Exploiting Cross Ambiguity Function Properties for Data Compression in Emitter Location Systems

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***Abstract*— In classical emitter location methods, pairs of sensors share the received data to compute the CAF and extract the ML estimates of TDOA/FDOA (time/frequency-difference-of-arrival). The TDOA/FDOA estimates are then transmitted to a common site where they are used to estimate the emitter location. In some recent methods, it has been proposed that rather than sending the TDOA/FDOA estimates, it is better to send the entire CAFs to the common site. Thus, it is desirable to use some methods to compress the data of the CAFs. In this paper, we will derive some beneficial properties and features of CAF that we then exploit to achieve a better CAF compression. Simulation results show that by exploiting these properties it is possible to improve the performance of the compression of CAFs and consequently the performance of location estimation.**

***Index Terms*— Cross Ambiguity Function (CAF), Embedded Zerotree Wavelet (EZW), Frequency Difference Of Arrival (FDOA), Time Difference Of Arrival (TDOA).**

I. INTRODUCTION

One of the most popular and common emitter location methods is based on TDOA/FDOA estimation. In the classical approach to this method, frequency-difference-of-arrival (FDOA) and time-difference-of-arrival (TDOA) are estimated from the cross-correlation of signals received by several pairs of sensors [1]; this is done by computing the cross ambiguity function (CAF) [2] and finding the peak of its magnitude surface. Then these TDOA/FDOA estimates are used in statistical processing to locate the emitter [3]. A challenge in such methods is the need to share large amounts of signal data between paired sensors prior to computation of the CAF for each pair, and has recently been addressed in [4], [5]; note that the subsequent sharing of the TDOA/FDOA estimates requires a very small amount of data transfer. However, new methods have been developed that dispense with explicitly estimating TDOA/FDOA from the CAF and instead share the entire CAFs from the sensors to estimate the location [6] – [9], and thus require the transfer of entire CAFs to a central processing node. The focus of this paper is on methods for compressing a CAF to enable reduction of the data needed to be transferred in these new methods. Some preliminary work in this vein has been presented by the present authors in [10], where the CAF is treated as a complex-valued image and the EZW algorithm [11] is modified to handle complex-valued images.

Under the so-called narrowband approximation the lowpass equivalent (LPE) model of the received signal will be:

$$\hat{s}_r(t) = e^{-jw_c\tau_d}\, e^{-jw_d t}\; \hat{s}(t-\tau_d) \qquad (1)$$

where $\hat{s}(t)$ is the LPE of the transmitted signal, w_d is the Doppler and τ_d is the delay for the received signal [12]. Now, suppose that two sensors Rx1 and Rx2 receive the LPE signals $\hat{s}_{r1}(t)$ and $\hat{s}_{r2}(t)$, respectively. Stein [1] showed that the maximum likelihood (ML) estimate for TDOA and FDOA can be obtained using the magnitude of cross ambiguity Function (CAF):

$$A_{12}(\tau,\omega) = \int_{-\infty}^{+\infty} \hat{s}_{r1}(t)\hat{s}_{r2}{}^{*}(t-\tau)e^{j\omega t}dt \qquad (2)$$

which measures the correlation between $\hat{s}_{r1}(t)$ and a Doppler-shifted by ω and delayed by τ version of $\hat{s}_{r2}(t)$.

Recently, some new methods based on TDOA/FDOA emitter location have been proposed that estimate the emitter location in one stage without extracting the TDOA/FDOA in a separate step. The goal of these methods is to improve the overall accuracy of the emitter location estimate. The main idea of the recent methods is that all pairs of sensors have to share their computed CAFs to each other or they have to send the CAFs to a common site to estimate the emitter location. Thus, there will be a large amount of data transmission and this leads to a need for methods to compress the CAFs. One of the recently proposed methods is named CAF-map method [6]. The main idea of the CAF-map method is to take each CAF magnitude and re-map its delay and Doppler axes into equivalent axes in *x-y* position (assuming location in only 2-D for simplicity). Then, the emitter's location is estimated as the *x-y* location that maximizes the average of all the CAF-map magnitudes [6]. Alternatively, Weiss and Amar [7], [8], [9] developed a single-stage ML method named direct position determination (DPD). The TDOA/FDOA based DPD [9] computes the CAF-map between every possible pairing of sensors. Then, it uses the CAFs to form a series of matrices

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and the location is estimated by computing the maximum eigenvalues of these matrices.

As mentioned before, the CAF is a two-dimensional function. Thus, we can consider the CAF to be an image and apply image compression methods to it. In particular, we use Embedded Zerotree Wavelet (EZW) [11] to compress the CAF. One of the most important reasons that encourage us to use EZW method is that EZW is an embedded algorithm - it attempts to provide a sequence of bits that if truncated anywhere gives the best distortion for that rate. Some simple methods for CAF data compression have been proposed by the present authors in [10]. In that paper, the detailed effects of lossy data compression on CAF and consequently, its effects on location estimation accuracy were assessed. Now, here we try to exploit some special properties of the CAF in data compression to get better results.

II. Properties of CAF for Data Compression

As mentioned above, we can consider the CAF as an image and we can apply image compression methods to compress it. However, not all of the CAF points have the same importance for location estimation, thus it is possible to assign different weights to different CAF points and therefore allocate larger number of data bits to transmit the more significant area, which contains the mainlobe area.

Price and Hofstetter [13] have done detailed research on the bounds of the ambiguity function volume distribution. Wilcox [14] showed that the contour of ambiguity function magnitude close to the peak is always an ellipse. This contour can be formed by the intersection of the mainlobe magnitude and a level plane. It is possible to find the approximate equation of this ellipse in terms of signal bandwidth, signal duration, signal energy and a specific level. The width of this ellipse along the TDOA axis is proportional to the reciprocal of the signal's rms bandwidth; likewise, the width of the ellipse along the FDOA axis is proportional to the reciprocal of signal's rms duration [15]:

$$\Delta\tau \propto (1/B_{rms})$$

$$\Delta\omega \propto (1/T_{rms})$$

where B_{rms} is rms value of signal bandwidth and T_{rms} is rms value of signal duration.

Thus, it is possible to determine the approximate significant area which is more important for the purpose of location estimation. Note that it is always possible to rotate the ambiguity function when it is tilted. An interesting property of ambiguity function is that the new function under the transformation that rotates the $\tau-\omega$ plane through some angle θ will be another ambiguity function. This new ambiguity function is corresponding to the new signals which are related to the old signals and the angle θ [15],[16].

The auto ambiguity function (AAF) (or just ambiguity function as in some papers) is defined as:

$$A_{uu}(\tau,\omega) = \int_{-\infty}^{+\infty} u(t)u^*(t-\tau)e^{j\omega t}dt \tag{3}$$

where $u(t)$ can be the LPE signal. In fact, auto ambiguity function shows the correlation between a signal and a Doppler-shifted by ω and delayed by τ version of itself. It is straight forward to show that AAF has a kind of symmetry around the origin [14], [15], [17].

$$A_{uu}(-\tau,-\omega) = A^*_{uu}(\tau,\omega)\, e^{j\tau\omega} \tag{4}$$

$$|A_{uu}(-\tau,-\omega)| = |A_{uu}(\tau,\omega)|$$

where $A^*_{uu}(\tau,\omega)$ is the complex conjugate of AAF and $|A_{uu}(\tau,\omega)|$ is the magnitude of AAF.

It is also simple to prove a similar property for CAF [15],[17].

$$A_{uv}(-\tau,-\omega) = A^*_{vu}(\tau,\omega)\, e^{j\tau\omega}$$

$$|A_{uv}(-\tau,-\omega)| = |A_{vu}(\tau,\omega)|$$

where $A_{uv}(\tau,\omega)$ is the CAF between signal $u(t)$ and $v(t)$. However, $A_{vu}(\tau,\omega)$ is the CAF between arbitrary signals $v(t)$ and $u(t)$, not specifically related by delay and Doppler to a single transmitted signal. To develop a result that we can exploit for our purpose we explore a similar result for the case when the signals are received from a transmitter. Then equation (1) gives

$$u(t) = e^{-j\omega_c\tau_1}\, e^{-j\omega_1 t}\, \hat{s}(t-\tau_1)$$

$$v(t) = e^{-j\omega_c\tau_2}\, e^{-j\omega_2 t}\, \hat{s}(t-\tau_2)$$

where τ_1 and τ_2 are the time delays and ω_1 and ω_2 are the Doppler shifts for the first and second received signals. Now, we can write one of them in terms of the other one,

$$v(t) = u(t+\tau_p)e^{j\omega_c\tau_p}\; e^{j\omega_p t}\; e^{j\omega_1\tau_p} \tag{5}$$

where $\tau_p = (\tau_1-\tau_2)$ is the TDOA and $\omega_p = (\omega_1-\omega_2)$ is the FDOA.

$$A_{uv}(\tau,\omega) = \int_{-\infty}^{+\infty} u(t)v^*(t-\tau)e^{j\omega t}dt \quad \overset{(5)}{\Rightarrow}$$

$$= e^{j\omega_p\tau - j\omega_c\tau_p - j\omega_1\tau_p} \int_{-\infty}^{+\infty} u(t)u^*\big(t-(-\tau_p+\tau)\big)\, e^{j(\omega-\omega_p)t}dt$$

$$= [e^{j\omega_p\tau - j\omega_c\tau_p - j\omega_1\tau_p}]\, A_{uu}\big(\tau-\tau_p, \omega-\omega_p\big) \tag{6}$$

Thus, the CAF is rewritten in terms of Auto Ambiguity Function. Then, by replacing the τ by $(\tau+\tau_p)$ and ω by $(\omega-\omega_p)$, the following equations are concluded,

$$A_{uv}\big(\tau+\tau_p, \omega+\omega_p\big) = e^{j\omega_p(\tau+\tau_p) - j\omega_c\tau_p - j\omega_1\tau_p}\, A_{uu}(\tau,\omega) \tag{7}$$

$$A^*_{uu}(\tau,\omega) = e^{j\omega_p(\tau+\tau_p)-j\omega_c\tau_p-j\omega_1\tau_p}\, A^*_{uv}(\tau+\tau_p, \omega+\omega_p) \quad (8)$$

Now, by negating the τ and ω in equation (7), we have:

$$A_{uv}(-\tau+\tau_p, -\omega+\omega_p) =$$

$$\overset{(7)}{\Rightarrow} = [e^{j\omega_p(-\tau+\tau_p)-j\omega_c\tau_p-j\omega_1\tau_p}]\, A_{uu}(-\tau,-\omega)$$

$$\overset{(4)}{\Rightarrow} = [e^{j\omega_p(-\tau+\tau_p)-j\omega_c\tau_p-j\omega_1\tau_p}\;\; e^{-j\omega\tau}]\, A^*_{uu}(\tau,\omega) \quad \overset{(8)}{\Rightarrow}$$

$$= [e^{j\omega_p(-\tau+\tau_p)-j\omega_c\tau_p-j\omega_1\tau_p}\;\; e^{-j\omega\tau}\;\; e^{j\omega_p(\tau+\tau_p)-j\omega_c\tau_p-j\omega_1\tau_p}\,]$$

$$.\, A^*_{uv}(\tau+\tau_p, \omega+\omega_p)$$

$$= \;[e^{(j2\omega_p\tau_p-j2\omega_c\tau_p-j2\omega_1\tau_p)}\;\; e^{-j\omega\tau}]\, A^*_{uv}(\tau+\tau_p, \omega+\omega_p)$$

$$= \; e^{-j(\omega\tau+\beta)}\, A^*_{uv}(\tau+\tau_p, \omega+\omega_p)$$

where β is defined as $(2\omega_p\tau_p - 2\omega_c\tau_p - 2\omega_1\tau_p)$ and finally,

$$A_{uv}(-\tau+\tau_p, -\omega+\omega_p) = e^{-j(\omega\tau+\beta)}\, A^*_{uv}(\tau+\tau_p, \omega+\omega_p) \quad (9)$$

$$|\, A_{uv}(-\tau+\tau_p, -\omega+\omega_p)\,| \; = |\, A_{uv}(\tau+\tau_p, \omega+\omega_p)\,| \quad (10)$$

which is the symmetry property we can exploit for data compression. This result provides a kind of symmetry of the CAF around the point (τ_p, ω_p) or the peak of CAF magnitude.

Now, it is possible to exploit this property in data compression. In practice, the received signals $u(t)$ and $v(t)$ are the delayed and Doppler-shifted version of transmitted signal plus noise. This noise perturbs the CAF a little bit from the perfect symmetry.

Thus, we rewrite (10) as,

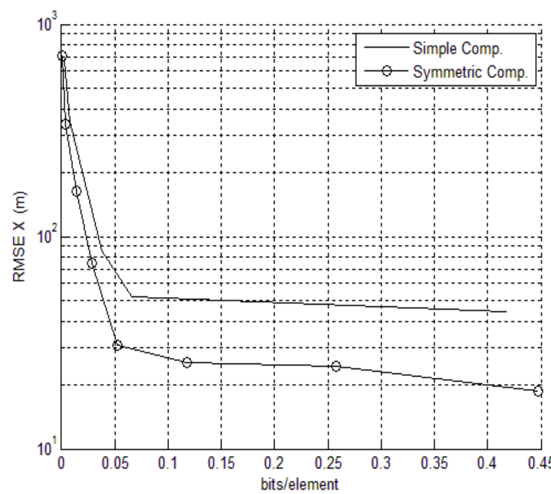
$$|\, A_{uv}(-\tau+\tau_p, -\omega+\omega_p)\,| \; = |\, A_{uv}(\tau+\tau_p, \omega+\omega_p)\,| \; + \; E \quad (11)$$

where E can be the error from perfect symmetry which is a negligible value. Thus, using the symmetry property, it is possible to extract the entire CAF magnitude by transmission of only half of the CAF magnitude plus the small residual amount of E. In this scheme we apply the EZW data compression method on only half of CAF as well as on E.

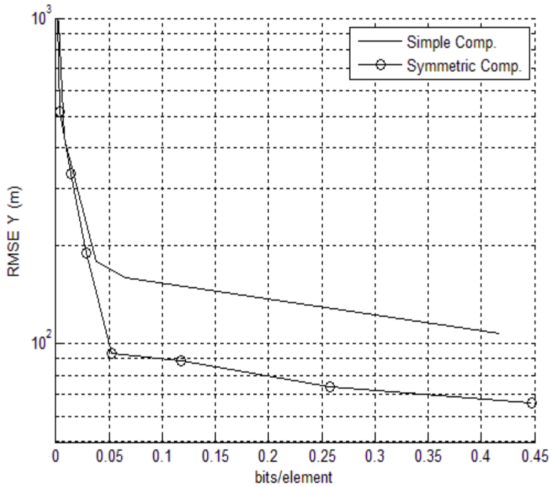
III. SIMULATION RESULTS

In this section we examine the performance of the proposed method and compare the results using Monte Carlo computer simulations (with 100 runs each time). In this simulation, the signals are BPSK, the sampling frequency = 400 kHz, SNR_1 = SNR_2 = -10 dB and the number of samples is equal to 65536. We assumed that 4 pairs of moving sensors receive the signals from one stationary emitter. Thus, there are four cross ambiguity functions which should be computed, compressed and transmitted to a common site to do the location estimation. The de-compressed CAFs were then used in the CAF map method [6] to compute an estimate of the X-Y location of the emitter. Two different compression methods have been examined in this simulation. In the first method, we just applied the EZW algorithm to compress the CAF (labeled "simple compression" in the figures). In the second method, we applied the EZW algorithm to compress the significant area of CAF and we used the symmetry property to reduce the amount of transmitted data (labeled "symmetric compression" in the figures).

The effect of data compression on RMS error of emitter location estimation for X and Y dimensions is illustrated in Fig.1 (a), (b). Also Fig.2 (a), (b) shows the effect of data compression on standard deviation for X and Y dimensions. Obviously, the RMS error and standard deviation will decrease by increasing the bit rate. Comparing the two curves in each plot shows that the symmetric compression method gives us much more accurate results for the same bit rates.



(a)

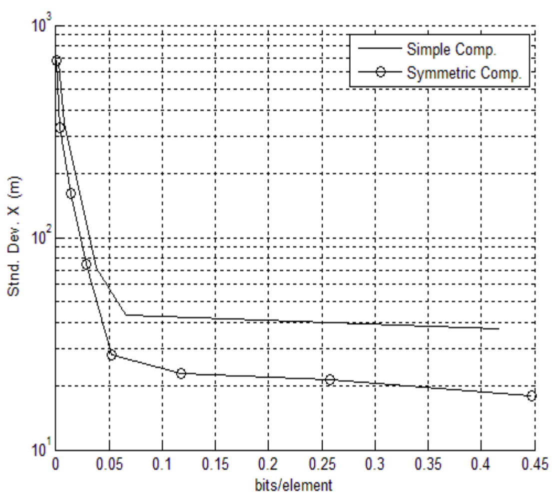


(b)

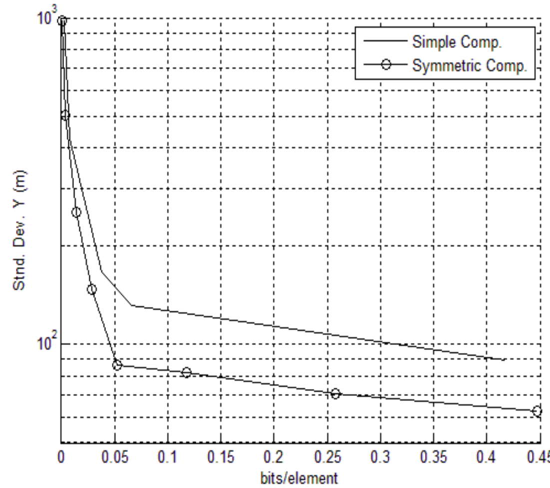
Fig. 1. RMS errors for X and Y versus bits/element.

IV. Conclusion

We applied the Embedded Zerotree Wavelet (EZW) algorithm to compress the two-dimensional CAF to reduce the amount of data which has to be shared between pairs of sensors. In this technique, we have supposed the two-dimensional CAF as an image. We also exploited some of the particular CAF properties and features, like symmetry around the peak and importance of points near the peak. We also focused on the area of the CAF that is more significant for the purpose of geolocation and allocated more bits there to reduce the amount of transmitted data. Finally, the simulations have been done for simple CAF compression and also for compression using the mentioned method. Fig. 1 and 2 shows the RMS error and standard deviation of emitter location estimation for X and Y dimensions for both methods. A comparison indicates that the compression performance is much better for the latter method.



(a)



(b)

Fig. 2. Standard Deviation in dB for X and Y versus bits/element.

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