BINGHAMTON
UNIVERSITY
State University of New York

EECE 301
Signals & Systems

Prof. Mark Fowler

Note Set #2

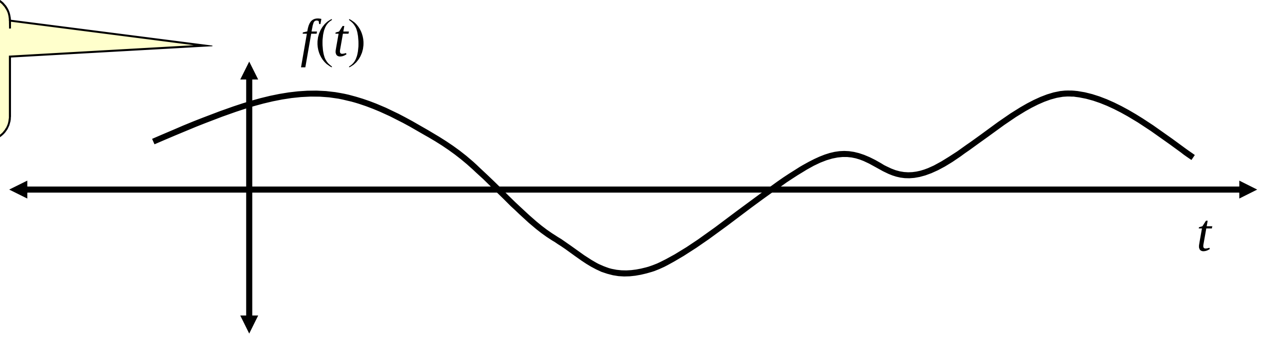
- What are Continuous-Time Signals???

Continuous-Time Signal

Continuous Time (C-T) Signal: A C-T signal is defined on the continuum of time values. That is: $f(t)$ for $t \in \Re$

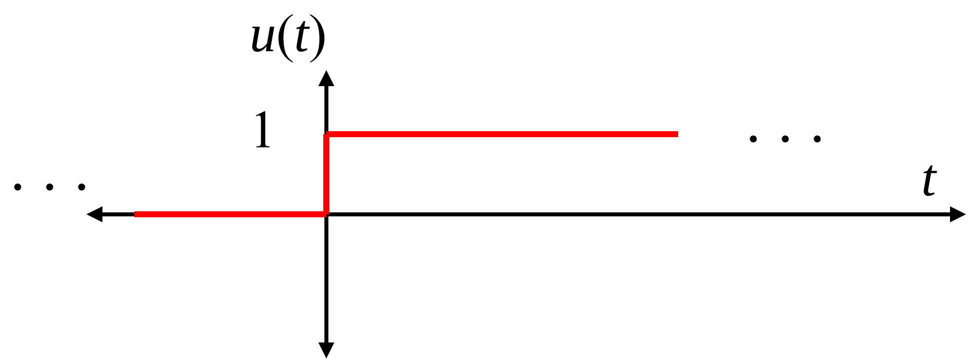
Real # Line

Time-Varying Voltage, Current, etc.



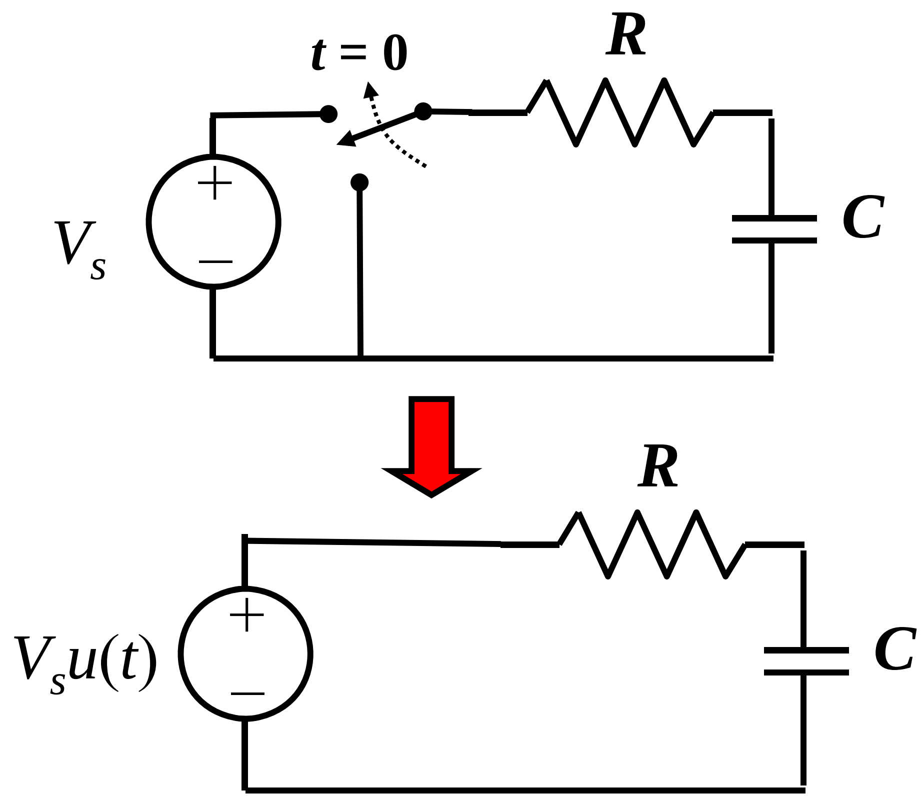
Unit Step Function *u*(*t*)

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



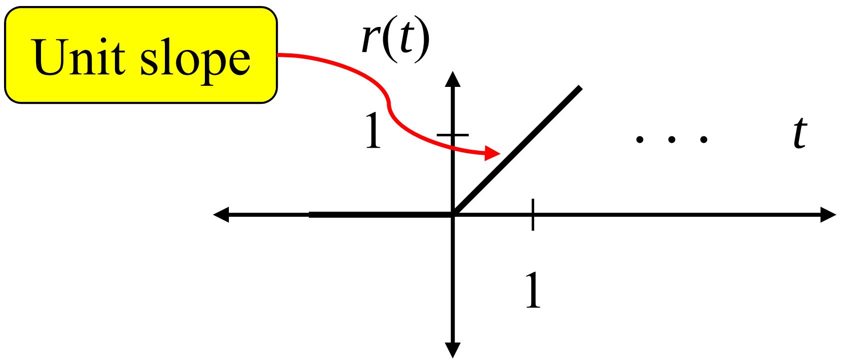
Note: A step of height A can be made from $Au(t)$

The unit step signal can model the act of switching on a DC source…



Unit Ramp Function *r*(*t*)

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Note: A ramp with slope m can be made from: $mr(t)$

$$mr(t) = \begin{cases} mt, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Time Shifting Signals

Time shifting is an operation on a signal that shows up in many areas of signals and systems:

- Time delays due to propagation of signals
  - acoustic signals propagate at the speed of sound
  - radio signals propagate at the speed of light
- Time delays can be used to "build" complicated signals
  - We'll see this later

Time Shift: If you know $x(t)$, what does $x(t - t_0)$ look like?

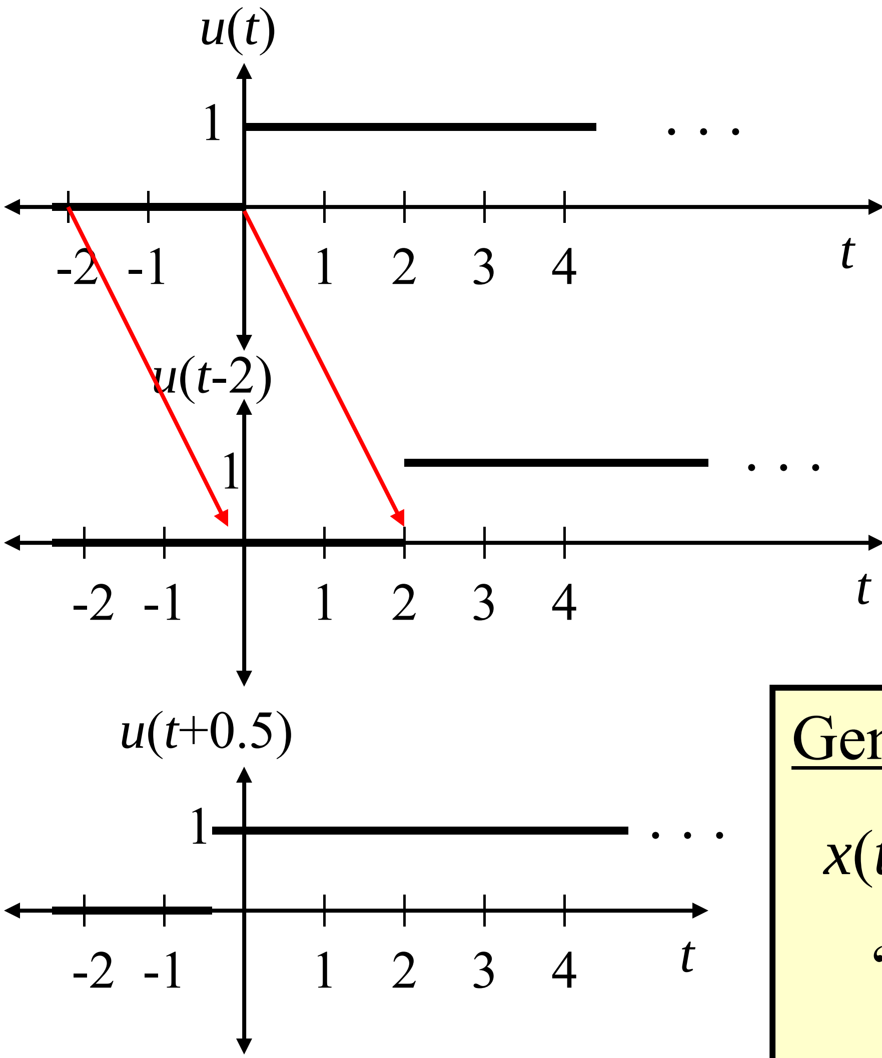
For example… If $t_0 = 2$:

$x(0-2) = x(-2)$ $\qquad$ $x(1-2) = x(-1)$

At $t = 0$, $x(t-2)$ takes the value of $x(t)$ at $t = -2$

At $t = 1$, $x(t-2)$ takes the value of $x(t)$ at $t = -1$

Example of Time Shift of the Unit Step $u(t)$:



General View:

$x(t \pm t_0)$ for $t_0 > 0$

"$+t_0$" gives Left shift (Advance)

"$-t_0$" gives Right shift (Delay)

The Impulse Function

Other Names: Delta Function, Dirac Delta Function

One of the most important functions for understanding systems!!

Ironically…it does not exist in practice!!

⇒ It is a theoretical tool used to understand what is important to know about systems!

But… it leads to ideas that are used all the time in practice!!

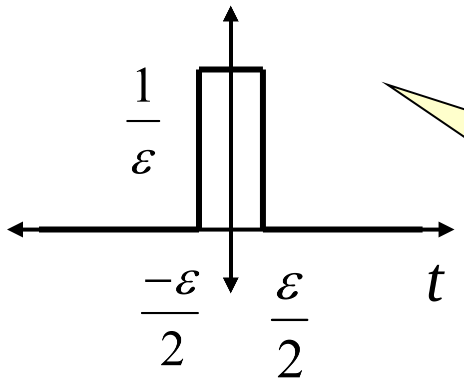
There are three views we'll take of the delta function:

Rough View: a pulse with:

Infinite height
Zero width
Unit area

"A *really* narrow, *really* tall pulse that has unit area"

Slightly Less-Rough View: Limit of pulse with width ε and height 1/ε



So as ε gets smaller the pulse gets higher and narrower but always has area of 1…

In the limit as ε gets smaller it "becomes" the delta function

Precise Idea: $\delta(t)$ is defined by its behavior inside an integral:

The delta function $\delta(t)$ is defined as something that satisfies the following two conditions:

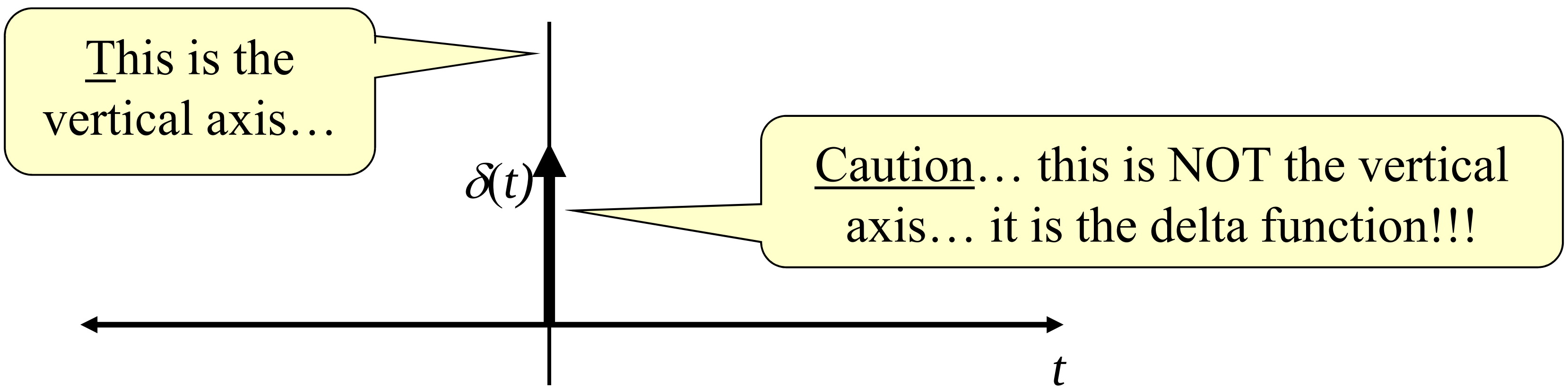
$$\delta(t) = 0, \quad \text{for any } t \neq 0$$

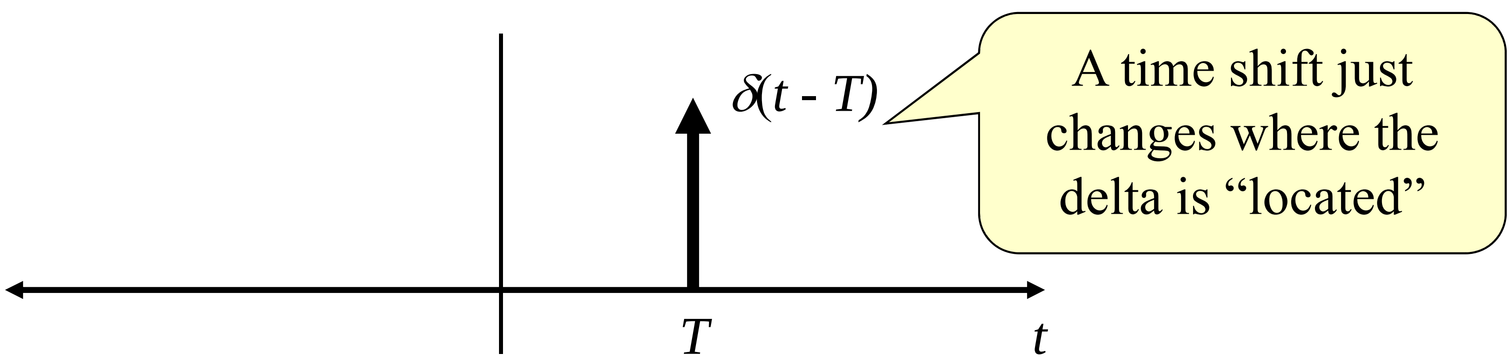
$$\int_{-\varepsilon}^{\varepsilon} \delta(t)\,dt = 1, \quad \text{for any } \varepsilon > 0$$

Showing Delta Function on a Plot:

We show $\delta(t)$ on a plot using an arrow…

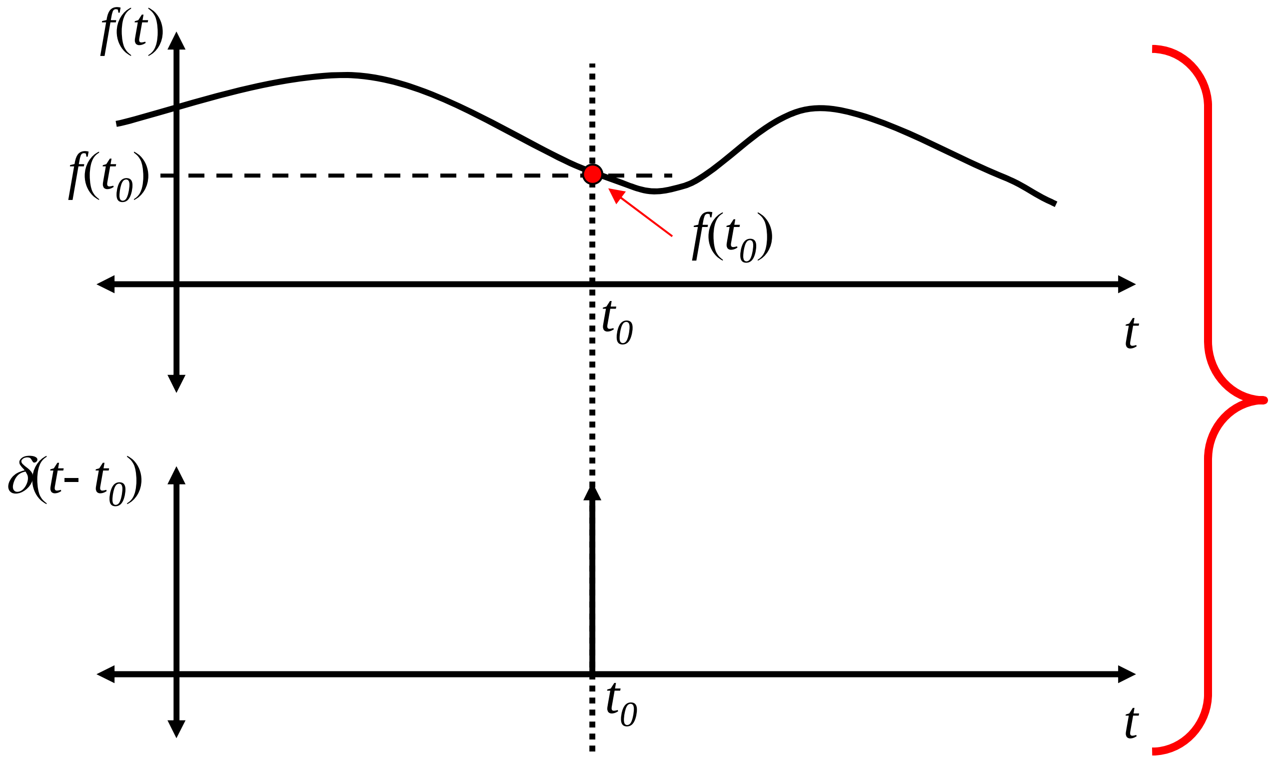
(conveys infinite height and zero width)





The <u>Sifting Property</u> is the most important property of $\delta(t)$:

$$\int_{t_0-\varepsilon}^{t_0+\varepsilon} f(t)\delta(t-t_0)dt = f(t_0) \quad \forall \varepsilon > 0$$



Integrating the product of $f(t)$ and $\delta(t - t_o)$ returns a single number… the value of $f(t)$ <u>at</u> the "location" of the shifted delta function

As long as the integral's limits surround the "location" of the delta… otherwise it returns zero

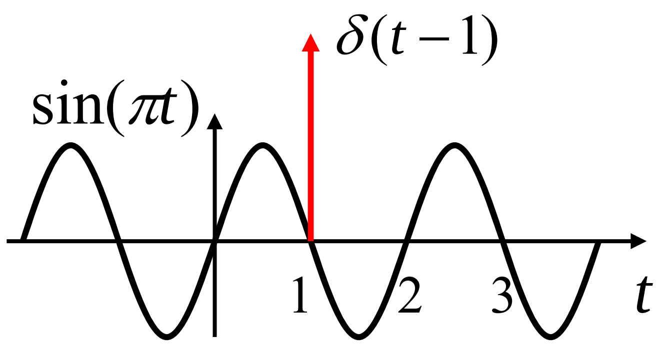
Steps for applying sifting property:

$$\int_{t_0-\varepsilon}^{t_0+\varepsilon} f(t)\delta(t-t_0)\,dt = f(t_0)$$

Step 1: Find variable of integration
Step 2: Find the argument of $\delta(\bullet)$
Step 3: Find the value of the variable of integration that causes the argument of $\delta(\bullet)$ to go to zero.
Step 4: If value in Step 3 lies inside limits of integration… Take everything that is multiplying $\delta(\bullet)$ and evaluate it at the value found in step 3; Otherwise… "return" zero

Example:

$$\int_{-4}^{7} \sin(\pi t)\delta(t-1)\,dt = ?$$



Step 1: t Step 2: $t-1$
Step 3: $t-1=0 \;\Rightarrow\; t=1$
Step 4: $t=1$ lies in [–4,7] so evaluate… $\sin(\pi\times 1)=\sin(\pi)=0$

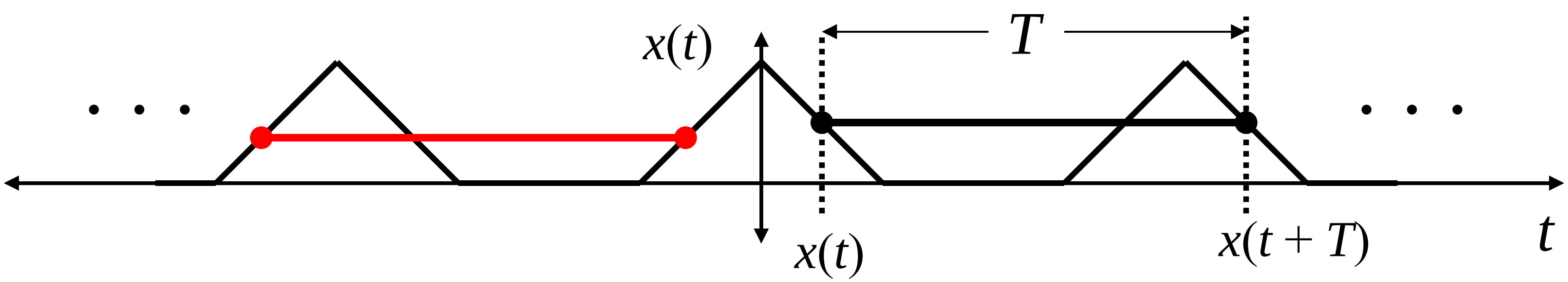
$$\int_{-4}^{7} \sin(\pi t)\delta(t-1)\,dt = 0$$

Periodic Signals

Periodic signals are important because many human-made signals are periodic. Most test signals used in testing circuits are periodic signals (e.g., sine waves, square waves, etc.)

A Continuous-Time signal $x(t)$ is periodic with period T

if: $x(t+T) = x(t) \quad \forall t$



Fundamental period = smallest such T

When we say "Period" we almost always mean "Fundamental Period"

Power and Energy of Signals

Imagine that signal $x(t)$ is a voltage.

If $x(t)$ drops across resistance R, the instantaneous power is $p(t) = \dfrac{x^2(t)}{R}$

Sometimes we don't know what R is there so we "normalize" this by ignoring the R value:

$$p_N(t) = x^2(t)$$

Once we have a specific R we can always un-normalize via $p_N(t)/R$

(In "Signals & Systems" we will drop the N subscript)

Recall: power = energy per unit time $\Rightarrow p(t) = \dfrac{dE(t)}{dt} \Rightarrow dE(t) = x^2(t)dt$

(1 W = 1 J/s)

differential increment of energy

$$\Rightarrow \text{Energy in one period } = \int_{t_0}^{T+t_0} dE(t) = \int_{t_0}^{T+t_0} x^2(t)dt$$

$$\text{The Total Energy } = \int_{-\infty}^{\infty} x^2(t)dt$$

$= \infty$ for a periodic signal

Note… if $x(t)$ is not periodic its energy may be finite if it falls off fast enough at its "ends"

Recall: power = energy per unit time

$$\text{Average power over one period} = \frac{\text{Energy in One Period}}{T}$$

$$P = \frac{1}{T}\int_{t_0}^{T+t_0} x^2(t)\,dt$$

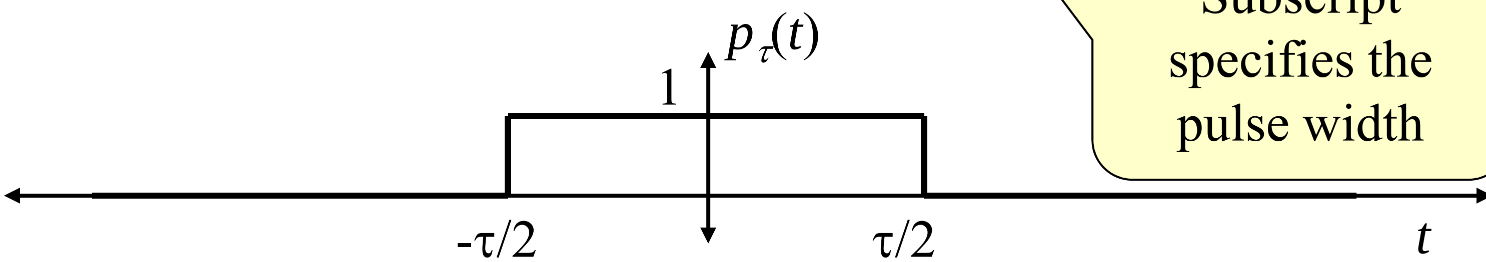
Often just called "Average Power"

For periodic signals we use the average power as measure of the "size" of a signal.

The Average Power of practical periodic signals is finite and non-zero.

(Recall that the total energy of a periodic signal is infinite.)

Rectangular Pulse Function: $p_\tau(t)$

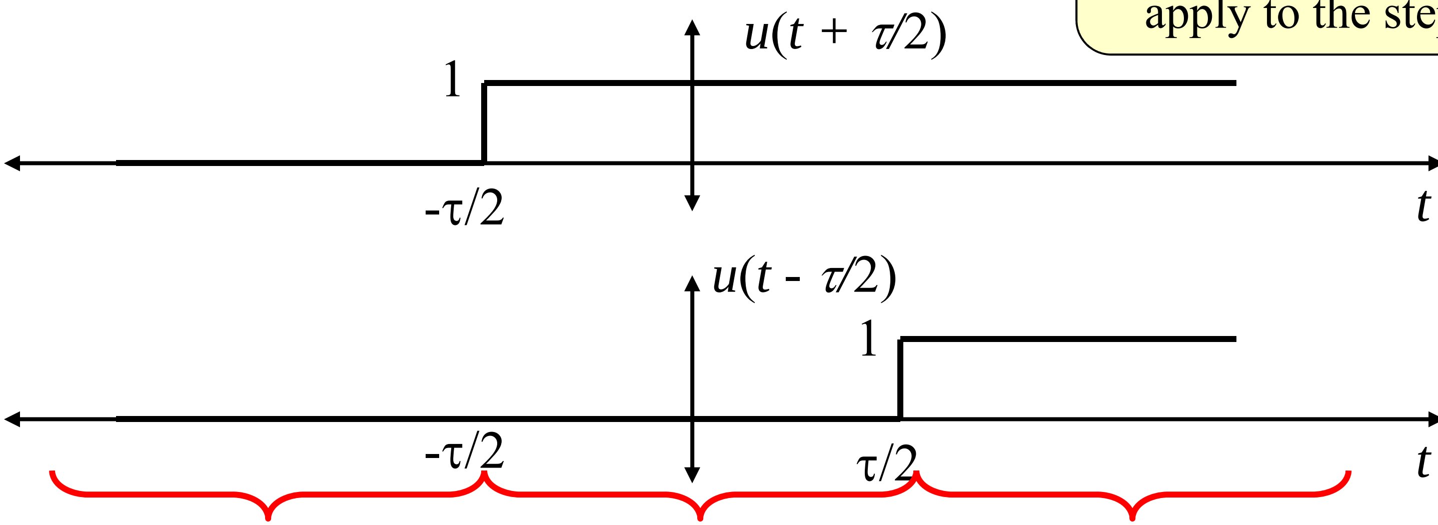


Subscript specifies the pulse width

We can build a Rectangular Pulse from Unit Step Functions:

$$p_\tau(t) = u(t + \tau/2) - u(t - \tau/2)$$

This is helpful because we will have lots of results that apply to the step function



0 – 0 = 0 1 – 0 = 1 1 – 1 = 0

Building Signals with Pulses: shifted pulses are used to mathematically "turn other functions on and off".