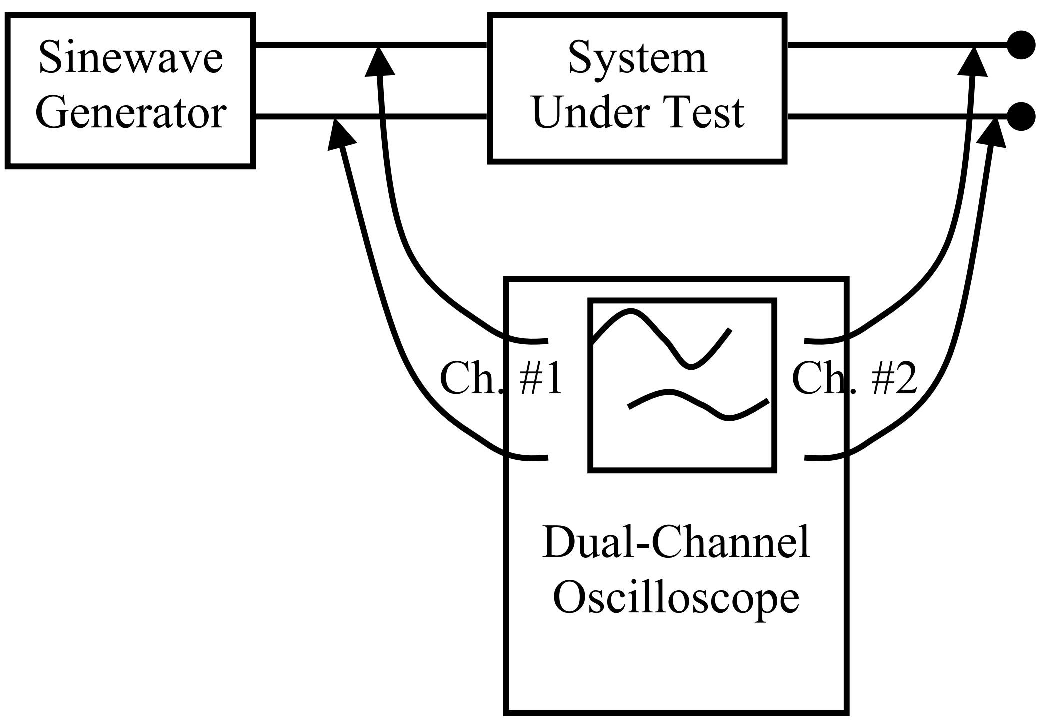
Measuring the Frequency Response of a System

Given some box containing an unknown system we wish to measure its frequency response in the lab. Note that by wishing to do this we are *assuming* that it is linear, time-invariant; otherwise the idea of frequency response doesn't exist. How do we do this? Well, remember that we know that $H(\omega)$ causes a multiplicative change in the input sinusoid's amplitude and an additive change in the input sinusoid's phase. Thus, if for a bunch of sinusoids at different frequencies we could measure:

1. the ratio of output amplitude to input amplitude
2. the phase shift between output and input

…then we could get a rough plot of $|H(\omega)|$ vs. ω and $\angle H(\omega)$ vs. ω. The setup would look like this:

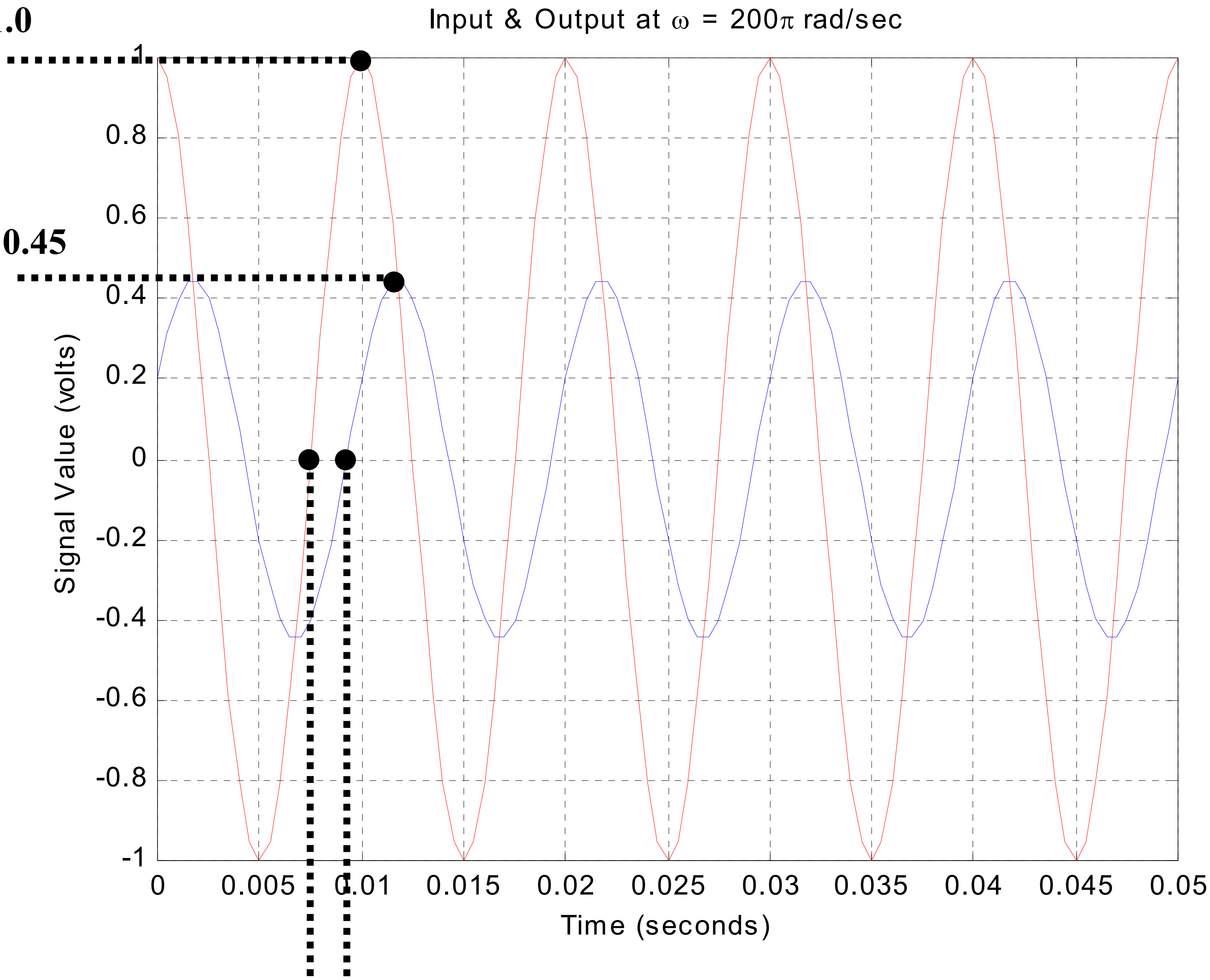


You would measure the output amplitude, the input amplitude, and the difference in time between the zero-crossings of the two sinusoids; then you would convert the time difference into a phase shift for the particular frequency being used.

The plots below show how this would look for two different frequencies; note that the input amplitude was set to be 1 to make computing the amplitude ratio easy.

Input = 1.0

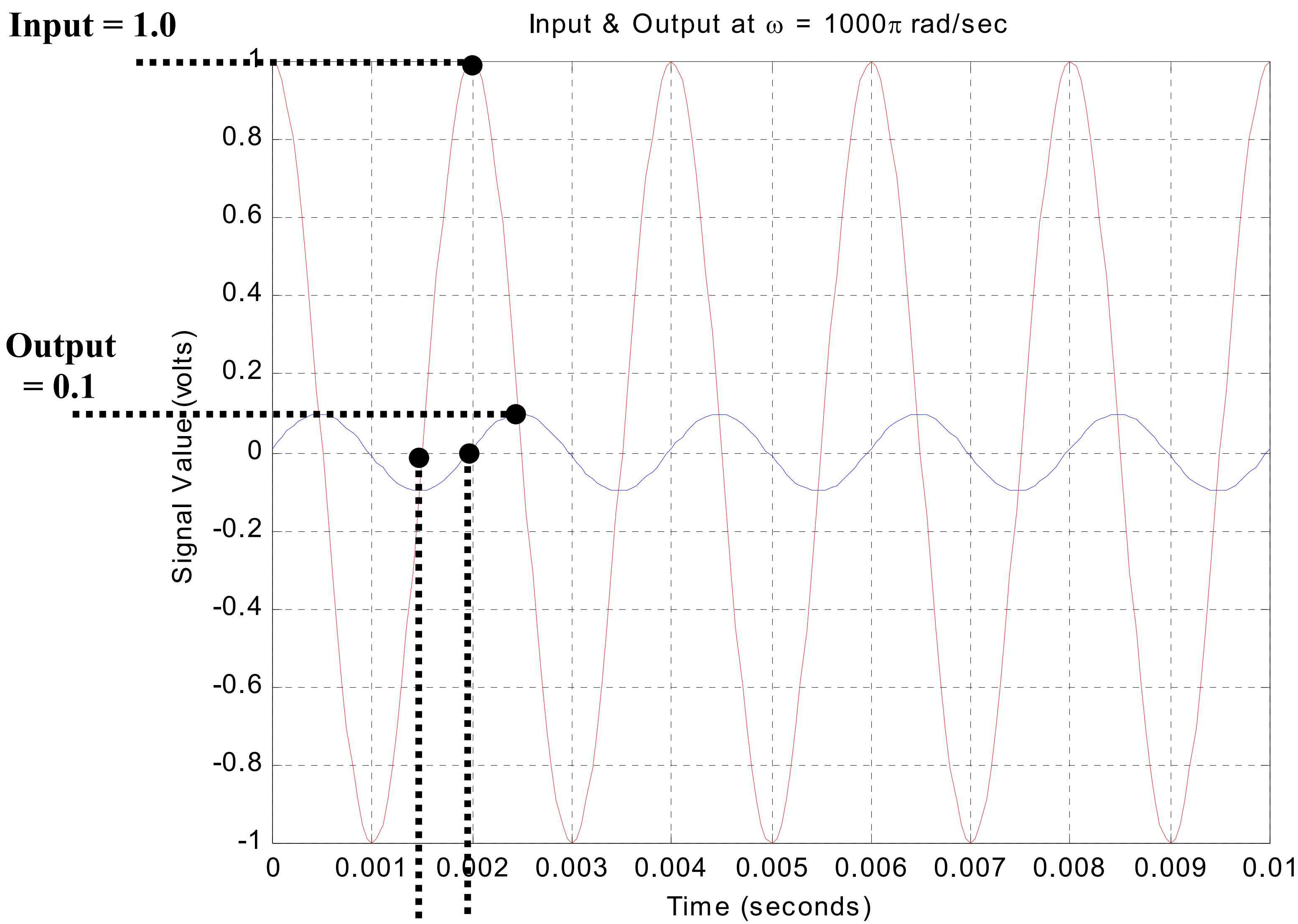
Output = 0.45



Δt = 0.00175 sec → **$\Delta\phi = \omega\Delta t$**

$=(200\pi)(0.00175) = 0.35\pi$ radians

Because the output <u>LAGS</u> the input,
the angle becomes negative: – 0.35π radians



$\Delta t = 0.00047$ sec

$\Delta\phi = \omega\Delta t$

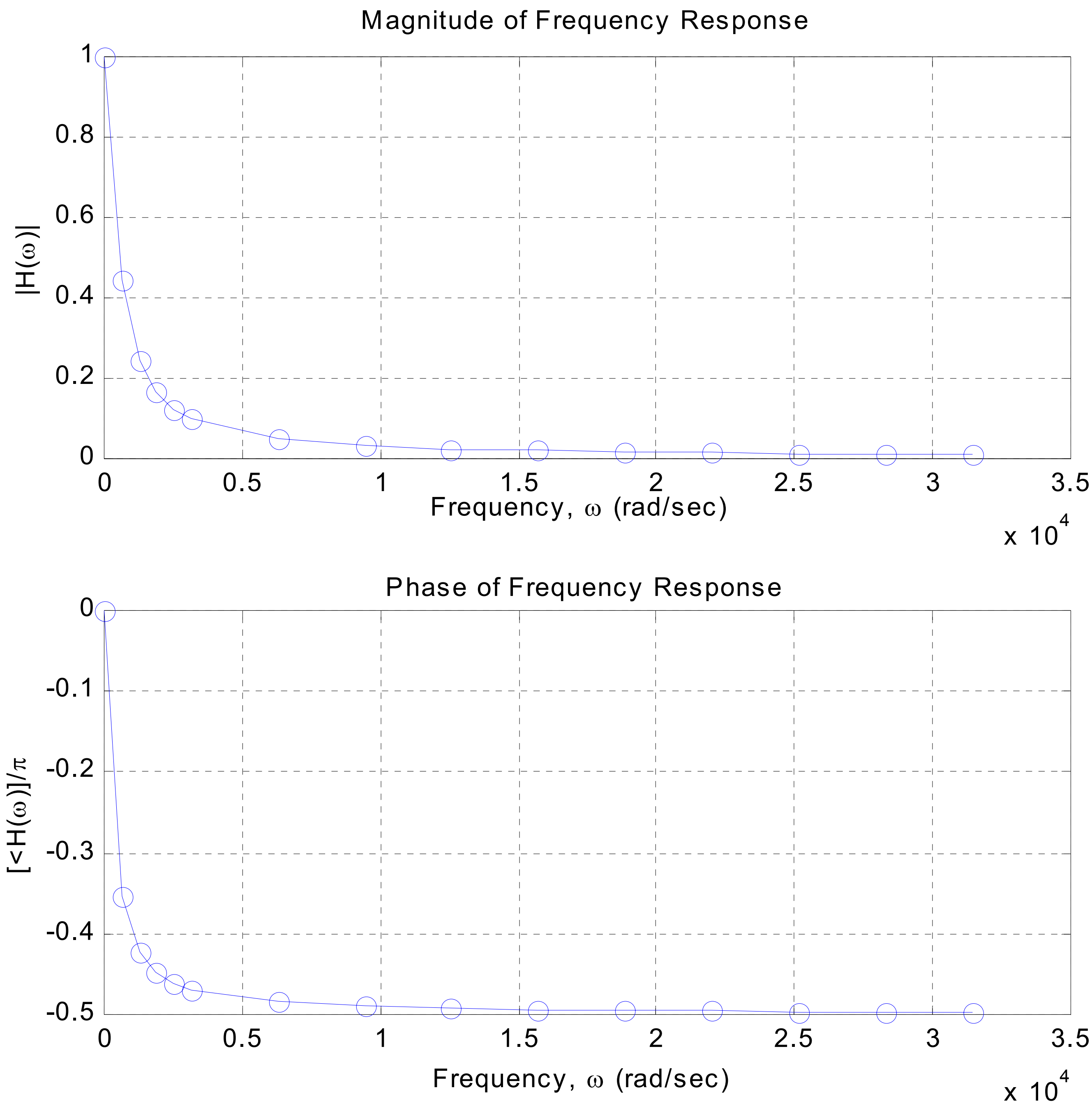
$=(1000\pi)(0.00047) = 0.47\pi$ radians

Because the output <u>LAGS</u> the input,
the angle becomes negative: -0.47π radians

The following table would result if you performed the above procedure at each of the frequencies listed in the table; the two highlighted rows are for the two cases shown above.

ω (rad/sec)	$\lvert H(\omega)\rvert$	$\angle H(\omega)$ (radians)
0	1.00	0
628	**0.45**	**-0.35π**
1257	0.24	-0.42π
1885	0.16	-0.45π
2513	0.12	-0.46π
3142	**0.10**	**-0.47π**
6283	0.05	-0.48π
9425	0.03	-0.49π
12566	0.03	-0.49π
15708	0.02	-0.49π
18850	0.02	-0.49π
21991	0.01	-0.50π
25133	0.01	-0.50π
28274	0.01	-0.50π
31416	0.01	-0.50π

If you plot these results and fill in between the plotted points with a smooth curve you get the plots shown below for the frequency response of the system. This plot gives an experimental characterization of the system's frequency response. You could use this to try to find an equation for $H(\omega)$ that would closely fit these experimental curves. You could then use that result for further analysis & design.



Magnitude of Frequency Response
|H(ω)|
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
Frequency, ω (rad/sec)
x 10^4
Phase of Frequency Response
[<H(ω)]/π
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
Frequency, ω (rad/sec)
x 10^4