EECE 301
Signals & Systems
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Note Set #14

- C-T Signals: Circuits with Non-Periodic Sources
Recall: Convolution Property  (The Most Important FT Property!!!)

\[ y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \quad \leftrightarrow \quad Y(\omega) = X(\omega)H(\omega) \]

Here… we will explore the real-world use of the right side of this result!

Recall: For Periodic Signal…

\[ x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0t} \quad \text{Input’s FS Coefficients} \]

\[ y(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0t} \quad \text{Output’s FS Coefficients} \]

\[ d_k = H(k\omega_0)c_k \]
Unlike for the FS case it is not easy to use these ideas numerically to find the actual $y(t)$... Rather, we usually use these ideas to help us "visualize" what we need in a circuit design.

Recall the definition of the frequency response:

\[
x(t) = \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega
\]

\[
y(t) = \int_{-\infty}^{\infty} H(\omega)X(\omega)e^{j\omega t} d\omega
\]

\[
Y(\omega) = H(\omega)X(\omega)
\]
So we have as a big picture view:

\[ x(t) \xrightarrow{H(\omega)} y(t) = \mathcal{F}^{-1}\{Y(\omega)\} \]

\[ Y(\omega) = X(\omega)H(\omega) \]

\[ |Y(\omega)| = |X(\omega)||H(\omega)| \]
\[ \angle Y(\omega) = \angle X(\omega) + \angle H(\omega) \]

So…

So…in general we see that the system frequency response re-shapes the input FT’s magnitude and phase.

⇒ **System can:**
   - emphasize some frequencies
   - de-emphasize other frequencies
Example Application of Time Shift Property: Room acoustics.

Practical Questions: Why do some rooms sound bad? Why can you fix this by using a “graphic equalizer” to “boost” some frequencies and “cut” others?

Very simple case of a single reflection:

Use linearity and time shift to get the FT at your ear:

\[
Y(\omega) = \mathcal{F}\{x(t) + \alpha x(t - c)\} = \mathcal{F}\{x(t)\} + \alpha \mathcal{F}\{x(t - c)\} \\
= X(\omega) + \alpha X(\omega)e^{-j\alpha c}
\]

This is the FT of what you hear…

It gives an equation that shows how the reflection affects what you hear!!!
The big picture!

\[ |Y(\omega)| = |X(\omega)| \left| 1 + \alpha e^{-j\omega} \right| \]

\[ \equiv |H(\omega)| \]

\[ |H(\omega)| \text{ changes shape of } |X(\omega)| \]

The room changes how much of each frequency you hear…

Let’s look closer at \(|H(\omega)|\) to see what it does…

\[ |H(\omega)| = \left| 1 + \alpha e^{-j\omega} \right| = \left| 1 + \alpha \cos(c \omega) - j\alpha \sin(c \omega) \right| \]

\[ = \sqrt{(1 + \alpha \cos(c \omega))^2 + \alpha^2 \sin^2(c \omega)} = \sqrt{1 + 2\alpha \cos(c \omega) + \alpha^2 \cos^2(c \omega) + \alpha^2 \sin^2(c \omega)} \]

\[ \text{mag} = \sqrt{(\text{Re})^2 + (\text{Im})^2} \]

Expand 1st squared term

Use Trig ID

\[ |H(\omega)| = \sqrt{1 + \alpha^2} + 2\alpha \cos(c \omega) \]
\[ |Y(\omega)| = |X(\omega)|\sqrt{(1 + \alpha^2) + 2\alpha \cos(\omega c)} \]

Effect of the room… what does it look like as a function of frequency?? The cosine term makes it wiggle up and down… and the value of \( c \) controls how fast it wiggles up and down.

Spacing = 1/\( c \) Hz

“Dip-to-Dip”

“Peak-to-Peak”

What is a typical value for delay \( c \)???

Speed of sound in air \( \approx 340 \text{ m/s} \)

Typical difference in distance \( \approx 0.167 \text{ m} \)

\[
\begin{align*}
  c &= \frac{0.167 \text{ m}}{340 \text{ m/s}} = 0.5 \text{ msec} \\
  \Rightarrow \text{Spacing} &= 2 \text{ kHz}
\end{align*}
\]
Attenuation: $\alpha = 0.2$ \hspace{1cm} Delay: $c = 0.5$ ms \hspace{1cm} (Spacing = $1/0.5\times10^{-3} = 2$ kHz)

- FT magnitude at the speaker (a made-up spectrum… but kind of like audio)
- $|H(\omega)|$… the effect of the room
- FT magnitude at your ear… room gives slight boosts and cuts at closely spaced locations

Longer delay causes closer spacing… so more dips/peaks over audio range!
Attenuation: $\alpha = 0.8$  
Delay: $c = 0.5$ ms  
(Spacing = $1/0.5\times10^{-3} = 2$ kHz)

Stronger reflection causes bigger boosts/cuts!!

**FT magnitude at the speaker**

**FT magnitude at your ear... room gives large boosts and cuts at closely spaced locations**
Attenuation: $\alpha = 0.2$  
Delay: $c = 0.1$ ms  
(Spacing = $1/0.1e-3 = 10$ kHz)

Shorter delay causes wider spacing… so fewer dips/peaks over audio range!
function room_delay(atten,delay)

f=0:100:20000; % Freq range: 0 Hz to 20 kHz
w=2*pi*f; % convert to rad/sec

H=abs(1 + atten*exp(-j*w*delay)); % Compute Room Effect

% Make up a fictitious audio spectrum
X=50000*w./((2*pi*2000+w)).^2;

% Now do plots
subplot(3,1,1) % splits figure into 3 subplots, pick 1st one
plot(f/1000,X) % note f converted into k Hz
xlabel('f (kHz)')
ylabel('Original Audio Spectrum')
axis([0 20 0 2]) % set axis ranges as desired
grid % put grid lines on

subplot(3,1,2) % splits figure into 3 subplots, pick 2nd one
plot(f/1000,H)
xlabel('f (kHz)')
ylabel('Room Effect')
axis([0 20 0 2])
grid

subplot(3,1,3) % splits figure into 3 subplots, pick 3rd one
plot(f/1000,H.*X)
xlabel('f (kHz)')
ylabel('Changed Audio Spectrum')
axis([0 20 0 2])
grid
Room boosts and cuts various frequencies… So, fix it using an “equalizer”

Equalizer

\[ x(t) \xrightarrow{H_{eq}(\omega)} \text{amp} \xrightarrow{X_2(\omega) = X(\omega)H_{eq}(\omega)} Y(\omega) = H_{room}(\omega)X_2(\omega) = H_{room}(\omega)H_{eq}(\omega)X(\omega) \]

Then:
\[
|Y(\omega)| = |X(\omega)|H_{eq}(\omega)|1 + \alpha e^{-j\omega\tau}| \\
\text{Recall: Peaks and dips} \\
\text{Want this whole thing to be } = 1 \text{ so } |Y(\omega)| = |X(\omega)| \]
Equalizer’s $|H_{eq}(\omega)|$ should peak at frequencies where the room’s $|H_{room}(\omega)|$ dips and vice versa.