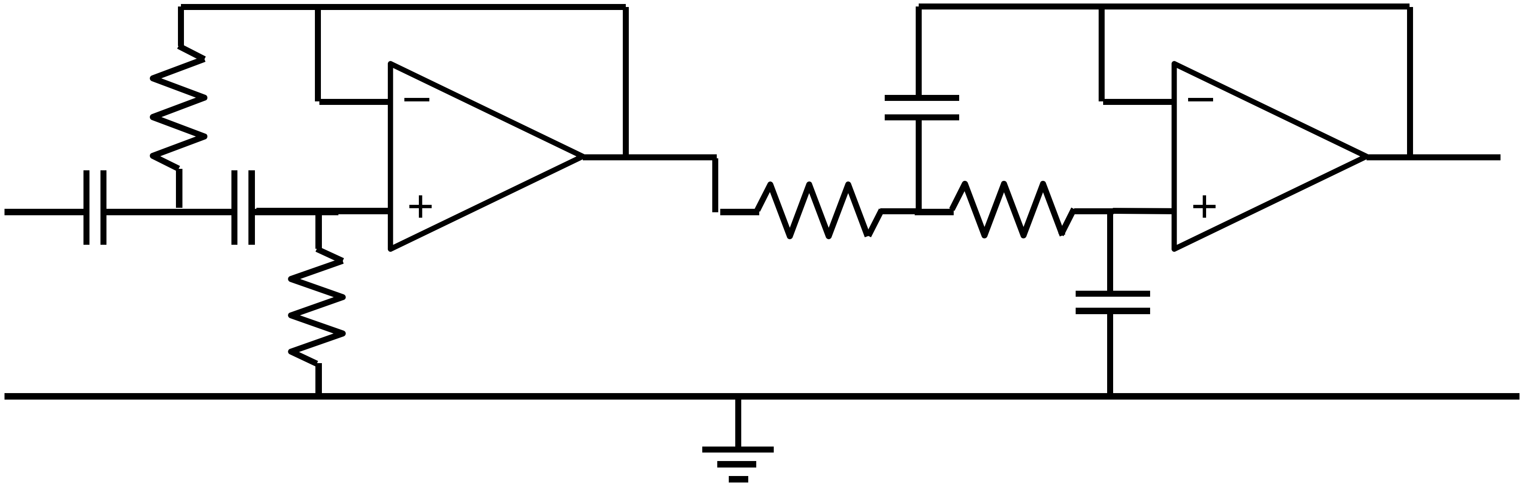
Another way to make a better BPF:

Here we must choose the components so that each stage has repeated poles.



Although these ideas lead to workable circuits they are not necessarily the best…
For one thing… they need inductors (which are big and can't be made in an IC!)
There are other forms… See this link for the form used below.

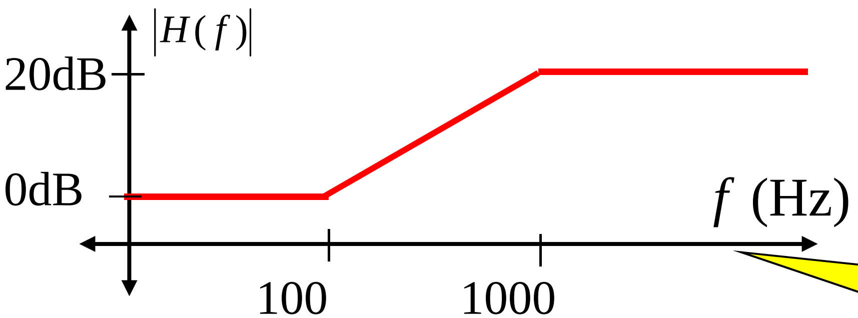
http://pdfserv.maxim-ic.com/en/an/AN1795.pdf



Design Example using Bode Plot Insight

Suppose you want to build a "treble booster" for an electric guitar.

You decide that something like this might work:



100Hz $\Rightarrow$ 628 rad/s = ω

1000Hz $\Rightarrow$ 6283 rad/s = ω

Notice that we are doing our rough "design thinking" in terms of Bode Plot approximations!!!

The A string on a guitar has a fundamental frequency of 110 Hz
The A note on 17th fret of the high-E string has a fundamental frequency of 880 Hz

From our Bode Plot Insight… we know we can get this from a single real pole, single real zero system… with the "zero first, then the pole":

$$H(s) = \frac{(1 + s/\omega_1)}{(1 + s/\omega_2)} \quad \Rightarrow \quad H(\omega) = \frac{(1 + j\omega/\omega_1)}{(1 + j\omega/\omega_2)}$$

with: ω_1 = 628 rad/s
ω_2 = 6280 rad/s

Now, how do we get a circuit to <u>do</u> this? Let's explore!

A series combination

…has impedance $Z(s) = R + 1/Cs$

Note: we could get $R + sL$ with an inductor but inductors are generally avoided when possible

So what do we get if we could some how form a ratio of such impedances?

$$\frac{Z_1(s)}{Z_2(s)} = \frac{R_1 + 1/C_1 s}{R_2 + 1/C_2 s} = \frac{C_2}{C_1}\frac{(sR_1C_1 + 1)}{(sR_2C_2 + 1)}$$

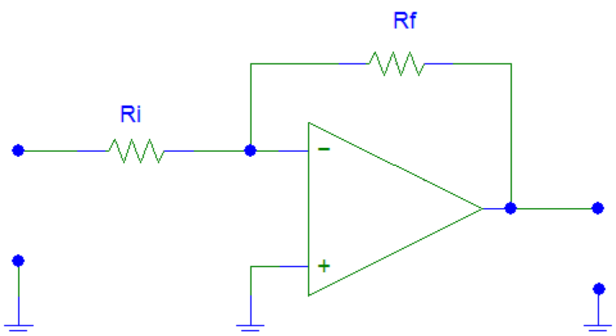
Aha!!! What we want!

$$\Rightarrow \text{Let: } \omega_1 = 1/R_1C_1$$

$$\omega_2 = 1/R_2C_2$$

$$\frac{Z_1(\omega)}{Z_2(\omega)} = \frac{C_2}{C_1}\frac{(1 + j\omega/\omega_1)}{(1 + j\omega/\omega_2)}$$

Okay…how do we build a circuit that has a transfer function that is a ratio of impedances?! **Recall the op-amp inverting amplifier!**

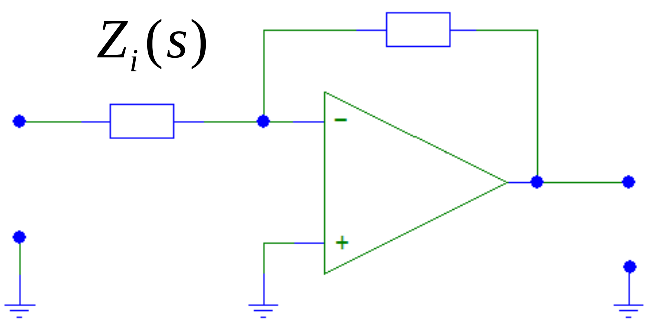


$v_{in}(t)$ $v_o(t)$

$$Gain = -\frac{R_f}{R_i}$$

Ratio of **resistances**

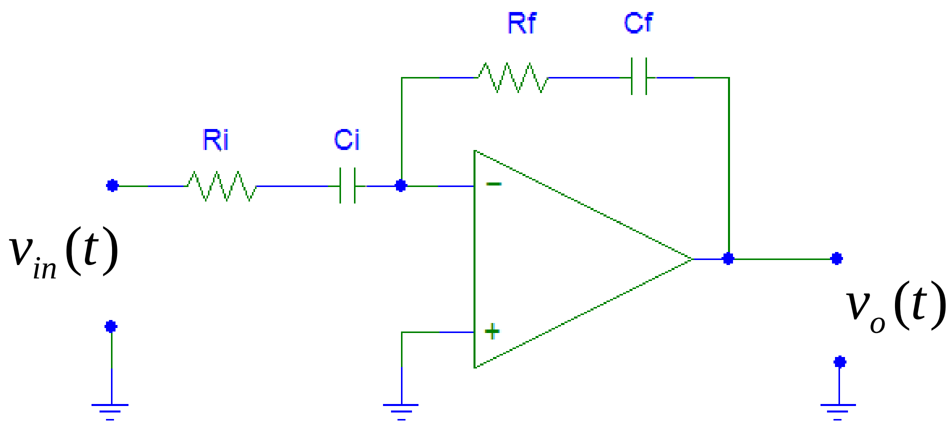
Extending the analysis to include impedances we can show that:



$Z_f(s)$ $Z_i(s)$ $v_{in}(t)$ $v_o(t)$

$$\Rightarrow \quad H(s) = -\frac{Z_f(s)}{Z_i(s)}$$

Won't affect our magnitude: $|H(\omega)|$



Now, you can choose the R's & C's to give the desired frequency points

p. 98, *The Art of Electronics*, Horowitz & Hill, Cambridge Press, 1980

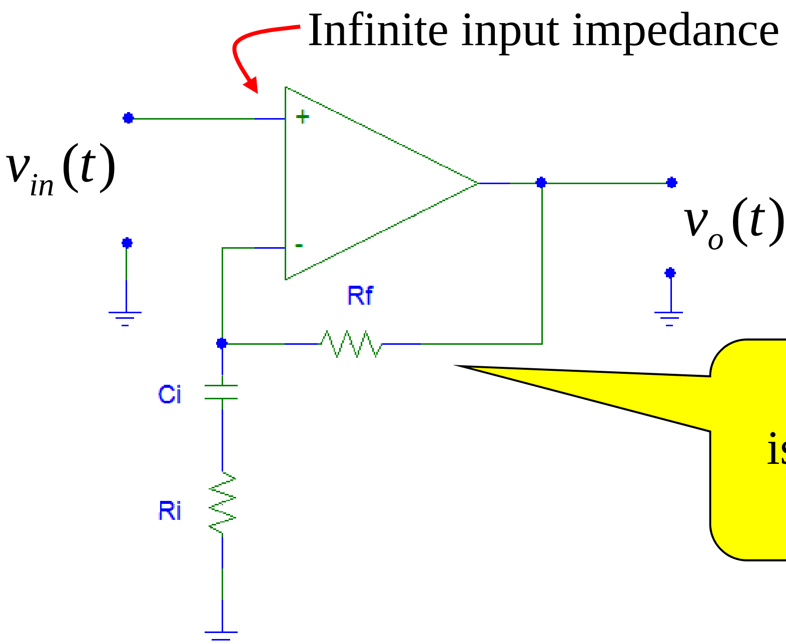
But wait!! You then remember that op amps must always have negative feedback at DC so putting C_f here is not a good idea…

So we have to continue…

We also might not like this circuit because it might not give us a very large input impedance… and that might excessively "load" the circuit that you plug into this (e.g., the guitar)

Back to the drawing board!!!

Okay, then you remember there is also Non-Inverting Op-Amp circuit…



$$Gain = 1 + \frac{R_f}{R_i}$$

We avoid the "no DC feedback" issue… but we're not yet sure we'll get what we want!

Applying this gain formula we get:

$$H(s) = 1 + \frac{R_f}{R_i + 1/C_i s} = \frac{(R_i + 1/C_i s) + R_f}{R_i + 1/C_i s} = \frac{(R_i + R_f) + 1/C_i s}{R_i + 1/C_i s}$$

Oh Cool!! We Get What We want!

$$H(s) = \frac{(R_f + R_i)C_i s + 1}{R_i C_i s + 1}$$

Set:

$$\omega_1 = \frac{1}{\left(R_i + R_f\right) C_i} = 628 \text{ rad/s}$$

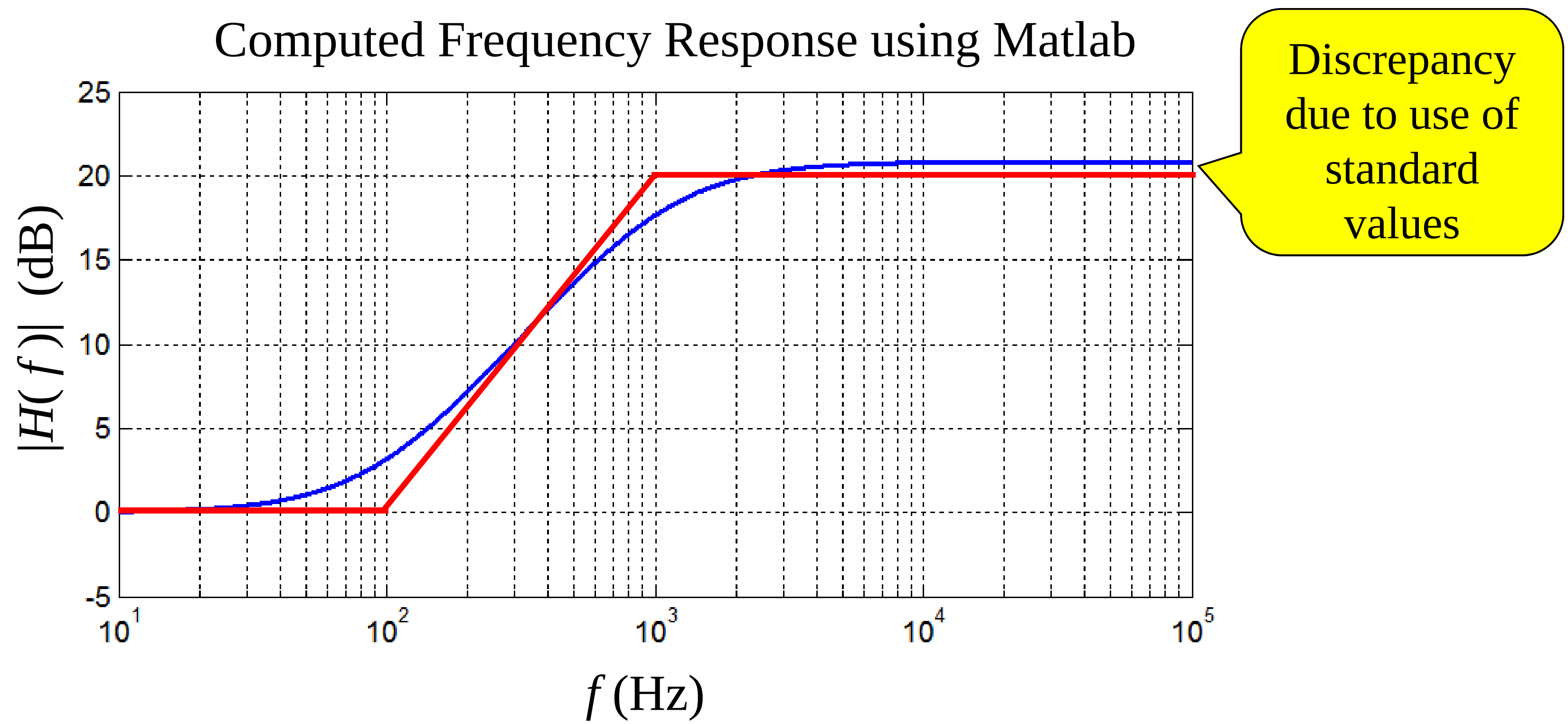
$$\omega_2 = \frac{1}{R_i C_i} = 6283 \text{ rad/s}$$

Choose:

$R_f = 15k\Omega$ $R_i = 1.5k\Omega$

$C_i = 0.1\mu\text{F}$

Using standard values



Computed Frequency Response using Matlab
25
20
15
10
5
0
-5
|H(f)| (dB)
10^1
10^2
10^3
10^4
10^5
f (Hz)
Discrepancy due to use of standard values

Summary of Bode-Plot-Driven Design Example

1. Through insight gained from knowing how to do Bode plots by hand… we recognized the kind of transfer function we needed
2. Through insight gained in circuits class about impedances we recognized a key building block needed: Series R-C
3. Through insight gained in electronics class about op-amps we found a possible solution… the inverting op-amp approach
4. We then scrutinized our design for any overlooked issues
   a. We discovered two problems that we needed to fix
5. We used further insight into op-amps to realize that we could fix the input impedance issue using a non-inverting form of the op-amp circuit
6. We didn't give up at first sign that the inverting form might not give us the form we want…
   a. Through mathematical analysis we showed that we did in fact get what we wanted!!!!!!