

Data compression for complex ambiguity function for emitter location

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ABSTRACT

The Complex Ambiguity Function (CAF) used in emitter location measurement is a 2-dimensional complex-valued function of time-difference-of-arrival (TDOA) and frequency-difference-of-arrival (FDOA). In classical TDOA/FDOA systems, pairs of sensors share data (using compression) to compute the CAF, which is then used to estimate the TDOA/FDOA for each pair; the sets of TDOA/FDOA measurements are then transmitted to a common site where they are fused into an emitter location. However, in some recently published methods for improved emitter location methods, it has been proposed that after each pair of sensors computes the CAF it is the entire CAFs that should be shared rather than the extracted TDOA/FDOA estimates. This leads to a need for methods to compress the CAFs. Because a CAF is a 2-D functions it can be thought of as a form of image – albeit, a complex-valued image. We apply and appropriately modify the Embedded Zerotree Wavelet (EZW) to compress the Ambiguity Function. Several techniques are analyzed to exploit the correlation between the imaginary part and real part of Ambiguity Function and comparisons are made between the approaches. The impact of such compression on the overall location accuracy is assessed via simulations.

Keywords: Complex Ambiguity Function (CAF), FDOA, TDOA, Embedded Zerotree Wavelet (EZW)

1. INTRODUCTION

There are several efficient methods to estimate the emitter location based on measuring one or more position-dependent parameters of the received signals. The standard approach used in all these methods is to first estimate these position-dependent parameters from many received signals (or in some methods, *pairs* of received signals) and then use the collection of estimated parameters in a second estimation stage to determine an estimate of the emitter's location [1]. A common emitter location method uses frequency-difference-of-arrival (FDOA) and time-difference-of-arrival (TDOA) estimates made from signals received at several pairs of sensors. In the first step, each pair of receivers estimates the TDOA and FDOA based on their received signals. Then, in the second step, the system estimates the emitter location using the enough number of computed TDOAs and FDOAs. The accuracy of the first step is governed by the Cramer-Rao lower bounds (CRLB) for TDOA and FDOA can be obtained from the following equations [2]:

$$\begin{aligned}\sigma_{FDOA} &\geq \frac{1}{2\pi\sqrt{2T_{rms}}\sqrt{BT \times SNR_{eff}}} \\ \sigma_{TDOA} &\geq \frac{1}{2\pi\sqrt{2B_{rms}}\sqrt{BT \times SNR_{eff}}}\end{aligned}\tag{1}$$

where B is the noise bandwidth at the receiver input, T is the collection time, B_{rms} and T_{rms} are the effective bandwidth and effective duration of the signal, respectively, given by

$$\begin{aligned}B_{rms} &= \left(\frac{\int f^2 |S(f)|^2 df}{\int |S(f)|^2 df} \right)^{1/2} \\ T_{rms} &= \left(\frac{\int t^2 |s(t)|^2 dt}{\int |s(t)|^2 dt} \right)^{1/2}\end{aligned}\tag{2}$$

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and SNR_{eff} is the effective SNR given by

$$\frac{1}{SNR_{eff}} = \frac{1}{SNR_1} + \frac{1}{SNR_2} + \frac{1}{SNR_1 SNR_2}, \quad (3)$$

where SNR_1 and SNR_2 are signal-to-noise ratio for the first and second signals of a given pair of sensors, respectively.

Stein [3] showed that the maximum likelihood (ML) estimate for TDOA and FDOA can be obtained using the Complex Ambiguity Function (CAF)

$$A(\omega, \tau) = \int_0^T \tilde{s}_1(t) \overline{\tilde{s}_2(t + \tau)} e^{-j\omega t} dt, \quad (4)$$

which measures the correlation between \tilde{s}_1 and a delayed by τ and Doppler-shifted by ω version of \tilde{s}_2 . Here, \tilde{s}_1 is the complex lowpass equivalent (LPE) signal (i.e., complex baseband signal or complex pre-envelope) of the signal received by the first sensor and \tilde{s}_2 is the complex LPE signal received by the second sensor. ω is frequency shift and τ is time shift parameters. The CAF is a two-dimensional complex-valued function of TDOA and FDOA and the values of delay τ and Doppler ω that maximize the CAF magnitude are ML estimate for TDOA and FDOA, respectively. In the standard method, after each pair extracts its own TDOA/FDOA estimate and only these TDOA/FDOA estimates from all the sensor pairs are used to estimate the location of the emitter using the nonlinear least-squares approach given in [1].

Recently some new methods for TDOA/FDOA-based emitter location have been proposed that abandon the process of using two stages of estimation. The goal of these methods is to improve the overall accuracy of the emitter location estimate and to avoid some of the challenges in finding the correct peak on the CAF. The first proposed approach of this type is called the CAF-map method [4]; according to [4] it was first proposed in 2002 in unpublished notes and was explored systematically in [4]. The main idea of the CAF-map method is to take each CAF and re-map its delay and Doppler axes into equivalent axes in x - y position (assuming location in only 2-D for simplicity) and then estimate the emitter's location is found to be the x - y location that maximizes the average all the CAF-map magnitudes. For this method there is no claim of optimality, yet there are some advantages over the standard two-stage approach. Weiss [5] and Weiss and Amar [6],[7] developed a single-stage ML method that they called direct position determination (DPD). The TDOA/FDOA version of DPD [7] essentially computes the CAF-map between every possible pairing of sensors and then uses them to form a series of matrices from which the location is estimated by computing the maximum eigenvalues of these matrices. It should be noted that this DPD method uses the complex-valued CAF-maps where as the method in [4] uses only the magnitudes of the computed CAF-maps.

Data compression for the standard method has been explored previously by one of the present authors (Fowler) together with Chen [8],[9] where a TDOA/FDOA-specific transform-coding method was developed to compress the raw signal data that must be transferred prior to computing the CAFs. In the standard method, once compression is used to efficiently transmit the signals for CAF computation, the remaining communication cost is very small since only the extracted TDOA/FDOA locations of each CAF peak must be transmitted to a central node for computation of the emitter location. However, even though in the single-stage methods of [4] and [7], raw signal compression is still needed to transfer the signal data to where the CAF can be computed, there may be a need to compress the CAFs for transmission. Granted, in principle it would be possible to simply send all the raw signal data (using the compression method of [8],[9]) to a single central node where all the required CAFs would be computed; this however, would result in an imbalance in computing requirements – the central node would solely need to compute all the required CAFs, which could be an excessive requirement. Alternatively, CAF computation could be distributed throughout the sensor network to balance the computational load across the network assets; however, that would then require transmission of the CAFs within the network, and with that need comes a desire to apply compression directly to the CAF. In this paper, we are focusing on a method for compressing the entire CAF to perform the geolocation. Although we *are* considering the compression of the *complex* valued CAF, we assess the impact only performance of only the method in [4] that requires

only *magnitudes* of the CAF. Our ongoing work is focused on methods for direct position determination (DPD), where compression of the complex valued CAF will be needed.

2. DATA COMPRESSION FOR CAF

As mentioned above, the CAF is a two-dimensional function. Thus, we can consider the CAF to be an image (although a complex-valued one) and apply image compression methods to the CAF. Even though the CAF is a complex function, we can decompose it into real functions to be able to use two-dimensional compression methods. In particular, we apply Embedded Zerotree Wavelet (EZW) [10] in an appropriately modified way to compress the CAF. One of the most considerable advantages of this algorithm is that EZW is an embedded code; i.e., truncating at any given point gives the lowest distortion representation using the number of bits sent prior to truncation. Thus, it is possible to terminate the compression process at any point which yields the desirable bit rate or distortion [10]. This characteristic is perfect for CAF compression because according to the desirable location estimation accuracy, the bit rate can be adjusted in a very simple manner.

The EZW compression causes two kinds of distortion on the data. The first distortion refers to the effect of data quantization which can be modeled by an additive noise. The second distortion is the result of throwing away the insignificant coefficients of the wavelet transform which can be roughly modeled as passing the data through a lowpass filter. Fig.1 shows a typical CAF; each curve on this plot shows the outline of the CAF along the delay axis at a specific Doppler value. Note that a typical CAF contains a large main lobe and (usually) various small side lobes; an important aspect for compression of the CAF is that it is a relatively slowly changing function over. The fast changing parts (which are equivalent to very high frequency points) come from the effect on the CAF of the additive noise of received signals. Thus, viewed as an image, it seems that the important part to be retained is a spatially low-pass type signal that should show up in the medium and low frequency parts of the wavelet transform used in EZW. Because most of the data concentrated in medium and low frequency parts, it helps to obtain lots of zero tree roots and that is another encouraging reason to use EZW method. Because the high frequency parts of the wavelet transform will contain mostly noise and will be discarded (except at the very highest data rate, when little or no truncation is done) the EZW algorithm will perform a denoising operation; thus, a small amount of compression is likely to actually improve results. In this paper we consider compressing the complex CAF *before* it is converted into the CAF-map form because the region of interest is more compact in the CAF (i.e., around the peak) than is the region of interest in the CAF-map (where the CAF peak is spread out over a wide range of x - y values).

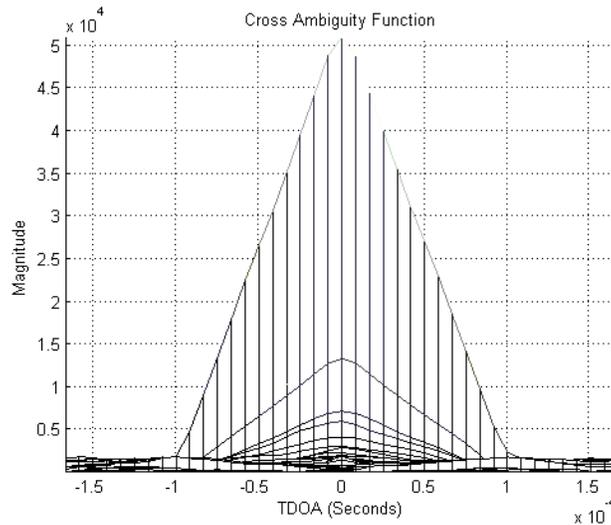


Fig.1. Magnitude of CAF in terms of TDOA; each curve shown represents the shape of the CAF magnitude along delay for a specific Doppler value

Too much compression causes a loss of the medium frequencies and consequently an undesirable effect on the significant peak of the CAF. In Fig. 2 (a), (b) the effects of the EZW compression on the CAF-map for two different compression rates are shown. Fig. 2 (a) shows the magnitude of the CAF-map when the corresponding CAF is

compressed with the rate of 4 (bits/element) and Fig. 2 (b) shows the same CAF-map magnitude when the CAF was compressed with the rate of 38 (bits/element). It is clear from Fig. 2 that the greater bit rate results in a narrower and consequently more accurate locus in terms of emitter location on the CAF-map.

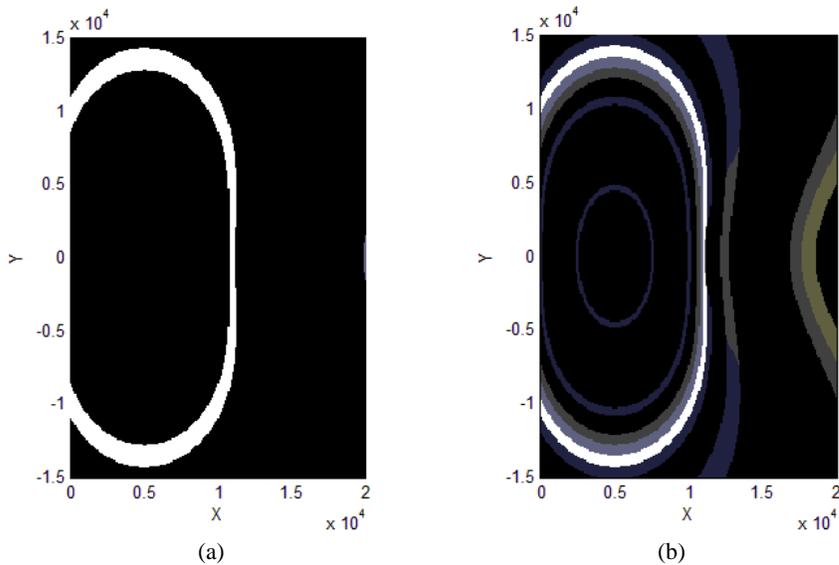


Fig.2. Magnitude of CAF-map in terms of emitter location. (a) CAF-map after CAF was compressed with the rate of 4 bits/element. (b) CAF-map after CAF was compressed with the rate of 38 bits/elements.

In Fig.3 (a), (b) the final results are shown for emitter location estimation after combining five CAF-maps. In these diagrams, the emitter location is estimated using signals collected by five sensor pairs. In Fig.3 (a), the five CAFs are compressed using EZW method with the rate of 4 bits/element, converted to CAF-maps, and then combined; in Fig.3 (b) the same steps were done except there they are compressed with the rate of 38 bits/element. In both Fig. 3 (a) and (b), the brightest area shows the emitter location estimation; the size of the brightest area gives an indication of the accuracy with which the location can be determined. Comparing two diagrams, it is obvious that the emitter location estimation in the Fig.3 (b), corresponding to the larger bit rate, is more accurate. Detailed numerical results for RMS error and standard deviation in terms of bit rate are shown in the following sections.

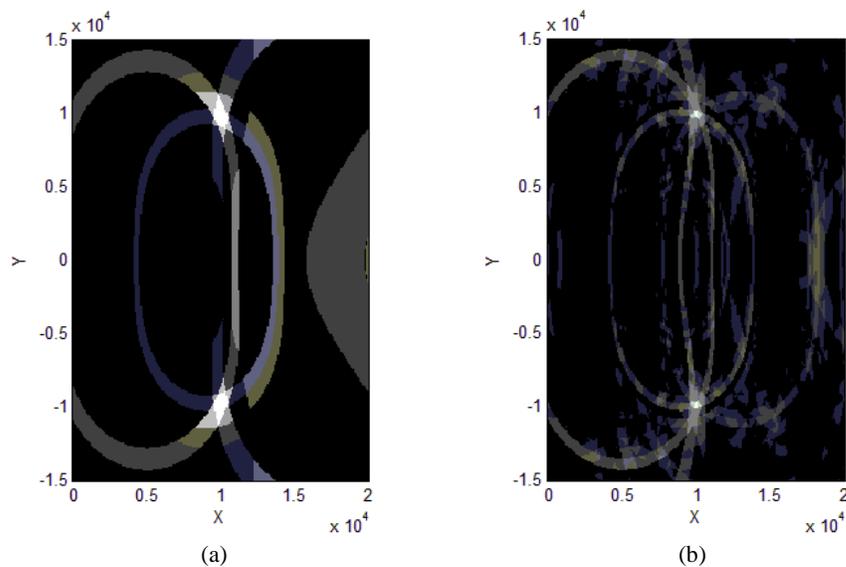


Fig.3. Average of five CAF-maps to provide emitter location estimation at the intersection marked by the brightest spot. (a) CAF's are compressed with the rate of 4 bits/elements. (b) CAF's are compressed with the rate of 38 bits/elements.

To use EZW algorithm to compress the complex CAF, in the simplest form, we can decompose the complex CAF into its real and imaginary parts and use the EZW for each part separately. However, to achieve more efficient compression we investigated several methods to exploit the correlation between real part and imaginary part of CAF.

2.1 Separate R/I Compression Method

In this method, the real part and the imaginary part of the CAF are compressed separately using EZW algorithm. Then each sensor pair sends the compressed CAF to the node that will perform the location estimation where the CAFs are decompressed, formed into CAF-maps, and then averaged to estimate the location. Simulations were run to illustrate the effect of compression and quantization of CAF on emitter location estimation. First, two signals s_1 and s_2 are generated based on the given locations and velocities of the emitter and two collectors. Next, a random additive white Gaussian noise (AWGN) having zero mean is added to each signal.

$$\begin{aligned} \hat{s}_1(k) &= s_1(k) + n_1(k) \\ \hat{s}_2(k) &= s_2(k) + n_2(k) \end{aligned} \tag{5}$$

For each signal, the signal to noise ratio (SNR) is defined as the power of the noise-less signal divided by the power of corresponding noise. Therefore, $SNR1 = P_{s1}/P_{n1}$ and $SNR2 = P_{s2}/P_{n2}$. Then, the complex analytic signals \tilde{s}_1 and \tilde{s}_2 are derived from the band pass signals to compute the Complex Ambiguity Function using the equation (4) and signals \tilde{s}_1 and \tilde{s}_2 . The goal is to evaluate the effect of compression and quantization of CAF on emitter location estimation. In the next step, the computed CAF is compressed with various bit rates using the separate R/I compression method described above and then the bit stream will be decompressed to estimate the location of emitter using the location estimator algorithm. By exploiting the embedded nature of the EZW method it is possible to easily assess the performance at a variety of bit rates.

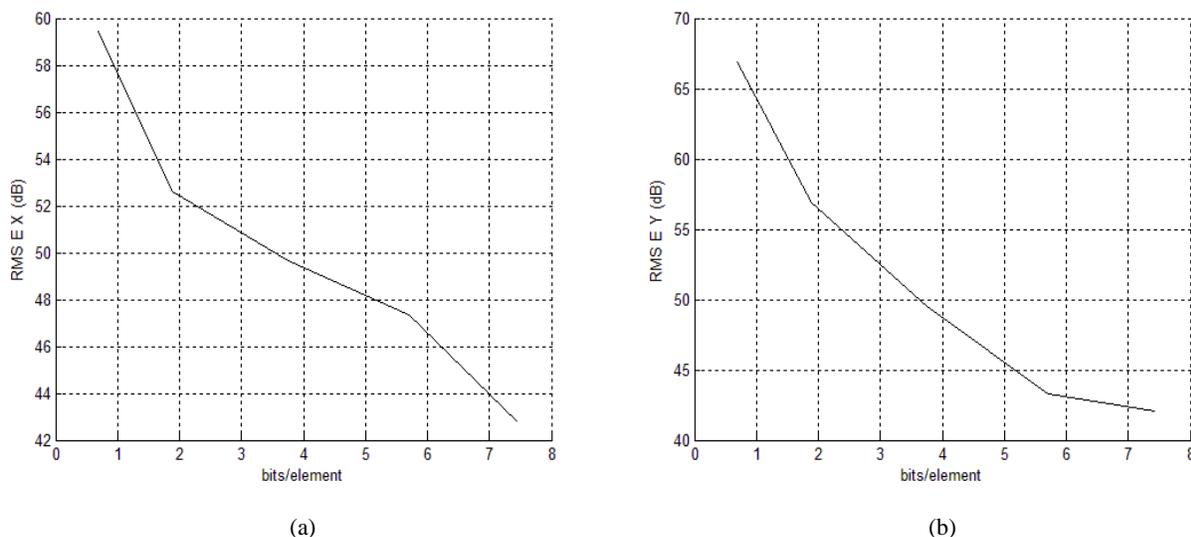


Fig. 4. RMS errors in dB for X and Y versus bits/element for first method: separate R/I compression and CAF 128 doppler cells x 41 delay cells .

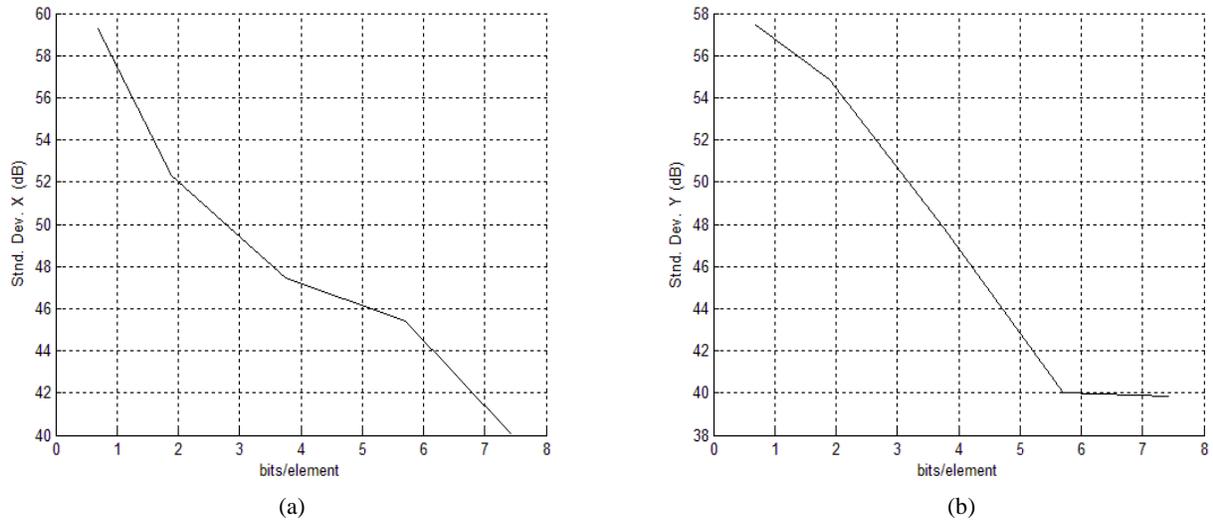


Fig. 5. Standard Deviation in dB for X and Y versus bits/element for first method: separate R/I compression.

In Fig. 4 (a), (b) the RMS errors and in Fig.5 (a), (b) the standard deviations of X and Y dimensions (dB) for different bit rates are shown. In this simulation, the signals are BPSK, the sampling frequency = 120 kHz, $SNR_1 = SNR_2 = 10$ dB, the number of samples is equal to 65536 and the CAF size is 128x41. We assumed two receivers moving on the x-axis with velocity of 100 m/s and distance of 2 km from each other and also one stationary emitter in position of (10km,10km) of x-y plane. Obviously, the RMS error and standard deviation will decrease by increasing the bit rate.

In another simulation, the CAF plane is interpolated by 2 to get more accurate result. Thus, the CAF size is 256x81. The tests are accomplished for various values of SNRs. It is possible to evaluate and compare the effects of received signal noise and compression quantization noise on emitter geolocation. The results are shown in Fig. 6 (a), (b) for RMS error of X and Y dimensions (dB) in terms of bits/element.

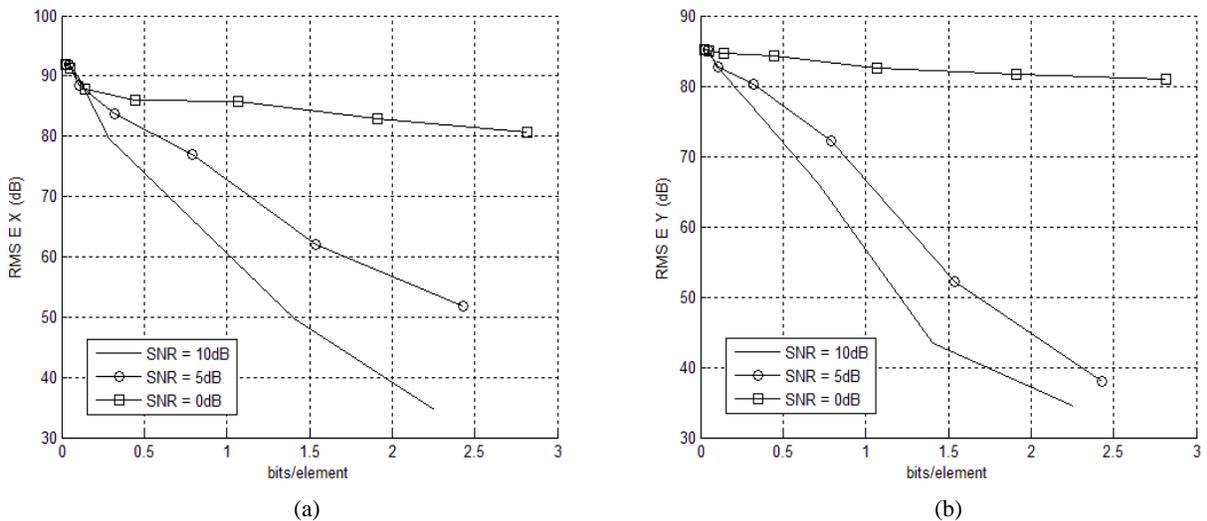


Fig. 6. RMS errors in dB for X and Y versus bits/element for first method (separate R/I compression) and CAF 256x81.

2.2 Combined R/I Compression Method

In the previous method, the real part and the imaginary part of the CAF were compressed using EZW algorithm separately and it has been assumed that the real and imaginary parts are independent. However, there are some correlations between imaginary and real parts and it is possible to exploit this correlation to achieve better results for compression.

In the Combined R/I Method, first, we decompose the CAF into its real part (Matrix R) and imaginary part (Matrix I):

$$\text{CAF} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots \\ a_{21} & a_{22} & a_{23} & \dots \\ a_{31} & a_{32} & a_{33} & \dots \\ \vdots & \vdots & \vdots & \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

$$\text{R} = \text{Real \{CAF\}} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \dots \\ r_{21} & r_{22} & r_{23} & \dots \\ r_{31} & r_{32} & r_{33} & \dots \\ \vdots & \vdots & \vdots & \\ \vdots & \vdots & \vdots & \end{bmatrix} \quad \text{I} = \text{Imaginary \{CAF\}} = \begin{bmatrix} i_{11} & i_{12} & i_{13} & \dots \\ i_{21} & i_{22} & i_{23} & \dots \\ i_{31} & i_{32} & i_{33} & \dots \\ \vdots & \vdots & \vdots & \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

Then, before taking Discrete Wavelet Transform (DWT), we construct a single matrix C by alternately filling its columns with the columns from the real part matrix and the imaginary part matrix:

$$\text{C} = \begin{bmatrix} r_{11} & i_{11} & r_{12} & i_{12} & r_{13} & i_{13} & \dots \\ r_{21} & i_{21} & r_{22} & i_{22} & r_{23} & i_{23} & \dots \\ r_{31} & i_{31} & r_{32} & i_{32} & r_{33} & i_{33} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$

Then we apply the EZW algorithm on Matrix C to compress it. Finally, the bit stream will be sent to emitter location estimator. After decompressing the C by estimator, the Complex Ambiguity Matrix will be reconstructed and used to emitter location estimation. It seems that there are some correlation between each element of matrix R and the corresponding element in matrix I. Thus, by putting them beside each other, some of the sharp variations become moderated. Thus, more low frequency elements and more zero tree roots will be obtained. Moreover, this correlation can help to get better result in lossless compression method used in the last step of EZW.

The same simulation was run to evaluate the effect of the combination, again using sampling frequency of 120 kHz, $SNR_1 = SNR_2 = 10$ dB, 65536 samples of the signal, and a CAF size 128x41. The results are shown in Fig. 7 (a), (b) for RMS errors of X and Y (dB) versus bit rate (bit/element). It is clear that in this method, we got less distortion in comparison with separate R/I compression, for same amount of bit rates. The fact that the performance gets slightly worse at the highest rates is something that needs to be explored further.

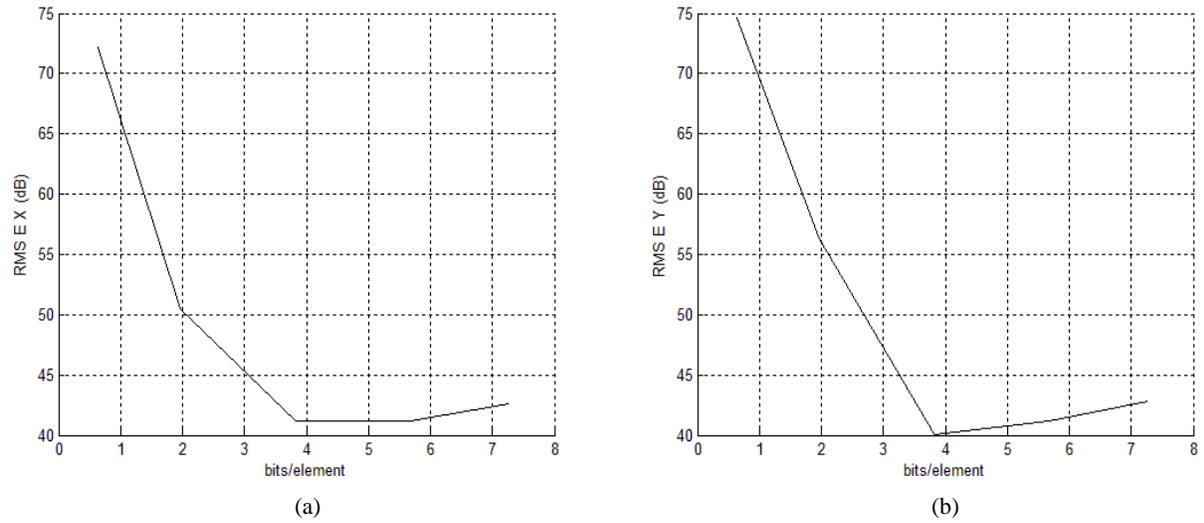


Fig.7. RMS errors in dB for X and Y versus bits/element for second method: combine R/I compression.

2.3 Add-Sub R/I Compression Method

In another method to exploit the correlation between the real and imaginary parts of the CAF, after decomposing the CAF to its real part (Matrix R) and imaginary part (Matrix I), we construct two new matrices. The first matrix is the sum of matrices R and I and is called *Add*. The second one is the result of subtracting I from R, and is called *Sub*:

$$Add = \begin{bmatrix} r_{11} + i_{11} & r_{12} + i_{12} & r_{13} + i_{13} & \dots \\ r_{21} + i_{21} & r_{22} + i_{22} & r_{23} + i_{23} & \dots \\ r_{31} + i_{31} & r_{32} + i_{32} & r_{33} + i_{33} & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$Sub = \begin{bmatrix} r_{11} - i_{11} & r_{12} - i_{12} & r_{13} - i_{13} & \dots \\ r_{21} - i_{21} & r_{22} - i_{22} & r_{23} - i_{23} & \dots \\ r_{31} - i_{31} & r_{32} - i_{32} & r_{33} - i_{33} & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

We then apply the EZW algorithm separately on *Add* and *Sub*. After decompressing the *Add* and *Sub* matrices, the location estimator will reconstruct the CAF and use it to emitter location estimation.

The results of the same simulation for this method are shown in Fig. 8 (a), (b) for RMS errors of X and Y (dB) versus bit rate (bit/element). Again in this method, for the same bit rates, this method gave lower distortion in comparison with separate R/I compression, but it also has the unusual effect that the performance degrades slightly at the highest bit rates.

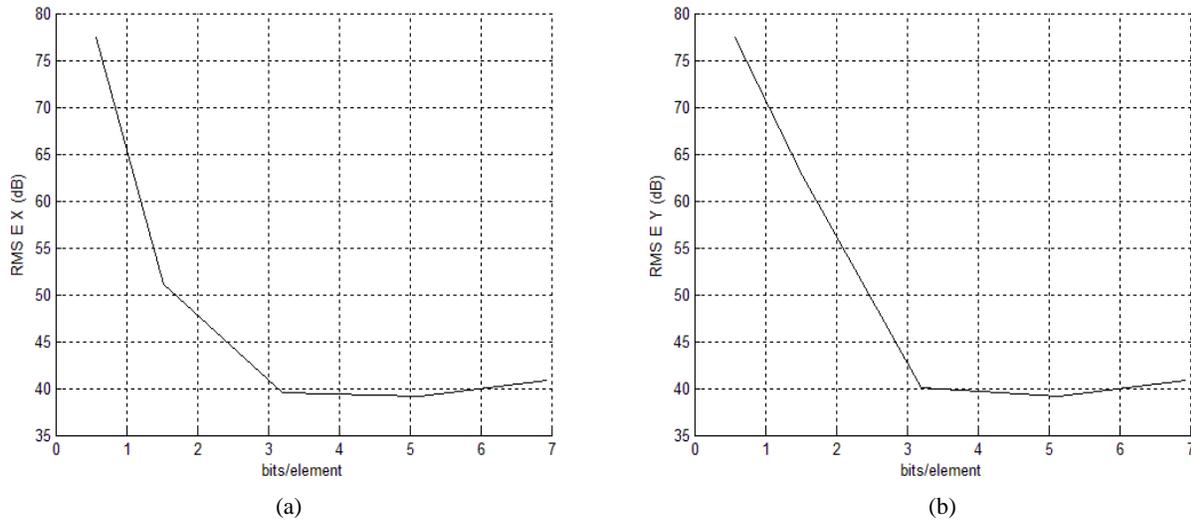


Fig. 8. RMS errors in dB for X and Y versus bits/element for second method: Add-Sub R/I compression.

3. COMPARING THE RESULTS AND CONCLUSION

The Embedded Zerotree Wavelet (EZW) algorithm has been applied to compress the two-dimensional Complex Ambiguity Function (CAF). The CAF is utilized in emitter location estimation methods based on frequency difference of arrival (FDOA) and time difference of arrival (TDOA). In this technique, we have supposed the two-dimensional CAF as a complex valued image. We have decomposed the CAF into its real and imaginary parts and apply three different methods to compress them. In the first method, the real and imaginary parts of CAF have been compressed separately. In the second method, a single new matrix C has been constructed by interleaving the real and imaginary matrices column by column. Finally, in the third method, two new matrices Add and Sub had been built by adding and subtracting the real and imaginary parts of CAF. The final results are shown in Fig. 9 (a), (b) and Fig. 10 (a), (b) for RMS errors and standard deviations of X and Y versus bits/element in individual graphs for three methods. A comparison indicates that for methods two and three (that try to exploit correlation between real and imaginary parts) the compression performance is better, except at the lower bit rates where the first method (that compresses real and imaginary separately) does better.

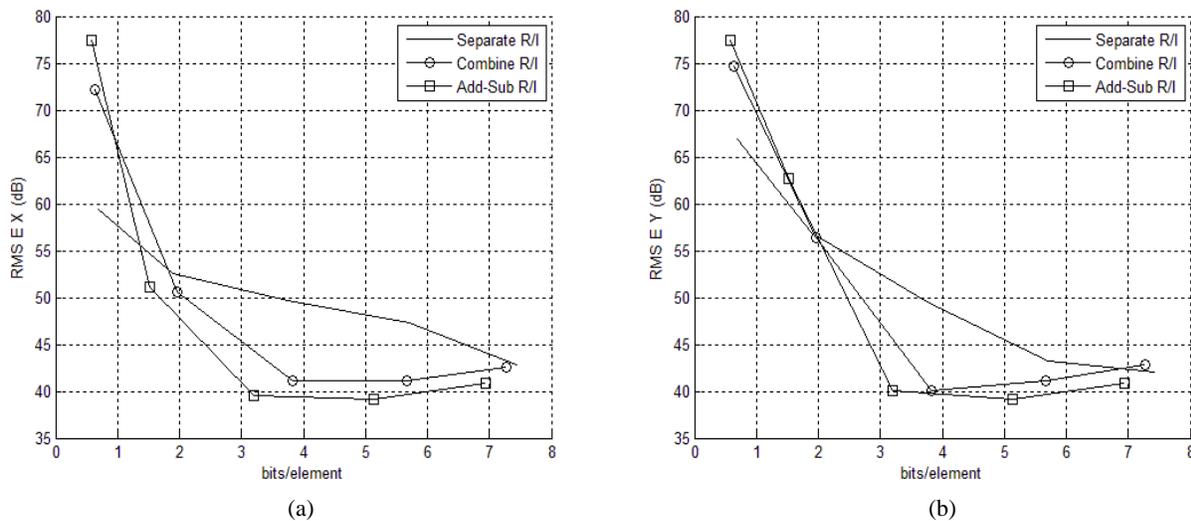


Fig. 9. RMS errors in dB for X and Y versus bits/element for three methods.

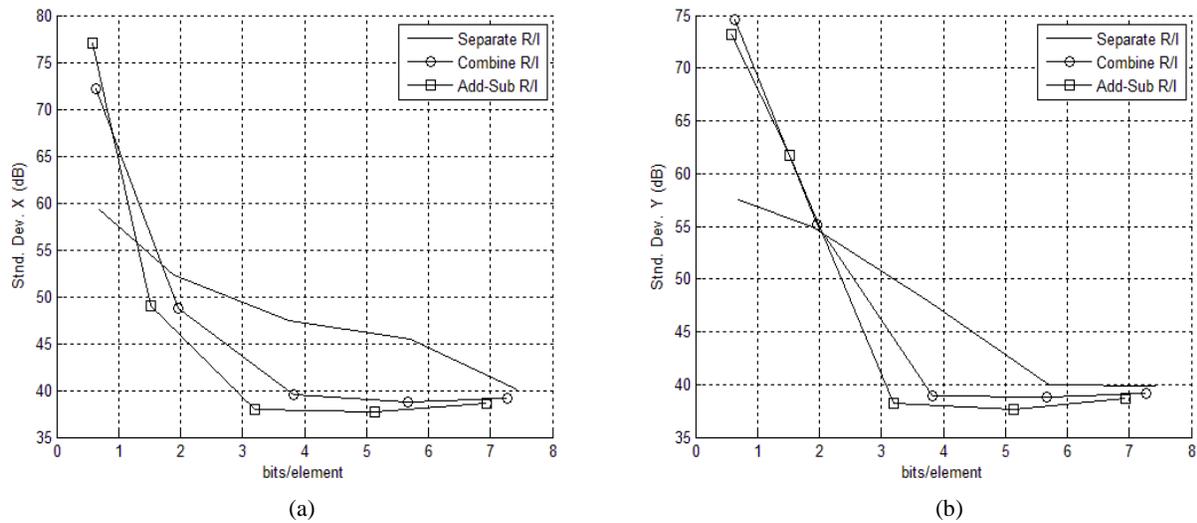


Fig. 10. Standard Deviation in dB for X and Y versus bits/element for three methods.

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