An SVD Approach for Data Compression in Emitter Location Systems

Mohammad Pourhomayoun and Mark L. Fowler

***Abstract*— In classical TDOA/FDOA emitter location methods, pairs of sensors share the received data to compute the CAF and extract the ML estimates of TDOA/FDOA. The TDOA/FDOA estimates are then transmitted to a common site where they are used to estimate the emitter location. In some recent methods, it has been proposed that rather than sending the TDOA/FDOA estimates, it is better to send the entire CAFs to the common site. Thus, it is desirable to use some methods to compress the CAFs. In this paper, we will propose an SVD (Singular Value Decomposition) approach for CAF data compression. We will see that SVD approach is a beneficial method for data compression and also it is a strong tool for de-noising. Simulation results show that by applying SVD Data Compression it is possible to perform accurate location estimation in spite of the fact that we transmit fewer bits. Also for smaller compression ratio, we even achieve an improvement in performance of location estimation compared to the case that we do not compress the data at all and that is because of the de-noising effect of the SVD.**

***Index Terms*— Singular Value Decomposition (SVD), Cross Ambiguity Function (CAF).**

I. Introduction

Passive emitter localization is a challenging discussion in statistical signal processing. The position can be estimated by measuring one or more location-dependent signal parameters. One of the most popular and common emitter location methods is based on TDOA (time-difference-of-arrival) and FDOA (frequency-difference-of-arrival) estimation. In the classical approach to this method, FDOA and TDOA are estimated from the cross-correlation of signals received by several pairs of sensors [1]; this is done by computing the cross ambiguity function (CAF) [2] and finding the peak of its magnitude surface. Then these TDOA/FDOA estimates are used in statistical processing to locate the emitter [3].

However A challenge in such methods is the need to share large amounts of signal data between paired sensors prior to computation of the CAF for each pair, and has recently been addressed in [4], [5]; note that the subsequent sharing of the TDOA/FDOA estimates requires a very small amount of data transfer.

Recently, some new methods based on TDOA/FDOA emitter location have been proposed that estimate the emitter location in one stage without extracting the TDOA/FDOA in a separate step. The goal of these methods is to improve the overall accuracy of the emitter location estimate. The main idea of the recent methods is that all pairs of sensors have to share their computed CAFs to each other or they have to send the CAFs to a common site to estimate the emitter location. Thus, there will be a large amount of data transmission and this leads to a need for methods to compress the CAFs. One of the recently proposed methods is named CAF-map method [6]. The main idea of the CAF-map method is to take each CAF magnitude and re-map its delay and Doppler axes into equivalent axes in *x-y* position (assuming location in only 2-D for simplicity). Then, the emitter's location is estimated as the *x-y* location that maximizes the average of all the CAF-map magnitudes [6]. Alternatively, Weiss and Amar [7], [8], [9] developed a single-stage ML method named direct position determination (DPD). The TDOA/FDOA based DPD [9] computes the CAF-map between every possible pairing of sensors. Then, it uses the CAFs to form a series of matrices and the location is estimated by computing the maximum eigenvalues of these matrices. Kay and Vankayalapati [10] also developed a single-stage method based on the detection point of view and they derived the same results. In this paper, we develop a method for compressing CAF to reduce the amount of data transmission and consequently, to facilitate the implementation of these new localization methods.

Cross Ambiguity Function (CAF) is a complex-valued 2-dimentional function of TDOA and FDOA:

$$A_{12}(\tau,\omega) = \int_{-\infty}^{+\infty} \hat{s}_{r1}(t)\hat{s}_{r2}^{\,*}(t-\tau)e^{j\omega t}dt \qquad (1)$$

where $\hat{s}_{r1}(t)$ is the lowpass equivalent (LPE) of the received signal at the first sensor and $\hat{s}_{r2}(t)$ is the LPE of the received signal at the second sensor. CAF measures the correlation between $\hat{s}_{r1}(t)$ and a Doppler-shifted by ω and delayed by τ version of $\hat{s}_{r2}(t)$.

As mentioned before, the CAF is a two-dimensional function. Thus, we can consider the CAF to be an image and apply image compression methods to it. Some preliminary work in this vein has been presented by the present authors in [11] and [12]. In these papers, we applied some image compression methods to compress CAF. We also exploited

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some special properties of the CAF in data compression to get better results. The detailed effects of lossy data compression on CAF and consequently, its effects on location estimation accuracy were assessed too.

Now, in this paper we develop an SVD (Singular Value Decomposition) approach for CAF data compression. We show that by applying SVD Data Compression it is possible to perform accurate location estimation in spite of the fact that we transmit fewer bits. Also for smaller compression ratio, we even achieve an improvement in performance of location estimation compared to the case that we do not compress the data at all and that is because of the de-noising effect of the SVD.

II. An SVD approach for CAF Data Compression

The singular value decomposition (SVD) is an important tool with many useful signal processing applications [13][14][15]. For a complex valued $M \times N$ matrix $\boldsymbol{X}$, the SVD representation will be

$$\boldsymbol{X} = \boldsymbol{U\Sigma V}^H = \sum_{i=1}^{r} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^H \tag{2}$$

where $\boldsymbol{U}$ is an $M \times M$ unitary matrix consisting of M *left singular vectors (LSV)* as its columns, $\boldsymbol{V}$ is an $N \times N$ unitary matrix consisting of N *right singular vectors (RSV)* as its columns and Σ is a pseudo-diagonal $M \times N$ matrix with nonnegative real *singular values* (σ_i) on the main diagonal ordered such that $\sigma_i \geq \sigma_{i+1}$. r is the number of non-zero singular values, $\boldsymbol{u}_i$ is the *i*th *left singular vector* and $\boldsymbol{v}_i^H$ is Hermitian of the *i*th *right singular vector*. By truncating the above summation to $k < r$ terms, we get a rank-k matrix $\boldsymbol{X_k}$ that approximates $\boldsymbol{X}$ better than any other rank-k matrix in the least square error sense [16], [17], [18]. This is the main idea of SVD data compression.

$$\boldsymbol{X}_k = \boldsymbol{U}_k \boldsymbol{\Sigma}_k \boldsymbol{V}_k^H = \sum_{i=1}^{k} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^H \tag{3}$$

The complex valued $M \times N$ matrix X contains *MN* complex values or equivalently *2MN* real values. But, in the truncated SVD representation of X, we have *kM* complex values to represent matrix U_k, *kN* complex values for matrix V_k, and *k* real values to represent the singular values. Thus, in approximation X by X_k, the compression ratio is:

$$CR = \frac{2MN}{2kM + 2kN + k}$$

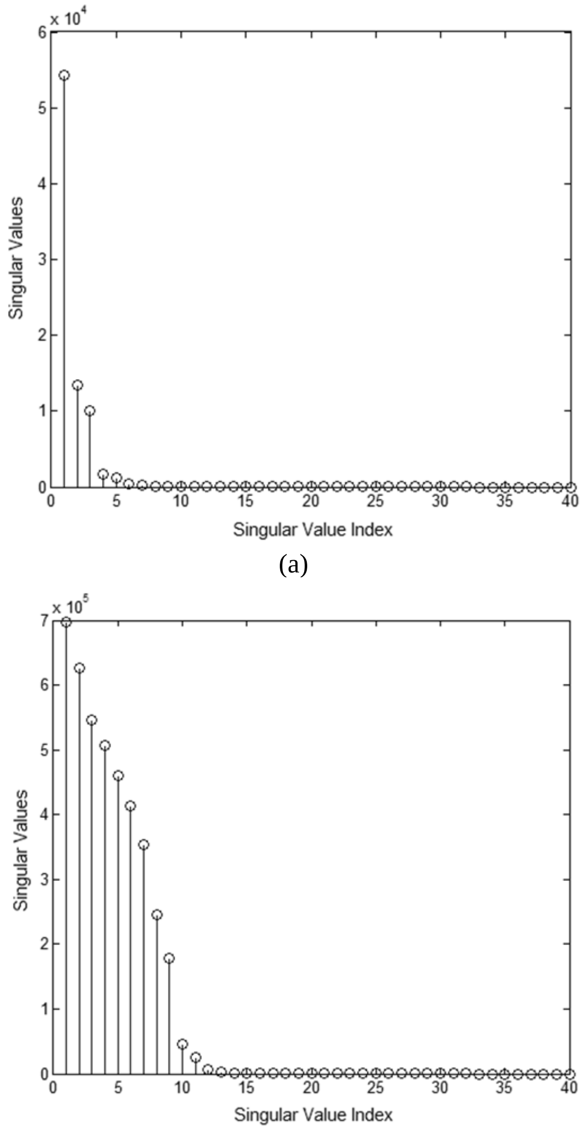
For example the compression ratio for a 128×32 matrix truncated by k=1 is 25:1.

As mentioned in [14], the singular value decomposition of an image is conceptually similar to its Karhunen-Loeve decomposition but in a different manner. The first difference is that Karhunen-Loeve decomposition basis are determined by the covariance matrix of the random process that generates the image but, SVD is defined on the raw data and the image itself. The second difference is that if both representations are truncated for the purpose of data compression, SVD is the best approximation in least square error sense, while Karhunen-Loeve is the best approximation in mean square error sense.

CAF magnitude is symmetric around the TDOA/FDOA point corresponding to the peak of that [11]. It usually contains a big main lobe and several small side lobes that if we slice each of them up at different points, we will always get a curve with a similar shape. It has been shown that for a time-frequency localization operator there are several large singular values at the beginning, followed by a sharp plunge in the values, with a final asymptotic decay to zero [13]. Since the cross Ambiguity function is considered to be a member of Cohen’s class of time-frequency representations [19], these properties imply that CAF is very close to a low rank matrix. Thus, most of the data is concentrated in the first few singular vectors and values.

In reality, the received signals are noisy. The received signal at the first sensor will be $\hat{s}_{r1}(t) + \hat{n}_{r1}(t)$ and the received signal at the second sensor will be $\hat{s}_{r2}(t) + \hat{n}_{r2}(t)$, where $\hat{n}_{r1}(t)$ and $\hat{n}_{r2}(t)$ are the noise terms. Thus, in equation (1) we will have three more terms which are corresponding to the correlation between $\hat{n}_{r1}(t)$ *and* $\hat{s}_{r2}(t)$, $\hat{n}_{r2}(t)$ *and* $\hat{s}_{r1}(t)$ and $\hat{n}_{r1}(t)$ *and* $\hat{n}_{r2}(t)$ and we can consider the sum of those terms as an additive noise ξ. The effect of the noise on the singular values is spread across all the singular values but, as mentioned before, most of the data is concentrated in the first few singular vectors and values. Thus, by SVD truncation we reduce the noise and equivalently we increase the signal to noise ratio (SNR) [18].

The singular values of a sample 128x32 CAF are illustrated in Fig.1 for two cases: (a) noiseless signals and (b) noisy signals. As we can see, there are only 3 to 5 significant singular values in the left figure showing that the CAF is very close to a low rank matrix. But, the right figure shows that in the noisy case the number of significant singular values increases to 12. Therefore, it is clear that the signal to noise ratio can increase by applying SVD data compression and retaining the first few singular values and discarding the rest.



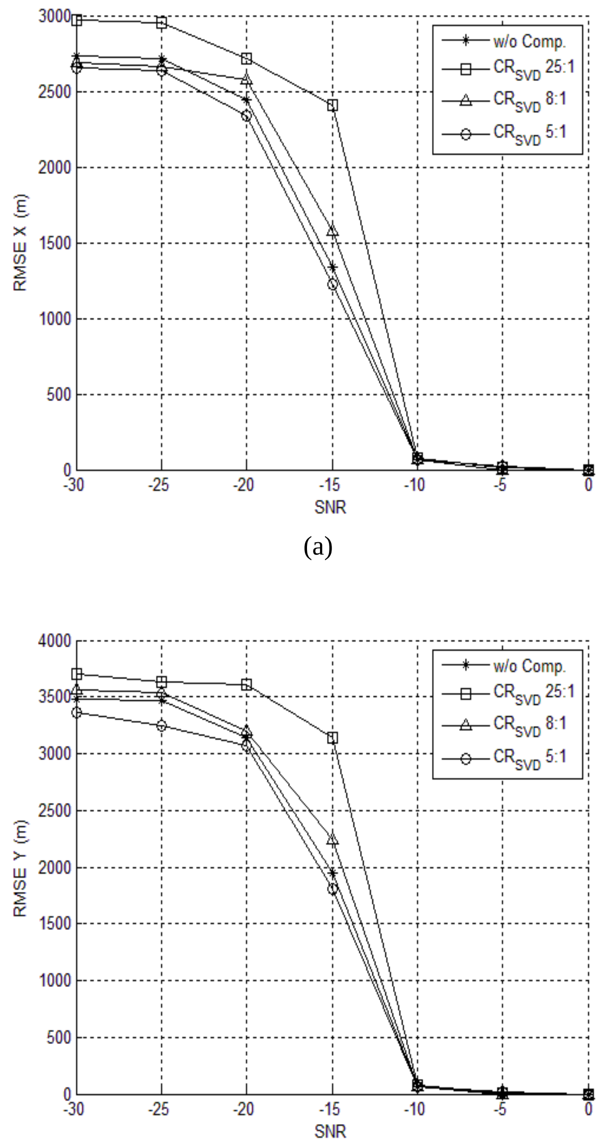
(a)

(b)

Fig. 1. Singular Values of CAF for two cases: (a) Noiseless signal, (b) Noisy signal

III. SIMULATION RESULTS

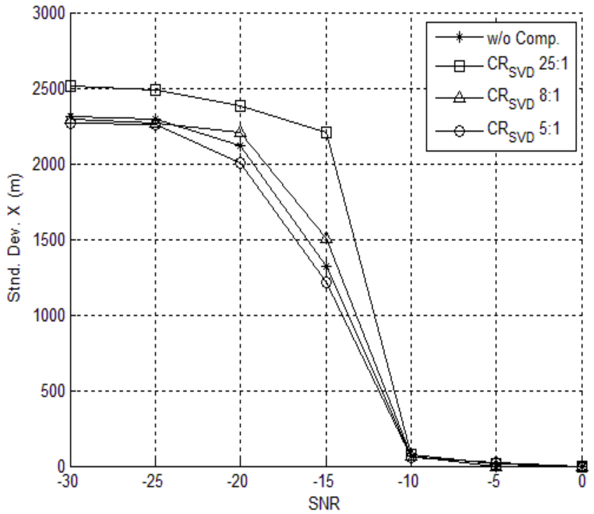
We examined the performance of the proposed method and compare the results using Monte Carlo computer simulations (with 500 runs each time). In this simulation, the signals are BPSK, the sampling frequency = 20 kHz and the number of samples is equal to 4096 and we used direct position determination method for location estimation [9]. We assumed that 4 moving sensors receive the signals from one stationary emitter and for each two of them there is a cross ambiguity function which should be computed, compressed and transmitted to a common site to do the location estimation. Fig.2 shows the effect of data compression on RMS error and Fig.3 shows the effect of data compression on standard deviation of emitter location estimation for X and Y dimensions. The four curves compare the cases (i) without compression, (ii) SVD-based compression with compression ratio of 25:1, (iii) SVD-based compression with compression ratio of 8:1, and (iv) SVD-based compression with compression ratio of 5:1. As we can see, even for high compression ratio of 25:1, the estimation accuracy is pretty close to the case without compression. Surprisingly, the case with the compression ratio of 5:1 (and even the case with the compression ratio of 8:1 in some points) yields more accurate results than without compression case and that is because of the de-noising property of SVD-based data compression.



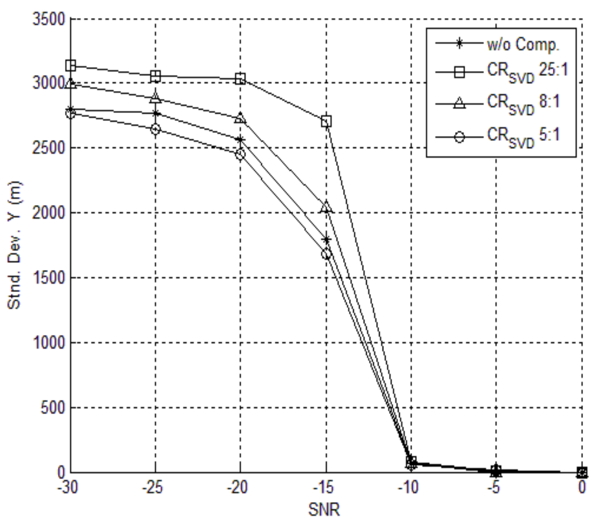
(a)

(b)

Fig. 2. Simulation results showing RMS error for X and Y when we compress CAF using SVD-based compression. The four curves compare the cases (i) without compression, (ii) SVD-based compression with compression ratio of 25:1, (iii) SVD-based compression with compression ratio of 8:1, and (iv) SVD-based compression with compression ratio of 5:1.



(a)



(b)

Fig. 3. Simulation results showing standard deviation for X and Y when we compress CAF using SVD-based compression. The four curves compare the cases (i) without compression, (ii) SVD-based compression with compression ratio of 25:1, (iii) SVD-based compression with compression ratio of 8:1, and (iv) SVD-based compression with compression ratio of 5:1.

IV. Conclusion

We developed an SVD (Singular Value Decomposition) approach to compress the two-dimensional CAF to reduce the amount of data which has to be shared in emitter location systems. In this technique, we have supposed the two-dimensional CAF as an image. We discussed that CAF is very close to a low rank matrix. Thus, it has several large singular values, followed by a sharp plunge in the values, with a final asymptotic decay to zero. We showed that in noisy cases, most of the data is concentrated in the first few singular vectors and values. However, the effect of the noise on the singular values is spread across all the singular values. Thus, by SVD truncation we reduce the noise and equivalently we increase the signal to noise ratio (SNR). Finally, Monte Carlo computer simulation results showed that it is possible to perform accurate location estimations applying SVD Data Compression in spite of the fact that we transmit fewer bits. As we see in Fig.2 and Fig.3, we can even achieve an improvement in performance of location estimation for smaller compression ratio, compared to the case that we do not compress the data at all and that is because of the de-noising effect of the SVD.

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