Emitter Location in the Presence of Information Injection

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Abstract—**One sensor network task of particular interest is estimating with maximum accuracy the location of an emitter. In this paper, we focus on the impact of a single *rogue* sensor which injects spurious information into a sensor network in order to maximally degrade location estimation accuracy. Our focus is on understanding and characterizing the impact of such a *rogue* sensor where as on-going work is focusing on methods to mitigate its impact. The goal is to exploit the nature of the shared wireless medium and sensitivity of localization methods to inaccurate sensor positioning. We find the false location that minimizes the accuracy of a sensor network tasked with estimating the location of an emitter. We assume a means for injecting the false location exists and that the network uses a time and frequency difference of arrival (TDOA/FDOA) localization method. We determine the best location to inject by formulating the problem as the minimization of the determinant of the Fisher Information Matrix (FIM). A numerical method for determining the false location is presented and we show that it significantly reduces the location estimate's accuracy independent of sensor-emitter geometry.**

Index Terms—**Emitter Location, TDOA/FDOA, Fisher Information, False Data, Information Injection**

I. Introduction

Advances in sensor technology hold large potential for sensor network based parameter estimation [1]. A collection of sensors makes measurements which are used to estimate an unknown parameter of interest. Of particular interest is the emitter location problem where a collection of sensors is used to estimate the location of an unknown emitter. A commonly used method for estimating emitter location is time and frequency difference of arrival (TDOA/FDOA) [2], [3]. Under the TDOA/FDOA method, sensors are first paired. Next, by cross correlating both sensors' measured signal data each pair computes a TDOA/FDOA estimate. Typically, the signal data is transferred from one of the sensors in the pair to the other over a data link. The TDOA/FDOA estimates of all pairs are then combined to estimate the emitter location.

Since sensor networks communicate using a shared wireless medium, it may be possible for a rogue sensor to infiltrate the network and thus influence the estimation accuracy of the network. Although encryption methods exist and could be used to prevent such unauthorized access, this scenario could still occur and is of interest. For example, encryption methods may be too costly and require too much overhead due to their need for encryption key exchange and distribution [4]. In addition, end-to-end encryption is in general unrealistic for large sensor networks since the number of unique encryption keys necessary is likely to exceed the sensors' storage capacity [4]. To avoid such storage limitations, a hop-by-hop method could be used where only the sensor's nearest neighbor's keys are stored [4]. However, if in this case a sensor was commandeered as a rogue sensor, then encryption would fail for all traffic passing through that sensor [4]. As illustrated, with or without encryption the scenario of a rogue sensor injecting spurious data into a network and subjecting the network to false data is a realistic one and is the focus of this work.

In this paper we focus on examining the impact of a rogue sensor on the location accuracy of a network. In particular, location methods are very sensitive to inaccurate sensor position information. The problem of false location injection is formulated such that it is assumed a false location can be injected into a single sensor thereby corrupting one pair in the emitter location network as in Figure 1. We present a method for determining the best false location that should be injected into a network tasked with estimating an emitter's location

*This work is supported in part by AFOSR LRIR 09RI02COR.

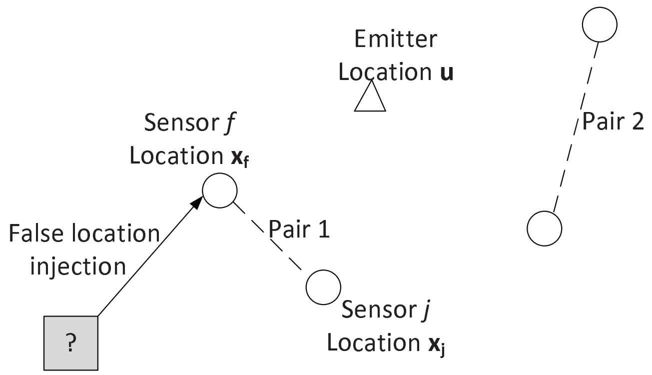


Fig. 1. System Model for False Location Injection: Two sensor pairs seek to estimate the location of an emitter. Sensor f is injected with a false location.

using TDOA/FDOA.

The Fisher Information Matrix (FIM) plays a key role in a wide range of parameter estimation tasks in which the geometry of the sensor network is of particular importance. For example, optimal sensor placement has been considered for various applications in [5], [6] where the determinant of the FIM is maximized. For emitter location estimation, the FIM is used over other distortion measures such as mean squared error because it intrinsically captures the sensor-emitter geometry [7]–[9]. In order to obtain a highly accurate location estimate, the FIM is maximized, where the more information is better. In [7], the trace of the FIM is maximized to find the optimal bit allocation for data compression and the trace of the FIM is maximized in [8] to find the best sensor pairings for location estimation. Similar to these previous works, we choose the FIM as our distortion criteria for the problem of false location injection. We minimize the determinant of the FIM to correctly capture the geometric relationship between the true and false sensor pairings to satisfy the objective of minimizing estimation accuracy.

The main contribution of this work is the formulation of the false location injection problem with the objective of minimizing the location estimation accuracy of a sensor network using TDOA/FDOA. Our focus is on exploring the impact that a single rogue sensor can have on the emitter location estimation accuracy. Ongoing work is addressing the issue of network-processing to mitigate the impact of such a sensor. A numerical method for obtaining the false location which results in lowest accuracy is presented. Further, we show that our approach reduces the estimation accuracy for different sensor-emitter geometries illustrating its robustness.

II. Problem Set-Up: Emitter location Estimation

In this section the system model of the emitter location sensing network and a brief review of TDOA/FDOA methods for emitter location are provided. Given a collection of N sensors the location of a stationary emitter, $\mathbf{u} = [x_e \;\; y_e]^T$ is sought, where x_e and y_e are the x-axis and y-axis locations of the emitter, respectively. For simplicity a two-dimensional scenario is considered. The sensors are paired apriori into $m = 1, \; ..., \; M = \frac{N}{2}$ pairs such that sensors have a constant velocity and no pair shares a common sensor.

TDOA/FDOA methods [2], [3] are commonly used for emitter location. The actual TDOA and FDOA of the m^{th} sensor pair which consists of sensors f and j, are

$$\tau_m = \frac{1}{c}\left(||\mathbf{x}_f - \mathbf{u}|| - ||\mathbf{x}_j - \mathbf{u}||\right) \quad (1)$$

$$\omega_m = \frac{f_e}{c}\left(\frac{(\mathbf{x}_f - \mathbf{u})^T \dot{\mathbf{x}}_f}{||\mathbf{x}_f - \mathbf{u}||} - \frac{(\mathbf{x}_j - \mathbf{u})^T \dot{\mathbf{x}}_j}{||\mathbf{x}_j - \mathbf{u}||}\right) \quad (2)$$

where $\mathbf{x}_f$, $\mathbf{x}_j$ and $\dot{\mathbf{x}}_f$, $\dot{\mathbf{x}}_j$ are the x-y locations and velocities of sensors f and j, respectively. The frequency of the emitter is f_e and c is the speed of light. Each sensor pair makes their TDOA/FDOA estimate, $\hat{\theta}_m = [\hat{\tau}_m \;\; \hat{\omega}_m]^T$ from cross correlating their measured signal data. Typically, the signal data will be transferred from one sensor in the pair to the other over a data link. The measurements are simulated by additive estimation errors

$$\hat{\theta}_m = \begin{bmatrix} \hat{\tau}_m \\ \hat{\omega}_m \end{bmatrix} = \begin{bmatrix} \tau_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} \Delta_{\tau_m} \\ \Delta_{\omega_m} \end{bmatrix} \quad \forall \; m \quad (3)$$

where Δ_{τ_m} and Δ_{ω_m} are the random TDOA/FDOA measurement errors of the m^{th} pair, respectively. The TDOA/FDOA measurements are obtained using the maximum likelihood (ML) estimator [2]. From the asymptotic properties of the ML estimator [10], the distribution of $[\Delta_{\tau_m} \;\; \Delta_{\omega_m}]^T$ is zero-mean Gaussian with covariance matrix $\mathbf{C}_m$ for $m = 1, ..., M$.

In order to assess the location accuracy the FIM [10] is used as the distortion criteria where more information is better [8] and is given by

$$\mathbf{J}_{\text{GEO}} = \mathbf{H}^T_{\text{GEO}}\mathbf{C}^{-1}_{\text{GEO}}\mathbf{H}_{\text{GEO}} \quad (4)$$

$$= \sum_{m=1}^{M} \mathbf{H}_m^T \mathbf{C}_m^{-1} \mathbf{H}_m \quad (5)$$

where $\mathbf{H}_{\text{GEO}} = [\mathbf{H}_1; \ldots; \mathbf{H}_M]$ is the Jacobian of the TDOA/FDOA with respect to the emitter location and $\mathbf{C}_{\text{GEO}} = diag\{\mathbf{C}_1 \ldots \mathbf{C}_M\}$ is the covariance matrix of the noise process that corrupts the TDOA/FDOA measurements. The Jacobian of the m^{th} pair is the derivative of the m^{th} pair's TDOA/FDOA with respect to the emitter location and is

$$\mathbf{H}_m = \frac{\partial \theta_m}{\partial \mathbf{u}} = \begin{bmatrix} \frac{\partial}{\partial \mathbf{u}} (\tau_m) \\ \\ \frac{\partial}{\partial \mathbf{u}} (\omega_m) \end{bmatrix} \tag{6}$$

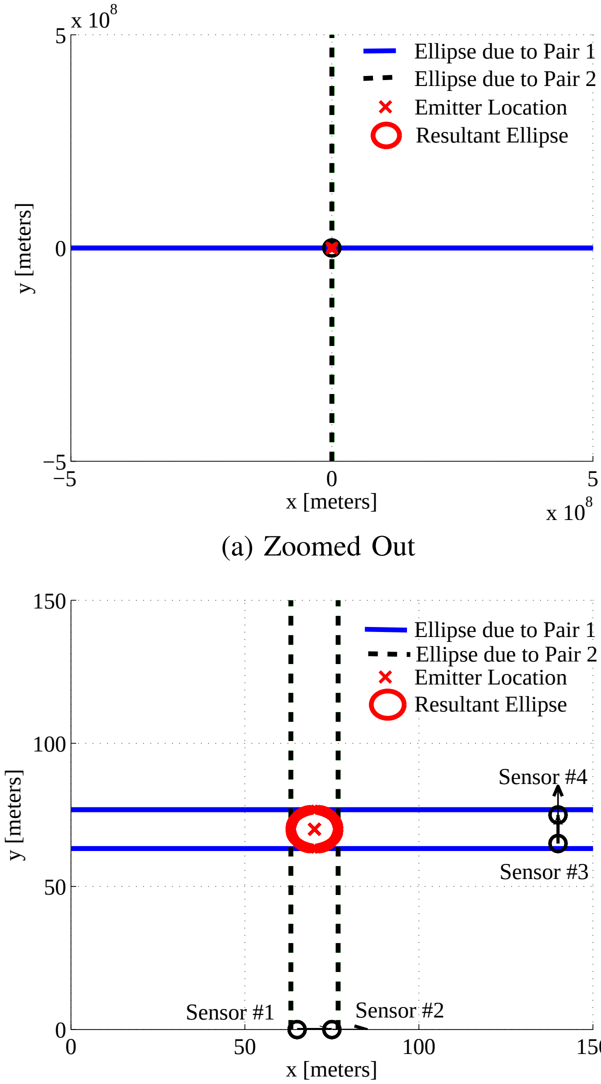
where

$$\frac{\partial (\tau_m)}{\partial \mathbf{u}} = \frac{1}{c} \left[\frac{\mathbf{x}_f - \mathbf{u}}{||\mathbf{x}_f - \mathbf{u}||} - \frac{\mathbf{x}_j - \mathbf{u}}{||\mathbf{x}_j - \mathbf{u}||} \right]^T \tag{7}$$

$$\begin{aligned} \frac{\partial (\omega_m)}{\partial \mathbf{u}} &= \frac{f_e}{c} \left[\frac{[\mathbf{x}_f - \mathbf{u}]^T \dot{\mathbf{x}}_f [\mathbf{x}_f - \mathbf{u}]^T}{||\mathbf{x}_f - \mathbf{u}||^3} - \frac{\dot{\mathbf{x}}_f^T}{||\mathbf{x}_f - \mathbf{u}||} \right] \\ &- \frac{f_e}{c} \left[\frac{[\mathbf{x}_j - \mathbf{u}]^T \dot{\mathbf{x}}_j [\mathbf{x}_j - \mathbf{u}]^T}{||\mathbf{x}_j - \mathbf{u}||^3} - \frac{\dot{\mathbf{x}}_j^T}{||\mathbf{x}_j - \mathbf{u}||} \right]. \end{aligned} \tag{8}$$

Equivalent to maximizing the FIM, the Cramer Rao Lower Bound (CRLB) can be minimized instead since the FIM and the CRLB matrices are inversely related [10].

In order to gain insights into the geometric aspects of this problem, we specify an ellipse showing how the location error is oriented in the x-y plane as in [11]. The ellipse interpretation of the FIM is used where the eigenvectors dictate the major and minor axes of the error ellipse and the reciprocal square roots of the eigenvalues dictate the lengths of the axes. Further, the error ellipse can be decomposed into a set of ellipses, where each ellipse represents an individual sensor pair's contribution. This geometric interpretation is shown in Figure 2 for a specific sensor-emitter geometry. In Figure 2, two pairs of sensors seek to locate the emitter. The error ellipses of each pair's contribution are shown in blue (Pair 1) and black (Pair 2) in Figure 2. Geometrically, the total resultant error ellipse $\mathbf{J}_{\text{GEO}}^{-1}$ is the ellipse inscribed in the intersection of the two individual pairs' ellipses as shown in red in Figure 2. Thus, for a highly accurate location estimate, the total resultant error ellipse should be small and correspond to a large FIM. Conversely, if the goal is to decrease accuracy, the false location should result in a large error ellipse indicating less Fisher Information.



(b) Zoomed In

Fig. 2. Ellipse Interpretation: The error ellipses due to Pair 1 (Sensors 1 & 2) and Pair 2 (Sensors 3 & 4) are shown in blue (solid) and black (dashed), respectively. The total resultant error ellipse (red) is inscribed in the intersection. SNR=10dB and $f_e = 3x10^9$.

With this motivation we now formulate our false location injection problem.

III. False Location Injection

The goal is to find the false location which minimizes the estimation accuracy of the sensor network. We assume a means for injecting a false location exists. Further, only a single false location is injected which corrupts one sensor pair's estimate.

A. Problem Statement

The optimal false location that should be injected which minimizes the location network's FIM is given by

$$\min_{\mathbf{x}_f} \qquad det\left\{\mathbf{J}_{\text{ GEO}}\right\} = det\left\{\mathbf{H}_{\text{GEO}}^T\mathbf{C}_{\text{GEO}}^{-1}\mathbf{H}_{\text{GEO}}\right\} \qquad (9)$$

where the sensor location to be falsified is $\mathbf{x}_f$. Geometrically, the total resultant error should be maximized, or similarly the partial ellipses due to the true and false pairs should have the largest intersection possible. As a result, minimizing the area of the ellipse is an intuitive choice for the objective function. Therefore, we choose the determinant of the FIM as it measures the area of an ellipse [7].

The Jacobian matrix, $\mathbf{H}_{\text{GEO}}$ is a function of the false location, $\mathbf{x}_f$. Since only one sensor is falsified only the corrupt pair's Jacobian matrix changes. To ensure that the non-corrupt pairs' Jacobian matrices do not change, an equality constraint is introduced. Further, since $\mathbf{H}_{\text{GEO}}$ is a function of the false location a change of variables is used to minimize the problem over $\mathbf{H}_{\text{GEO}}$ instead of $\mathbf{x}_f$. Thus, the problem is reformulated as

$$\min_{\mathbf{H}_{\text{GEO}}} \qquad det\left\{\mathbf{H}_{\text{GEO}}^T\mathbf{C}_{\text{GEO}}^{-1}\mathbf{H}_{\text{GEO}}\right\} \qquad (10)$$

$$\text{s.t.} \qquad \mathbf{D}\mathbf{H}_{\text{GEO}} = \mathbf{E} \qquad (11)$$

where both $\mathbf{D}$ and $\mathbf{E}$ are constant matrices specifying the fixed entries of $\mathbf{H}_{\text{GEO}}$. We solve for the problem in (10)-(11) numerically using a grid-based approach.

IV. Numerical Results

The $det(\mathbf{FIM})$ is evaluated over a fine grid, except at the actual emitter location where $\mathbf{x}_f = \mathbf{u}$. Figure 3 shows the value of the $det\left\{\mathbf{H}_{\text{GEO}}^T\mathbf{C}_{\text{GEO}}^{-1}\mathbf{H}_{\text{GEO}}\right\}$ at each grid location on a log scale for the geometry in Figure 2. The location with the minimum value of the $det(\mathbf{FIM})$ is chosen as the false location to be injected. The ten false locations $\mathbf{x}_f$ which yield the smallest values of the $det\left\{\mathbf{H}_{\text{GEO}}^T\mathbf{C}_{\text{GEO}}^{-1}\mathbf{H}_{\text{GEO}}\right\}$ are identified in Figure 3 by an '**X**'.

A. Decreasing Accuracy for a Specific Sensor-Emitter Geometry

The selection of the false location is examined for the sensor-emitter geometry shown in Figure 2. To gain insight into the behavior of the falsified FIM the individual sensor pair's error ellipses are plotted. As shown in Figure 4(a) the location which minimizes the FIM is

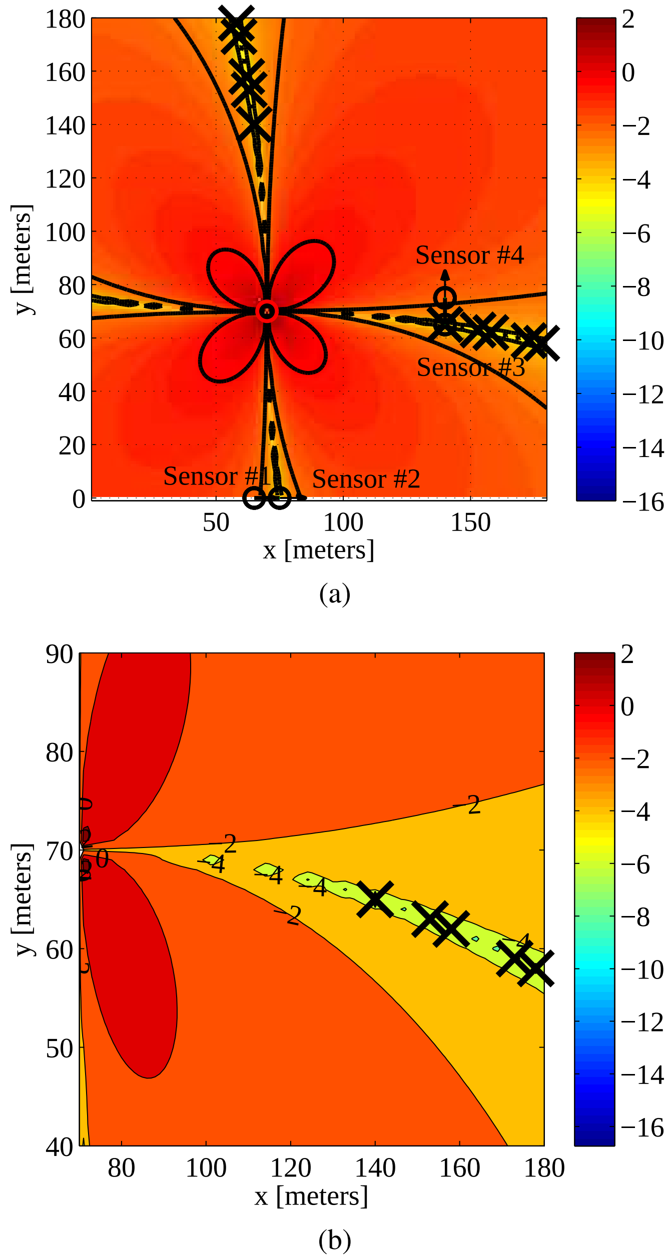
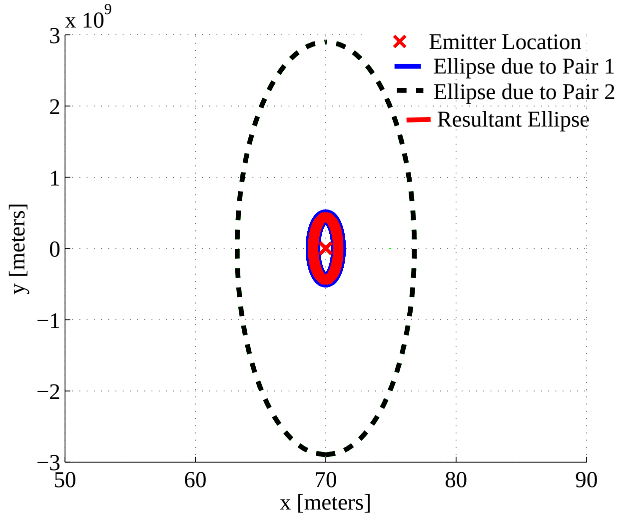


Fig. 3. Evaluation of $det\,(\mathbf{FIM})$ over a grid on a log scale. SNR=10dB and $fe = 3x10^9$ (a) Normal view (b) Close-Up

the one which maximizes the resultant error ellipse, or similarly the false pair's ellipse should have as much area in common with the ellipse of the true pair.

For this geometry, the false pair's ellipse aligns towards the true pair's ellipse as shown in Figure 4. In Figure 5 the ellipses with and without information injection are overlaid. Note that the resultant error ellipse under information injection approaches a line, while the resultant ellipse without information injection constitutes a smaller area indicating the location accuracy has been



(a)

Fig. 4. Injecting a False Location: Sensor 1 is falsified by injecting the false location $[140 \;\; 65]$. The error ellipses due to the false pair (solid blue) and true pair (dashed black) are aligned.

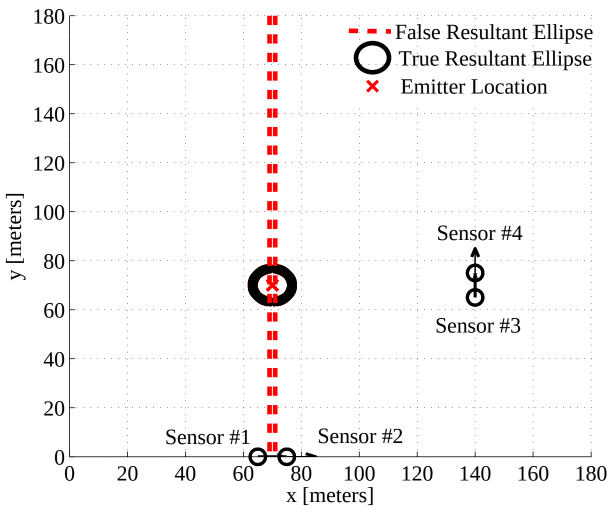


Fig. 5. Comparison of error ellipses with (red dashed) and without (black solid) information injection.

substantially decreased.

B. Decreasing Accuracy across Sensor-Emitter Geometries

Our method is able to decrease emitter location estimation accuracy across sensor-emitter geometries of varying quality as measured by the geometric dilution of precision (GDOP). GDOP indicates the quality of a particular sensor-emitter geometry and is given by $\frac{\sqrt{trace\{\mathbf{J}_{\text{GEO}}\}}}{c\sigma_s}$ where $c\sigma_s$ is the square root of the mean square ranging error [3]. Smaller values of GDOP indicate better location accuracy [12]. For 500 random sensor-emitter geometries uniformly generated in a 200m x 200m field, with values of GDOP ≤ 6, the determinant of the FIM was evaluated over a fine grid. For each geometry, the relative percent error between the $det(\mathbf{FIM})$ with and without false information injection was computed and averaged according to its value of GDOP. In Figure 6, the average relative percent error in the $det(\mathbf{FIM})$ is plotted versus GDOP and shows that our method is able to significantly decrease the location accuracy for both high and low quality geometries.

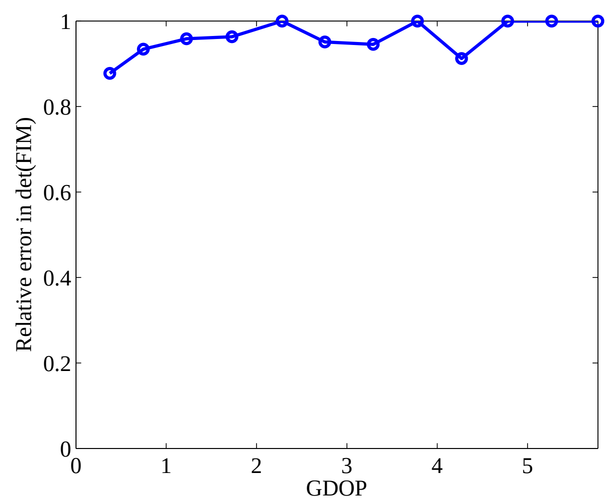


Fig. 6. Relative percent error in location accuracy for different geometries with varying GDOP. SNR=10dB, $c\sigma_s = 1$.

V. Conclusion

This work begins to explore how a rogue sensor with the ability to inject spurious data into a network can influence the estimation accuracy of an emitter location network utilizing TDOA/FDOA. By injecting only a single false location we show that our solution can significantly reduce estimation accuracy for a variety of geometries. By examining the geometry of the individual sensor pairs' error ellipse, we maximize the total falsified error ellipse thereby minimizing location accuracy. We illustrate the appropriateness of our false location solution in terms of the FIM where the less information the better. Consideration of methods which mitigate the effect of a rogue sensor are being investigated in ongoing work.

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