Sensor Network Distributed Computation for Direct Position Determination

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***Abstract*—Classical localization systems based on TDOA/FDOA use a two-stage estimation approach. In the first stage, pairs of sensors share data to estimate TDOA/FDOA. Then, the extracted TDOA/FDOA measurements are used to estimate the emitter location. In some recently published methods, an optimal single-stage approach named Direct Position Determination (DPD) has been proposed to improve the position estimation accuracy. However, unlike the classical two-stage method where the TDOA/FDOA estimation can be distributed across all sensors, DPD processes all the received signals together at a single sensor node. However, when sensors have limited computational capabilities it is desirable to distribute the computation across all sensors. Furthermore, concentrating all the processing into a single node makes the location system less robust to the loss of sensors. In this paper, we develop a distributed localization method with the goal of reducing the computational load on each sensor and increasing the reliability of the system.**

I. INTRODUCTION

One of the most accurate and common methods for passive radio signal localization is based on frequency-difference-of-arrival (FDOA) and time-difference-of-arrival (TDOA) estimation. The classical approach to this method uses two stages to estimate the signal position. In the first stage, TDOA and FDOA are estimated from the cross-correlation of signals received by several pairs of sensors [1]; this is done by computing the cross ambiguity function (CAF) [2] and finding the peak of its magnitude surface [1], [2]. In [4] and [5], a Fisher Information based data compression method has been suggested to reduce the amount of data transmission and improve the communication performance between each pair of sensors. In the second stage of the classic method the TDOA/FDOA estimates are used in statistical processing to locate the emitter [3]. Suppose that the lowpass equivalent (LPE) model of the received signal is

$$\hat{s}_r(t) = \alpha e^{j\omega_d t}\hat{s}(t-\tau_d) + \nu(t), \qquad (1)$$

where $\hat{s}(t)$ is the LPE of the transmitted signal, ω_d is the Doppler, τ_d is the delay for the received signal, α is a complex number and $\nu(t)$ is the LPE of the noise [10]. Now, suppose that two sensors R1 and R2 receive the LPE signals $\hat{s}_{r1}(t)$ and $\hat{s}_{r2}(t)$, respectively. Stein [1] showed that the maximum likelihood (ML) estimate for TDOA and FDOA can be obtained by finding the peak of the magnitude of the cross ambiguity Function (CAF) given by

$$CAF_{12}(\tau,\omega) = \int_{-\infty}^{+\infty} \hat{s}_{r1}(t)\hat{s}_{r2}^{*}(t-\tau)e^{i\omega t}dt, \qquad (2)$$

which measures the correlation between $\hat{s}_{r1}(t)$ and a Doppler-shifted-by-ω and delayed-by-τ version of $\hat{s}_{r2}(t)$. Stein [2], Wax [12], Fowler and Hu [13] and Yeredor and Angel [14] have derived formulas for the CRLB on TDOA and FDOA.

Recently, a new method named Direct Position Determination (DPD) based on TDOA/FDOA emitter location has been proposed to estimate the emitter location in one stage without extracting the TDOA/FDOA in a separate stage [9]. Weiss and Amar [7], [8], [9] showed that the two-stage method is not necessarily optimal because in the first stage of these methods, the TDOA and FDOA estimates are obtained by ignoring the fact that all measurements should be consistent with a single emitter location. In other words, although each TDOA/FDOA estimation is optimal in the first stage and the second stage is optimal (given the results of the first stage), the whole two-stage method is not optimal. In related work, Kay and Vankayalapati [11] developed the generalized likelihood ratio (GLR) detector based on the received signals from all sensors, and the DPD location result naturally appears as the ML estimate used in the GLR. This shows another advantage of DPD over the classical two-stage method: the classical method can't make use of the data from a CAF whose peak is undetectable due to low SNR – yet the DPD method can.

In the DPD method, we need all the received signals together at a single point to start the location estimation processing. Consequently, all sensors have to transmit their received signals to a common site, which usually is one of the sensors so we will refer to this as the common sensor. It would be also desirable to use some data compression methods to reduce the amount of data transmission [16][18]. The common sensor then uses the received signals to form a series of matrices (one for each point on an x-y location grid)

This work was partially supported by the Air Force Research Lab, Rome, NY under contract FA8750-09-2-0068. Approved for Public Release; Distribution Unlimited: 88ABW-2012-0924, 8-Mar-2012.

and computes the maximum eigenvalue of each of these matrices; the location estimate is the grid point that produced the largest of these maximum eigenvalues.

As mentioned above, the one-stage DPD method achieves more accurate results compared to classic two-stage methods. However, in the published papers, the authors did not address the issues of computation and data transmission for DPD, which create some difficulties that may limit DPD application in practice. The first problem is the large amount of computations that are to be done by only the common sensor. As mentioned above, in DPD method we need all received signals together to start the estimation process. Thus, all sensors should send their received signals to one common sensor to start the estimation process. The common sensor will do all mathematical computations having all received signals. This leads to a large computational load on only one point in the network and no computational load on other members of the sensor network, which requires any one sensor in the system to be computationally capable of doing the complete set of computations needed to locate an emitter. In scenarios where the amount of computational capabilities any one sensor possesses is limited, this centralized approach is not desirable. The other problem is the high dependence of the whole network on the common sensor. In this scenario, if we lose the common sensor during computations, we will lose everything. In other words, it is not desirable to rely on only one point in the sensor network for all computations and data collection, because if we lose that sensor for any reason, then we will lose all intermediate and final results.

In this paper, we develop a method to increase the flexibility and feasibility of the one-stage DPD method. In this method, we use distributed data computations in the sensor network to reduce the computational load on any one sensor.

II. BACKGROUND

In this section we provide some more details about the one-stage localization method (DPD) and its formulation [9]. Suppose that there are L moving sensors in a sensor network intercepting the transmitted signal in one short snapshot. The complex signal observed by the l^{th} sensor is

$$\hat{s}_{rl}(t) = \alpha_l s(t-\tau_l) e^{j2\pi f_l t} + w_l(t),$$

where $s(t)$ is the transmitted signal, α_l is an unknown complex path attenuation, f_l is the Doppler shift, τ_l is the signal delay and $w_l(t)$ is a white, zero mean, complex Gaussian. Assume that each sensor collects N time samples sampled with sampling frequency $F_s = 1/T_s$. Then, we have

$$\begin{aligned}
&\hat{\boldsymbol{s}}_{\boldsymbol{rl}} = \alpha_l \boldsymbol{W}_l \boldsymbol{D}_l \hat{\boldsymbol{s}} + \boldsymbol{w}_l \\
&\boldsymbol{W}_l \triangleq diag\{e^{j2\pi f_l t_1},\ e^{j2\pi f_l t_2},\ \ldots,\ e^{j2\pi f_l t_N}\} \\
&\hat{\boldsymbol{s}}_{\boldsymbol{rl}} \triangleq [\hat{s}_{rl}(t_1),\ \hat{s}_{rl}(t_2),\ \ldots,\ \hat{s}_{rl}(t_N)]^T \\
&\boldsymbol{w}_l \triangleq [w_l(t_1),\ w_l(t_2),\ \ldots,\ w_l(t_N)]^T,
\end{aligned}$$

where $\hat{\boldsymbol{s}}_{\boldsymbol{rl}}$ is N samples of the received signal at l^{th} sensor, $\hat{\boldsymbol{s}}$ is N samples of the transmitted signal, f_l is the Doppler shift and $\boldsymbol{D}_l$ is the time sample shift operator by $n_l = (\tau_l / T_s)$ samples. We can write $\boldsymbol{D}_l = \boldsymbol{D}^{n_l}$ where $\boldsymbol{D}$ is an $N \times N$ permutation matrix defined as $[\boldsymbol{D}]_{ij} = 1$ if $i = j+1$, $[\boldsymbol{D}]_{0,N-1} = 1$ and $[\boldsymbol{D}]_{ij} = 0$ otherwise.

According to [9] and [11], the estimated transmitter's position in TDOA/FDOA-based one-stage method is found as follows. Let $\{\mathbf{p}_i\}_{i=1}^{G}$ be grid points of possible emitter locations. For each grid point form the matrix

$$\boldsymbol{V}_{\boldsymbol{p}_i} \triangleq \left[\boldsymbol{D}_1^H \boldsymbol{W}_1^H \hat{\boldsymbol{s}}_{r1},\ \ \boldsymbol{D}_2^H \boldsymbol{W}_2^H \hat{\boldsymbol{s}}_{r2},\ \ \ldots,\ \ \boldsymbol{D}_L^H \boldsymbol{W}_L^H \hat{\boldsymbol{s}}_{rL}\right],$$

where the delay and Doppler operators correspond to the path between the grid point and the respective sensor [9]. For each grid point form the $L \times L$ matrix $\boldsymbol{Q}_{\boldsymbol{p}_i} = \boldsymbol{V}_{\boldsymbol{p}_i}^H \boldsymbol{V}_{\boldsymbol{p}_i}$ and find its largest eigenvalue. The location estimate is then the grid point that maximizes the largest eigenvalue, that is

$$\hat{\boldsymbol{p}} = \arg\max_{\boldsymbol{p}_i} \{\lambda_{\max}(\boldsymbol{Q}_{\boldsymbol{p}_i})\}\ . \tag{3}$$

It is clear that a large amount of computation must be done to find the location and all of it is done at the common sensor.

It is interesting to mention that in the single-stage DPD method, the (*i*,*j*)th element of the matrix $\boldsymbol{Q}$ is the value of the CAF between the signals received by sensors *i* and sensor *j* [9], [11] and that is why in [11] this matrix is called the Cross Ambiguity Matrix (CAM) and is given by

$$[\boldsymbol{Q}]_{ij} = [\boldsymbol{V}^H \boldsymbol{V}]_{ij} = \hat{\boldsymbol{s}}_{ri}^{\ H} \boldsymbol{W}_i \boldsymbol{D}_i \boldsymbol{D}_j^{\ H} \boldsymbol{W}_j^{\ H} \hat{\boldsymbol{s}}_{rj} = [\boldsymbol{CAF}_{ij}]_{(\tau,\omega)}, \tag{4}$$

where τ and ω are the corresponding TDOA and FDOA between sensors *i* and *j* and emitter located at a grid point. Note that the diagonal elements in the CAM are the Auto-Ambiguity Function of the received signals at TDOA = 0 and FDOA = 0 which is equal to the energy of the received signals.

III. APPROXIMATED DPD

In this section, we develop a method to distribute and reduce the amount computation based on eigenvalue approximation. This then increases the flexibility and feasibility of one-stage geolocation method.

Definition (Gershgorin's disc) [15]: Assume that $\boldsymbol{A}$ is an $n \times n$ complex-valued matrix with entries a_{ij} and $P_i = \sum_{j \neq i} |a_{ij}|$ is the summation of the absolute values of all non-diagonal elements of the i^{th} row. Then, the set $D_i = \{z \in \mathbb{C} : |z - a_{ii}| \leq P_i\}$ is called the i^{th} Gershgorin's disc of $\boldsymbol{A}$. This disc contains the interior and boundary points of a circle with radius of P_i and centered at a_{ii} in complex plane.

Theorem 1 (Gershgorin's Theorem) [15]: Every eigenvalue of matrix $\boldsymbol{A}=\left[a_{ij}\right]\in\mathbb{C}^{n\times n}$ lies within at least one of the Gershgorin discs. In other words, every eigenvalue λ of matrix $\boldsymbol{A}$ satisfies

$$\forall\lambda,\exists i \quad \left|\lambda-a_{ii}\right|\le P_i \quad ; \quad P_i=\sum\nolimits_{j\neq i}\left|a_{ij}\right| \ . \tag{5}$$

Theorem 2 [15]: Assume that $\boldsymbol{A}$ is an $n\times n$ complex valued matrix with entries a_{ij}, $R_i=\sum_j\left|a_{ij}\right|$ is the summation of the absolute values of all elements in the *i*th row, and $T_j=\sum_i\left|a_{ij}\right|$ is the summation of the absolute values of all elements in the j^{th} column. Let $R=\max_i R_i$ and $T=\max_j T_j$. Then, the absolute value of each eigenvalue λ of matrix $\boldsymbol{A}$ satisfies

$$\forall\lambda, \quad \left|\lambda\right|\le\min(R,T) \ . \tag{6}$$

As mentioned above, in TDOA/FDOA-based one-stage method, the $(i,j)^{\text{th}}$ element of the cross ambiguity matrix (CAM) or $\boldsymbol{Q}$ in equation (3) is the value of CAF between the signals received by sensors *i* and sensor *j* ([9],[11]) and we name it as CAF_{ij}. The emitter location is estimated by computing the maximum eigenvalues of the CAM (or Q) at each grid point. Since the CAM is Hermitian and positive definite, the eigenvalues of CAM are real and positive. Moreover, since CAM is a Hermitian matrix, we have, $R=T$ in Theorem 2 and consequently, $\min(R,T)=R$. Thus, for the CAM, the inequality in (6) can be replaced by

$$\begin{aligned}&\forall\lambda, \quad \lambda\le\max_i(CAF_i)\\ &CAF_i\triangleq\sum_j\left|CAF_{ij}\right|\\ &\hat{\lambda}_{\max}\triangleq\max_i(CAF_i),\end{aligned} \tag{7}$$

where $\hat{\lambda}_{\max}$ is the upper bound on eigenvalues of the CAM.

Suppose that we have *L* receiving sensors and each one of them broadcasts its received signal to all other sensors in the sensor network. Then, each sensor *i* is able to compute all CAF_{ij} for $j=1...L$ and consequently, it is able to compute $CAF_i=\sum_{j=1}^{L}\left|CAF_{ij}\right|$. Now, if we approximate the largest eigenvalue of CAM by the upper bound on the eigenvalues ($\lambda_{\max}\approx\hat{\lambda}_{\max}$), then the location estimation will be determined by the point having the largest $\hat{\lambda}_{\max}$.

Here is the scenario:

1- Each sensor broadcasts its received signal in the sensor network.
2- Each sensor *i* computes CAF_{ij}'s in TDOA/FDOA plane and then maps them from the TDOA-FDOA plane to the X-Y (emitter position) plane. The mapping will be done very easily knowing the position and velocity of the sensors and also the grid point position.
3- Each sensor *i* computes $CAF_i=\sum_{j=1}^{L}\left|CAF_{ij}\right|$ by adding up the CAF_{ij}'s and then finds the peak of CAF_i (named $CAF_{i,peak}$) and its location ($x_{i,peak}$, $y_{i,peak}$) and then transfers the three numbers $x_{i,peak}$, $y_{i,peak}$ and $CAF_{i,peak}$ to a common sensor (or to all other sensors since there are just three numbers and there is no communication load to transfer them). Note that this step is motivated by Gershgorin's Theorem.
4- According to (3) and (7), the emitter location estimate is taken as the ($x_{i,peak}$, $y_{i,peak}$) corresponding to the largest $CAF_{i,peak}$ over all *i*.

Note that in the original DPD method, we need to re-compute and form the matrix CAM (or $\boldsymbol{Q}$) *for each grid point* and find the largest eigenvalue of that matrix each time, which leads to a huge amount of computation especially when the number of receiving sensors gets larger. Moreover, all of these computations would be done at one single point. But, the method outlined above does not need to form the matrix CAM (or $\boldsymbol{Q}$) at all nor does it need to do computationally expensive computations of the largest eigenvalue each time. Thus, in the new method, not only has the costly eigenvalue computation been removed, but also the process is distributed among all receiving sensors. When implementing DPD in a scenario where each sensor has limited computational abilities, it is desirable to minimize the amount of computation done by <u>*each sensor*</u> rather than minimize the total computational complexity. To compare the computational load suppose that there are *L* sensors trying to estimate the emitter location in an $M\times M$ grid plane. In the original DPD method, the common sensor needs to compute the CAM for each grid point. Since CAM is a Hermitian $L\times L$ matrix formed by CAFs, the common sensor just needs to find all the entries on and above the main diagonal. This is equivalent to computing $L(L\text{-}1)/2$ CAFs (as non-diagonal elements) and *L* signal energies (as diagonal elements). Moreover, the common sensor needs to calculate the largest eigenvalue of the matrix for each grid point (N^2 times). On the other hand, in the suggested method, each sensor just needs to find (L-1) CAFs and one signal energy. In addition, they don't need to form the matrix CAM and find its eigenvalues. Thus, rather than having one sensor compute $L(L\text{-}1)/2$ CAFs as in the original DPD, in the method propose here each sensor computes only (L-1) CAFs; furthermore, each sensor performs a simple Gershgorin estimation rather than a complex eigenvalue computation.

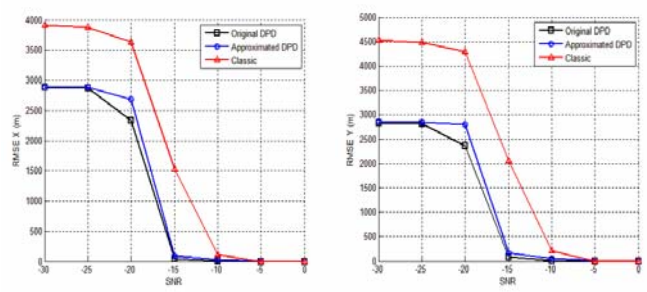
It is worth saying that in the proposed method, if we lose any one of the sensors or even if we lose a couple of them, it may reduce the accuracy of estimation because of missing some data, but the rest of the receivers can continue the estimation process with no interruption.

In the proposed method, if we ignore the maximum operator term in equation (7) and just take $\hat{\lambda}_{\max}=CAF_i$ for only one arbitrary sensor *i*, then the results will be equivalent to a method named CAF-MAP in [6] which has less quality

compared to DPD and Approximated DPD. In [11], we can also see another approach named as *pair-wise maximum CAF detector* that is based on comparing the value of $\max_j \max_{\tau,\omega} \left|CAF_{ij}\right|$ with a threshold γ_{CAF} for only one arbitrary i as reference sensor (τ, ω are TDOA and FDOA). The results in [11] showed that this method also has much lower quality in detection compared to the GLRT detector based on largest eigenvalue; no results were provided in [11] on the location accuracy of the pair-wise maximum CAF method.

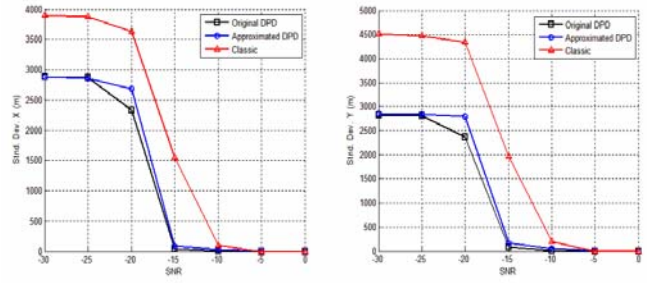
IV. SIMULATION

The simulation results for many different cases show that the eigenvalue upper bound is very close to the true largest eigenvalue. However, this approximation lowers the quality of the estimation slightly. We examined the effect of the proposed approximation on the estimation accuracy using Monte-Carlo computer simulations (with 500 runs each time). In this simulation, we assumed that a set of 8 moving sensors receive the signal from one stationary emitter. There exists a cross ambiguity function for each two of the sensors. The sampling frequency is 80 kHz and the number of samples is equal to 4096. Fig.1 and Fig.2 show the effect of eigenvalue approximation on RMS error and standard deviation of emitter location estimation for X and Y dimensions.



(a) (b)

Fig. 1. RMS errors for X and Y versus SNR.



(a) (b)

Fig. 2. Standard Deviation for X and Y versus SNR.

V. CONCLUSION

The DPD method is a major new development in TDOA/FDOA-based emitter location that provides significant improvement in performance at low SNR levels. However, that improvement comes at a cost of significantly more computational complexity. Worse, as DPD was proposed, that complexity is all concentrated at one computing node, which is different from the classical method where the computations are distributed evenly among the sensors. Furthermore, unlike the classical method, the location processing is highly complex (requiring the computation of eigenvalues for each grid point). The approximate DPD method proposed here exploits the simplicity of Gershgorin's theorem to approximately compute the largest eigenvalue without the high cost of exactly computing it. This enables each sensor to locally make its best estimate of the location based on that data it has. These locally-generated estimates are then transmitted to a central location where a final decision is made. This development allows DPD to be implemented in a decentralized manner where no single sensor is required to do an unfair share of the computations, yet the performance improvement of DPD is not sacrificed.

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