

Laplace Transform Table

Time Signal	Laplace Transform
$u(t)$	$1/s$
$u(t) - u(t-c), \quad c > 0$	$(1 - e^{-cs})/s, \quad c > 0$
$t^N u(t), \quad N = 1, 2, 3, \dots$	$\frac{N!}{s^{N+1}}, \quad N = 1, 2, 3, \dots$
$\delta(t)$	1
$\delta(t-c), \quad c \text{ real}$	$e^{-cs}, \quad c \text{ real}$
$e^{-bt} u(t), \quad b \text{ real or complex}$	$\frac{1}{s+b}, \quad b \text{ real or complex}$
$t^N e^{-bt} u(t), \quad N = 1, 2, 3, \dots$	$\frac{N!}{(s+b)^{N+1}}, \quad N = 1, 2, 3, \dots$
$\cos(\omega_o t) u(t)$	$\frac{s}{s^2 + \omega_o^2}$
$\sin(\omega_o t) u(t)$	$\frac{\omega_o}{s^2 + \omega_o^2}$
$\cos^2(\omega_o t) u(t)$	$\frac{s^2 + 2\omega_o^2}{s(s^2 + 4\omega_o^2)}$
$\sin^2(\omega_o t) u(t)$	$\frac{2\omega_o^2}{s(s^2 + 4\omega_o^2)}$
$e^{-bt} \cos(\omega_o t) u(t)$	$\frac{s+b}{(s+b)^2 + \omega_o^2}$
$e^{-bt} \sin(\omega_o t) u(t)$	$\frac{\omega_o}{(s+b)^2 + \omega_o^2}$
$t \cos(\omega_o t) u(t)$	$\frac{s^2 - \omega_o^2}{(s^2 + \omega_o^2)^2}$
$t \sin(\omega_o t) u(t)$	$\frac{2\omega_o s}{(s^2 + \omega_o^2)^2}$
$A e^{-\zeta \omega_n t} \sin\left[\left(\omega_n \sqrt{1-\zeta^2}\right)t\right] u(t)$ where: $A = \frac{\alpha}{\omega_n \sqrt{1-\zeta^2}}$	$\frac{\alpha}{s^2 + 2\zeta \omega_n s + \omega_n^2}$
$A e^{-\zeta \omega_n t} \sin\left[\left(\omega_n \sqrt{1-\zeta^2}\right)t + \phi\right] u(t)$ $A = \beta \sqrt{\frac{(\alpha - \zeta \omega_n)^2}{\omega_n^2 (1-\zeta^2)} + 1} \quad \phi = \tan^{-1}\left(\frac{\omega_n \sqrt{1-\zeta^2}}{\alpha - \zeta \omega_n}\right)$	$\beta \frac{s + \alpha}{s^2 + 2\zeta \omega_n s + \omega_n^2}$
$t e^{-bt} \cos(\omega_o t) u(t)$	$\frac{(s+b)^2 - \omega_o^2}{((s+b)^2 + \omega_o^2)^2}$
$t e^{-bt} \sin(\omega_o t) u(t)$	$\frac{2\omega_o (s+b)}{((s+b)^2 + \omega_o^2)^2}$

Laplace Transform Properties

Property Name		Property
Linearity	$ax(t) + bv(t)$	$aX(s) + bV(s)$
Right Time Shift (Causal Signal)	$x(t - c), \quad c > 0$	$e^{-cs} X(s)$
Time Scaling	$x(at), \quad a > 0$	$\frac{1}{a} X(s/a), \quad a > 0$
Multiply by t^n	$t^n x(t), \quad n = 1, 2, 3, \dots$	$(-1)^n \frac{d^n}{ds^n} X(s), \quad n = 1, 2, 3, \dots$
Multiply by Exponential	$e^{at} x(t), \quad a \text{ real or complex}$	$X(s - a), \quad a \text{ real or complex}$
Multiply by Sine	$\sin(\omega_0 t)x(t)$	$\frac{j}{2} [X(s + j\omega_0) - X(s - j\omega_0)]$
Multiply by Cosine	$\cos(\omega_0 t)x(t)$	$\frac{1}{2} [X(s + j\omega_0) + X(s - j\omega_0)]$
Time Differentiation 2 nd Derivative n^{th} Derivative	$\dot{x}(t)$ $\ddot{x}(t)$ $x^{(N)}(t)$	$sX(s) - x(0)$ $s^2 X(s) - sx(0) - \dot{x}(0)$ $s^N X(s) - s^{N-1}x(0) - s^{N-2}\dot{x}(0) - \dots - sx^{(N-2)}(0) - x^{(N-1)}(0)$
Time Integration	$\int_{-\infty}^t x(\lambda)d\lambda$	$\frac{1}{s} X(s)$
Convolution in Time	$x(t) * h(t)$	$X(s)H(s)$
Initial-Value Theorem	$x(0) = \lim_{s \rightarrow \infty} [sX(s)]$ $\dot{x}(0) = \lim_{s \rightarrow \infty} [s^2 X(s) - sx(0)]$ $x^{(N)}(0) = \lim_{s \rightarrow \infty} [s^{N+1} X(s) - s^N x(0) - s^{N-1} \dot{x}(0) - \dots - sx^{(N-1)}(0)]$	
Final-Value Theorem	If $\lim_{t \rightarrow \infty} x(t)$ exists, then	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$