Binghamton University
State University of New York

EECE 301
Signals & Systems
Prof. Mark Fowler

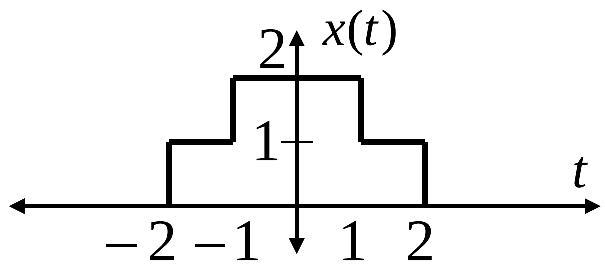
Note Set #16

- C-T Signals: Using FT Properties

Recall that FT Properties can be used for:

1. **Expanding use of the FT table**
2. **Understanding real-world concepts**

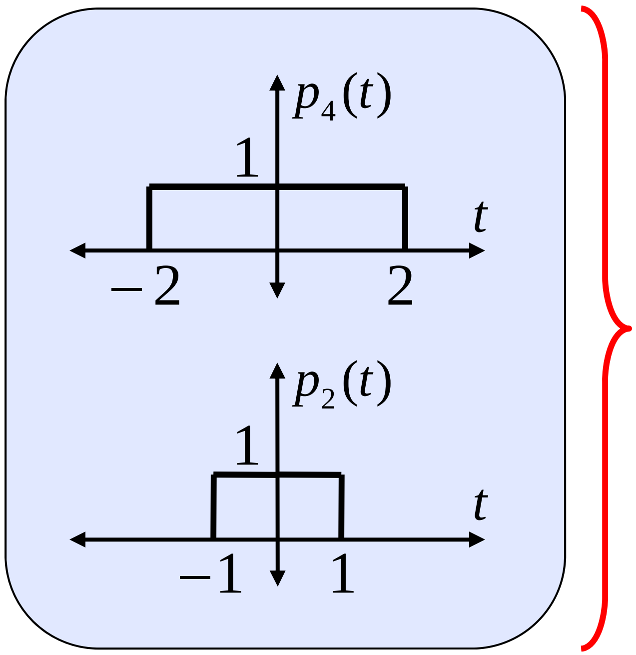
<u>Example Application of “Linearity of FT”:</u> Suppose we need to find the FT of the following signal…



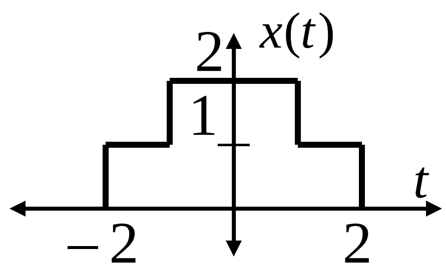
We don’t see this on our table… so we should think brainstorm ways to use FT properties to tackle it…

- One way is to break $x(t)$ down into a sum of signals on our table!!!

Break a complicated signal down into simple signals before finding FT:



Add to get



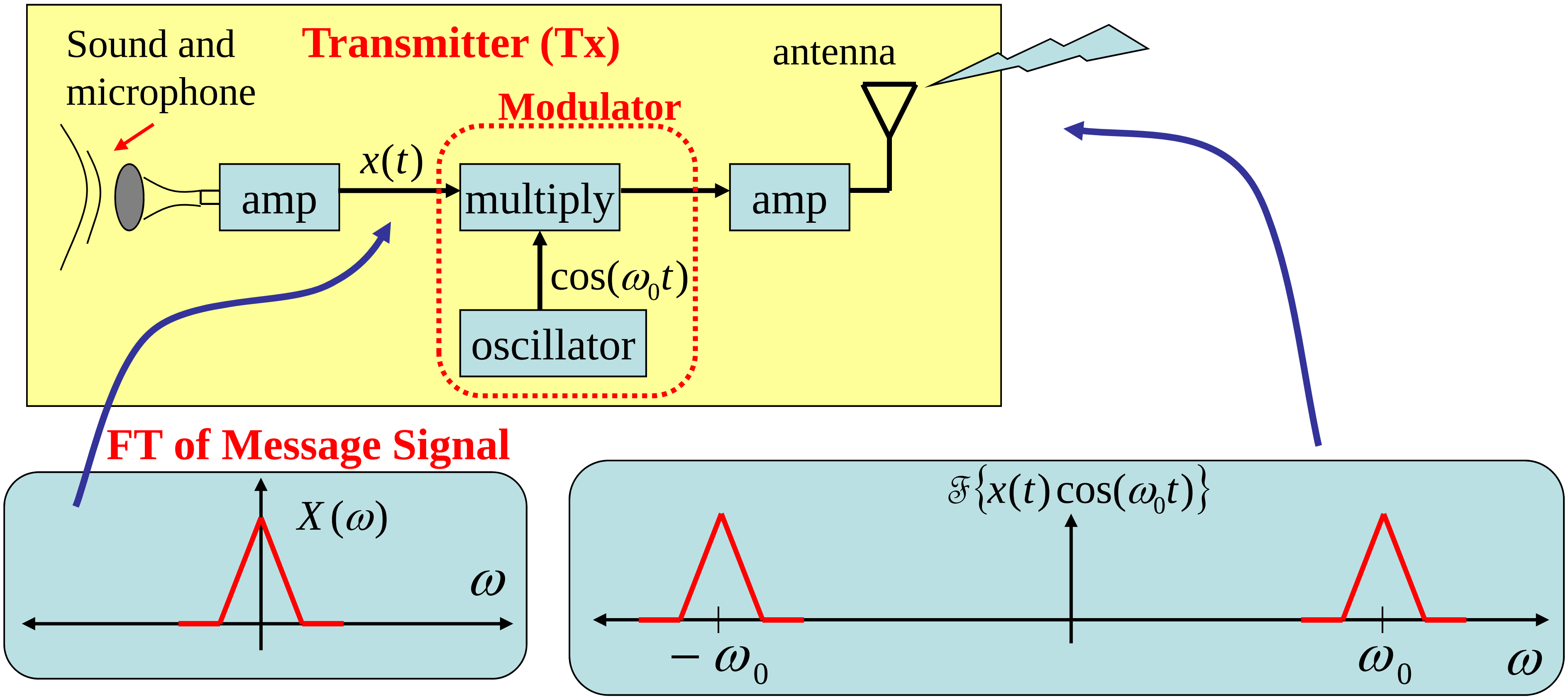
Mathematically we write: $x(t) = p_4(t) + p_2(t)$ → $X(\omega) = P_4(\omega) + P_2(\omega)$

From FT Table we have a <u>known</u> result for the FT of a pulse, so…

$$X(\omega) = 4\,\text{sinc}\left(\frac{2\omega}{\pi}\right) + 2\,\text{sinc}\left(\frac{\omega}{\pi}\right)$$

Application of Modulation Property to Radio Communication

FT theory tells us what we need to do to make a **simple** radio system… *then* electronics can be built to perform the operations that the FT theory calls for:

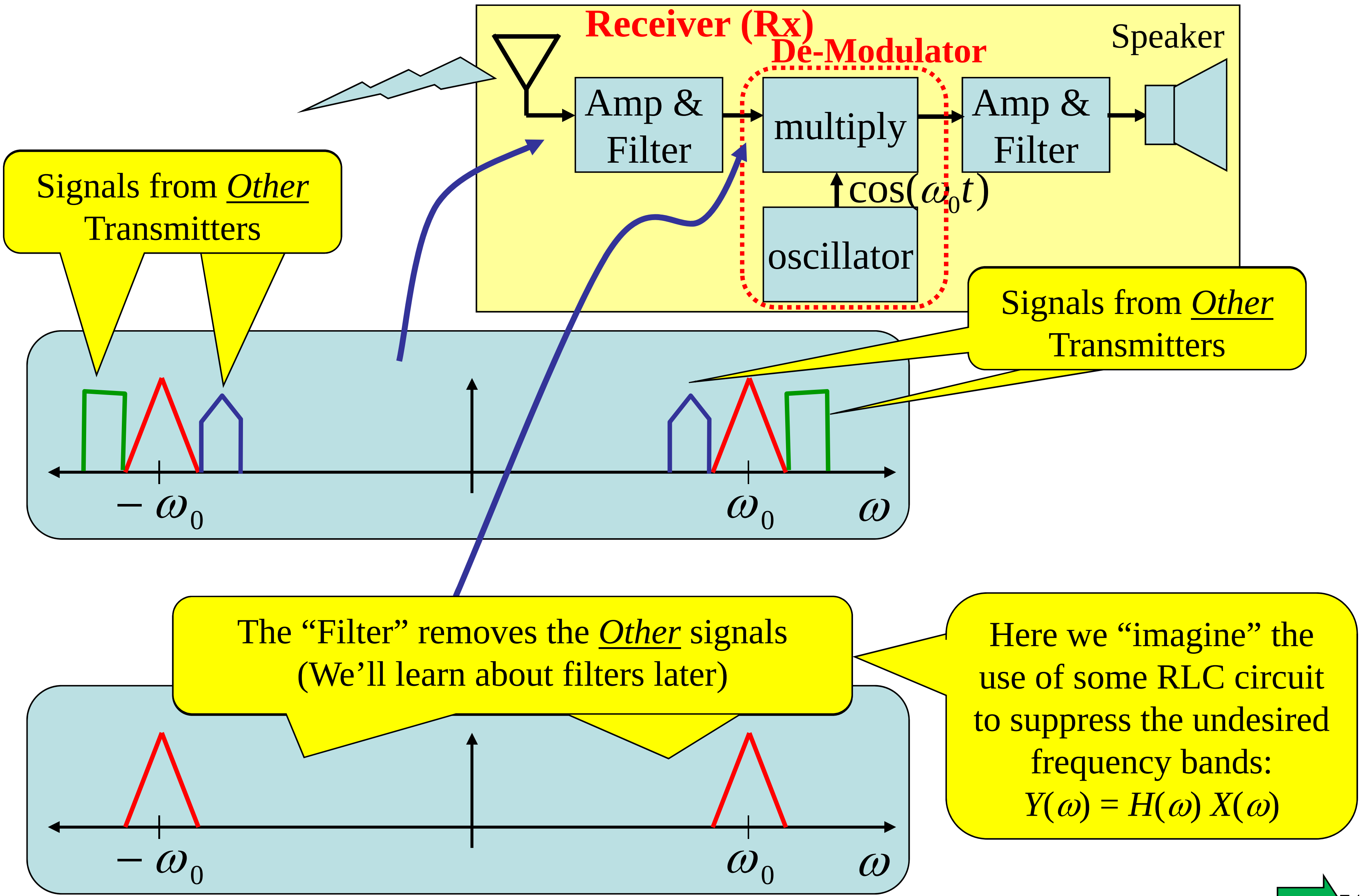


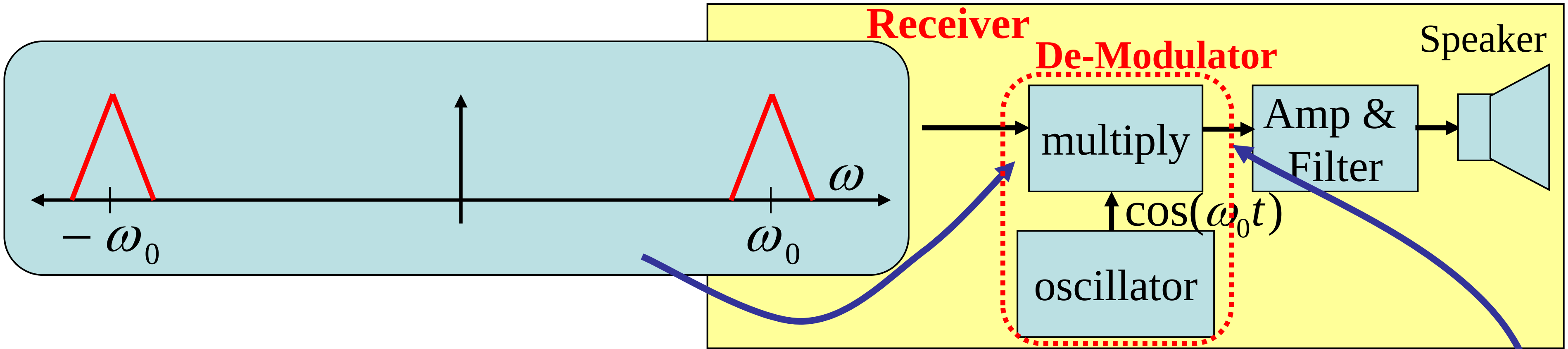
Choose f_0 > 10 kHz to enable efficient radiation (with $\omega_0 = 2\pi f_0$)

AM Radio: around 1 MHz   FM Radio: around 100 MHz

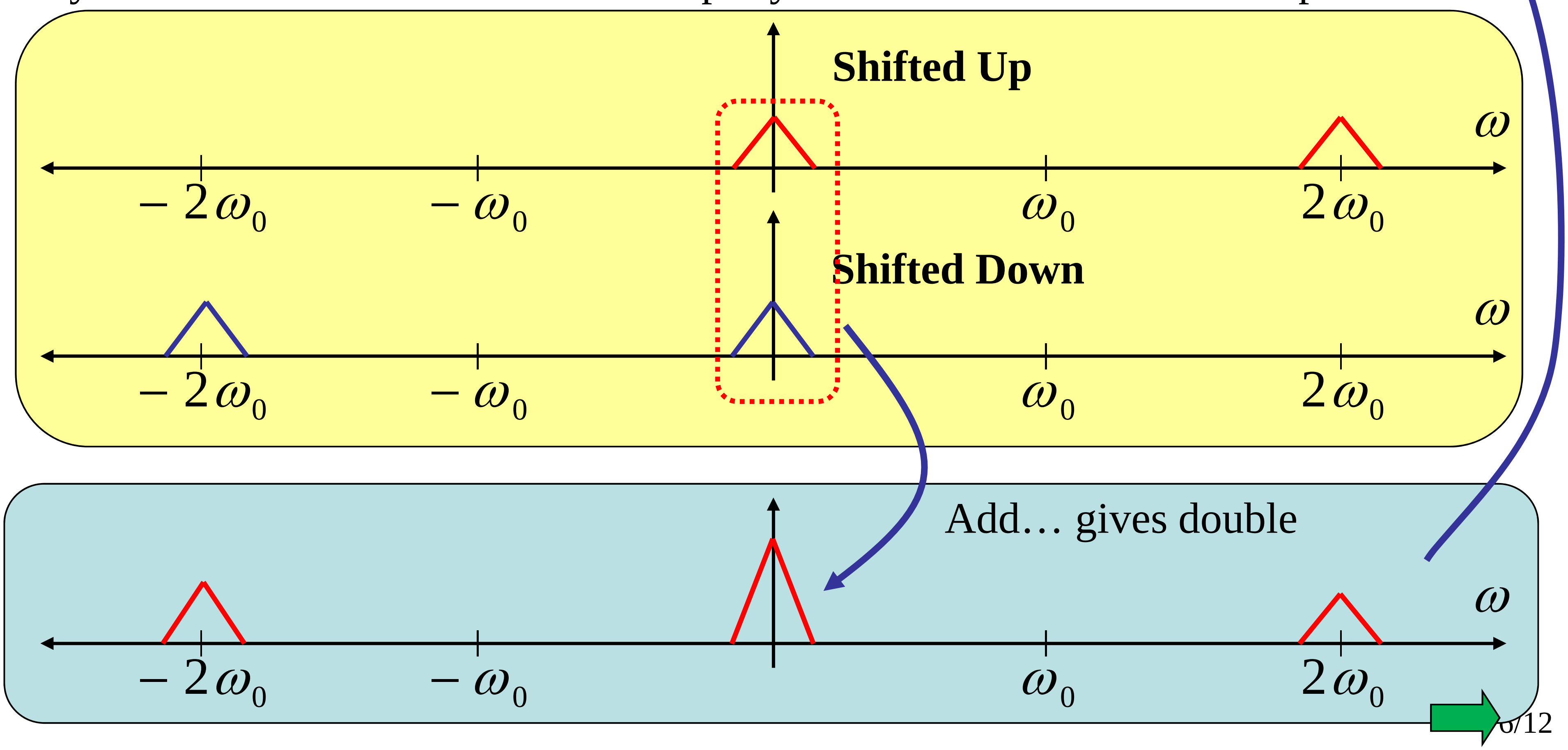
Cell Phones: around 900 MHz, around 1.8 GHz, around 1.9 GHz etc.

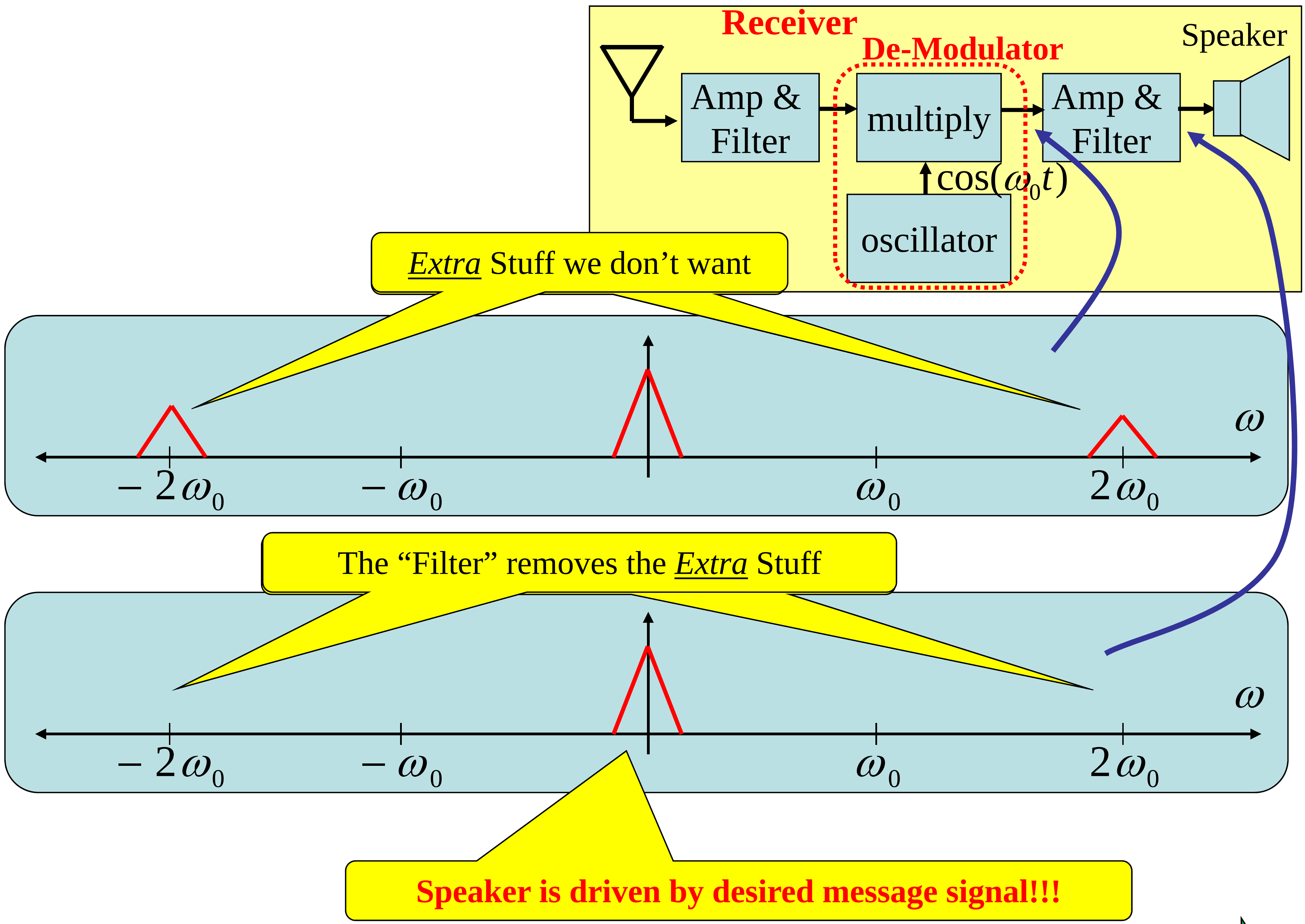
The next several slides show how these ideas are used to make a receiver:





By the Real-Sinusoid Modulation Property… the De-Modulator shifts up & down:




Receiver
De-Modulator
Speaker
Amp & Filter
multiply
Amp & Filter
$\cos(\omega_0 t)$
oscillator
Extra Stuff we don't want
ω
$-2\omega_0$
$-\omega_0$
ω_0
$2\omega_0$
The "Filter" removes the *Extra* Stuff
ω
$-2\omega_0$
$-\omega_0$
ω_0
$2\omega_0$
Speaker is driven by desired message signal!!!

So… what have we seen in this example:

Using the Modulation property of the FT we saw…

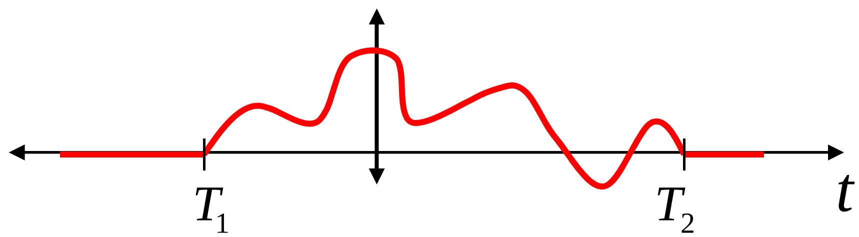
1. Key Operation at Transmitter is up-shifting the message spectrum:
   a) FT Modulation Property tells the theory then we can build…
   b) "modulator" = oscillator and a multiplier circuit
2. Key Operation at Transmitter is down-shifting the received spectrum
   a) FT Modulation Property tells the theory then we can build…
   b) "de-modulator" = oscillator and a multiplier circuit
   c) But… the FT modulation property theory also shows that we need filters to get rid of "extra spectrum" stuff
      i. So… one thing we still need to figure out is how to deal with these filters…
      ii. Filters are a specific "system" and we still have a lot to learn about Systems…
      iii. That is the subject of much of the rest of this course!!!

Bandlimited and Timelimited Signals

Now that we have the FT as a tool to analyze signals, we can use it to identify certain characteristics that many practical signals have.

A signal $x(t)$ is **timelimited** (or of finite duration) if there are 2 numbers T_1 & T_2 such that:

$$x(t) = 0 \quad \forall t \notin [T_1, T_2]$$

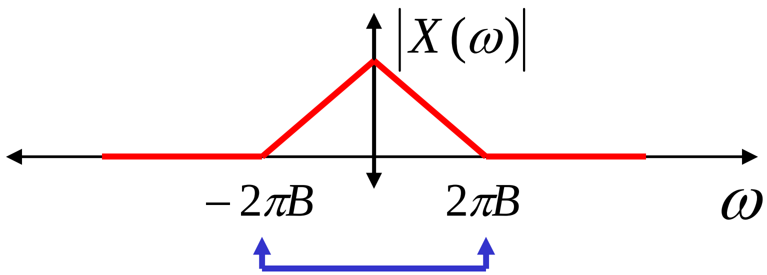


A (real-valued) signal $x(t)$ is **bandlimited** if there is a number B such that

$$|X(\omega)| = 0 \quad \forall \omega > \underbrace{2\pi B}$$

$2\pi B$ is in rad/sec

B is in Hz



Recall: If $x(t)$ is real-valued then $|X(\omega)|$ has "even symmetry"

FACT: A signal can not be both timelimited and bandlimited

$\Rightarrow$ Any timelimited signal is not bandlimited

$\Rightarrow$ Any bandlimited signal is not timelimited

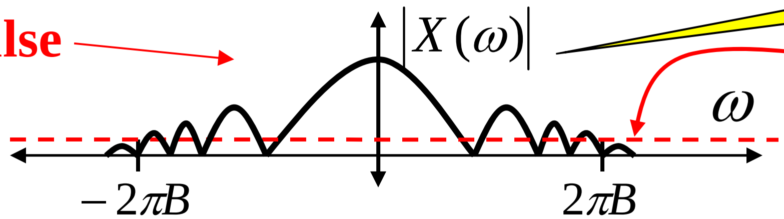
Note: All practical signals must "start" & "stop"

$\Rightarrow$ timelimited $\Rightarrow$ Practical signals are not bandlimited!

But… engineers say practical signals are effectively bandlimited because for almost all practical signals $|X(\omega)|$ decays to zero as ω gets large

Note: In our exploration above of radio Rx & Tx we ignored this issue and just drew the "spectra" as perfectly bandlimited!!!

Common "First Cut" Approach

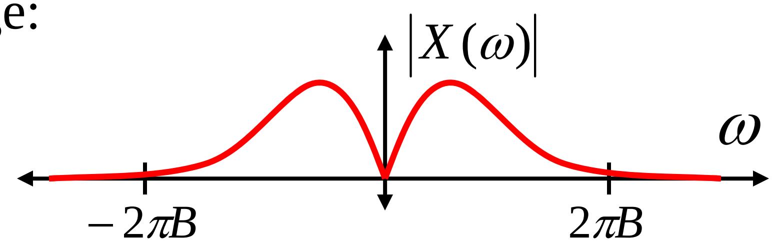


This signal is effectively bandlimited to B Hz because $|X(\omega)|$ falls below (and stays below) the specified level for all ω above $2\pi B$

Bandwidth (Effective Bandwidth)

Abbreviate Bandwidth as "BW"

For a lot of signals – like audio – they fill up the lower frequencies but then decay as ω gets large:



Signals like this are called "lowpass" signals

We say the signal's BW = B in Hz if there is "negligible" content for $|\omega| > 2\pi B$

Must specify what "negligible" means

For Example:

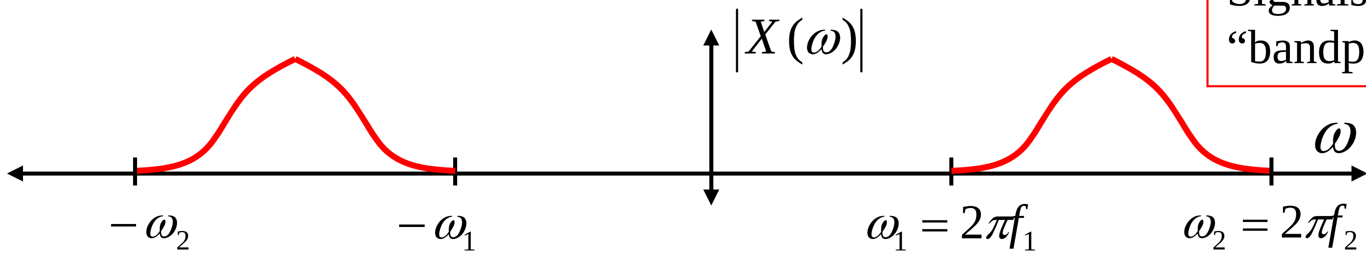
1. High-Fidelity Audio signals have an accepted BW of about 20 kHz
2. A speech signal on a phone line has a BW of about 4 kHz

Early telephone engineers determined that limiting speech to an effective BW of 4kHz still allowed listeners to understand the speech.

As we've seen… such limiting of the bandwidth can be done using RLC circuits.

For other kinds of signals – like "radio frequency (RF)" signals – they are concentrated at high frequencies

Signals like this are called "bandpass" signals



If the signal's FT has negligible content for $|\omega| \notin [\omega_1, \omega_2]$ then we say the signals BW = $f_2 - f_1$ in Hz

<u>For Example</u>:

1. The signal transmitted by an FM station has a BW of 200 kHz = 0.2 MHz
    a. The station at 90.5 MHz on the "FM Dial" must ensure that its signal does not extend outside the range [90.4, 90.6] MHz
    b. Note that: FM stations all have an odd digit after the decimal point. This ensures that adjacent bands don't overlap:
        i. FM90.5 covers [90.4, 90.6]
        ii. FM90.7 covers [90.6, 90.8], etc.
2. The signal transmitted by an AM station has a BW of 20 kHz
    a. A station at 1640 kHz must keep its signal in [1630, 1650] kHz
    b. AM stations have an even digit in the tens place and a zero in the ones