# A LBI Based Emitter Location Estimator with Platform Trajectory Optimality

Ran Ren and Mark L. Fowler
Department of Electrical and Computer Engineering
State University of New York at Binghamton
Binghamton, New York 13902
Email: {rran1, mfowler}@binghamton.edu

***Abstract*—When trying to estimate the location of a non-cooperative non-coherent emitter using intercepted signal measurements from a single airborne platform, the doppler-based techniques, such as the *Frequency of Arrival* (FOA) is not applicable. In this paper, we propose a single platform long baseline interferometry (LBI) based emitter location estimator which achieves optimal estimation accuracy on average without any prior information such as a rough emitter location estimate or a reference point. The novelty of the proposed approach is that it tackles the "phase wrapping" problem inherited in the LBI by exploring the spatial diversity and requiring the platform to fly along a spiral-shaped trajectory. We demonstrate that an arbitrary platform trajectory can be evaluated in terms of the ability of getting accurate estimation using an entropy-based diversity measure. The robustness of the proposed scheme is also explored. This work intends to provide a different angle for single platform emitter location estimation accuracy improvements.**

## I. Long Baseline Interferometry

Emitter location estimation based on the long baseline interferometry (LBI) is a classical technique for finding geolocation of a non-cooperative emitter. A LBI based location estimator calculates phase difference measurements between the received signals from two antennas(apertures) that have been spatially separated on a single platform. The calculated phase differences are then used as data measurements to further estimate the location of the emitter using a Least Squares estimator.

In LBI terminology, the platform that performs the estimation task is called the "baseline"; the baseline length, denoted as $L$, is the distance between two antennas(apertures) on the platform and "long baseline" refers to the case where the two antennas are placed at a distance greater than half of the signal wavelength $\lambda$, i.e., $L \geq \lambda/2$. On the contrary, in the short baseline interferometry (SBI) scenario, two antennas are placed at a distance less than $\lambda/2$.

Unlike other single platform methods such as the Frequency of Arrival (FOA), LBI does not require the emitting signal to have certain frequency and/or timing coherency which makes LBI more widely applicable. However, LBI based methods are generally less accurate because they suffer from the "phase wrapping" effect.

From the classical signal processing knowledge, if two antennas are more than half wavelength apart, ambiguity in the phase difference measurements will be introduced due to the cyclic nature of phase measurements. Higher the emitting signal frequency, severer the ambiguity becomes. Lots of research efforts [1]–[11] focused on removing the ambiguities in order to improve the estimation accuracy of LBI based emitter location methods. Among other ambiguity resolving methods, [4] proposed a self-resolving technique which relies on a grid search over a cost surface followed by an iterative least squares convergence over the local neighborhood of the selected trial grid point. However, the granularity of the grid search satisfying the unimodal assumption on the cost surface was not studied in [4]. Moreoever, as will be shown below, LBI cost surfaces are often characterized by a slim ridge over which the surface is extremely multimodal. Simple grid searching over the entire solution space might not be able to provide adequate estimation accuracy and requires extensive computational overload at the same time.

## II. Estimation Accuracy From the Platform Trajectory Point of View

It has been shown in [12] that relative geometry between the emitter and the platform greatly affects the Cramér-Rao bound of an emitter location estimator. Previous research also demonstrated that in many emitter location estimators the flying trajectory of the platform has a crucial impact on the final estimation accuracy. A trajectory which maximizes the time portion when the platform flies perpendicular to the emitter is most likely to lead to optimal estimate in terms of estimation accuracy. However in practical cases, due to the lack of apriori information about the true emitter location, estimation accuracy suffers dramatic fluctuations as the true emitter location varies. Certain trajectory may triumph when the emitter lies in certain spatial area while performs poorly or even shows inability to estimate when the emitter is located somewhere else. Hence in situations when prior information about the true emitter location is not provided, online trajectory design is impossible in general. A reasonable question to ask instead is, would it be possible to design a universally optimal platform trajectory which maximizes the estimation performance on average without any prior information of the true emitter location?

In order to study the connection between trajectory pattern and estimation accuracy, we use an entropy-based diversity measure to capture the degree of trajectory angular variation which is proportional to the trajectory's ability to obtain accurate estimation on average without prior information about the emitter location. We found out that a universal optimal trajectory which on average maximize the accuracy of the location estimate is an Archimedes spiral one.

### A. Entropy-Based Angular Diversity Measure

The angle of arrival (AoA) of the emitting signal to the platform alone significantly influences the estimation accuracy of the emitter location estimation. Therefore it is heuristically desirable for the designed trajectory to have high angular diversity, i.e. the platform should follow a trajectory which thoroughly explore its spatial neighborhood in order to obtain accurate estimation.

The angular diversity of a trajectory is defined in the form of the entropy of a discrete random variable:

$$D_a = -\sum_i p_\theta^i \log p_\theta^i \tag{1}$$

where $360$ degree is sliced into $s$ angular intervals and $p_\theta^i$, $i = 1, 2, \cdots, s$ is the probability mass evaluated as the number of occurrences in the $i$th interval divided by the total number of measurements of the angle of arrival $\theta$. For a large set of AoA measurements, the angular diversity characterizes the spatial variation of a trajectory. The maximum diversity is achieved when the trajectory demonstrates uniform angular histogram, i.e. $p_\theta^1 = p_\theta^2 = \cdots = p_\theta^s$ in which case a trajectory maximizes the portion flying perpendicular to the target bearing.

### B. Spiral Shaped Trajectory

We claimed above that a trajectory with high angular diversity tends to do a better job in finding accurate location estimates, the goal is to find such trajectories which maximize the angular diversity defined in (1). Obviously to achieve maximum evenness in the histogram of the angle of arrival to the emitting signal, the trajectory must be self-revolving in nature. The circular trajectory however introduces spatial redundancy after the platform flies a close loop and therefore phase difference data collected thereafter becomes redundant and does not contribute to further estimation accuracy improvement.

A trajectory shape which combines evenness in angular distribution and non-overlapping path is the spiral. Circular in nature, every point on a spiral is getting progressively further away as it revolves around the origin. A particular category of spirals called the Archimedean Spirals are spirals defined in polar form as follows [13],

$$r = a\theta^{1/n} \tag{2}$$

where $a$ is a constant that determines the spatial separation between loops, $r$ is the radial distance, $\theta$ is the polar angle and $n$ is a constant that determines the tightness in shape of the spiral. A particular type of the Archimedean spiral with $n = 1$ is called the Archimedes' Spiral. [14] proposed the idea of eliminating the systematic bias in direction finding estimations by a particularly designed trajectory which after theoretical derivation is a logarithmic spiral, but the paper did not address the spiral's impact on overall estimation accuracy. To the best of our knowledge, estimation performance improvement in LBI based emitter location from the aspect of the optimality of the trajectory has not been explored in the literature so far.

The proposed spiral trajectory reduces the amount of redundancy by attaining more spatial diversity, and at the same time, approximately achieves the uniformness in the angular distribution, thus it preserves the optimality in terms of the angular diversity. Therefore, the spiral based trajectory is *optimal on average* because it maximizes the angular diversity defined in (1) thus maximizes the time portion when the trajectory is perpendicular to the bearing angle.

## III. Performance Evaluation

We demonstrate the performance improvements by applying the spiral shaped trajectory compared to three other widely researched counterparts:

1) Sinusoidal Wiggling;
2) Constant Acceleration Turn and
3) Constant Velocity.

In the sinusoidal wiggling case, the platform is designed to fly sinusoidally along the horizontal axis in the 2-D plane with the maximum vertical acceleration $A_max = 3g$ where $g = 9.8m/s^2$. In the case of constant acceleration turn, the platform performs a turn with constant acceleration $A = 3g$. And in constant velocity case, the platform flies along the horizontal axis

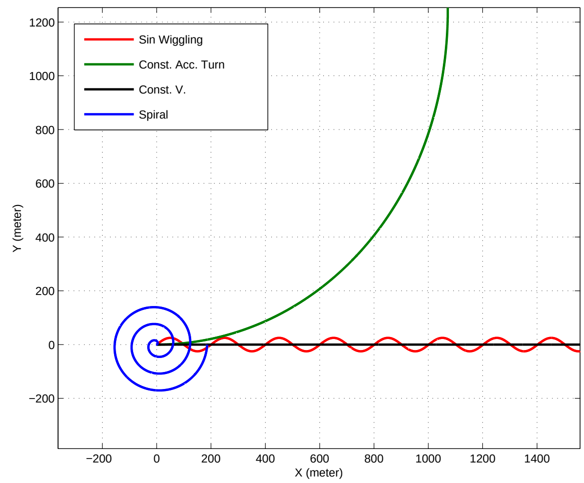



Fig. 1. Four Types of Platform Trajectories

with constant speed $v = 50m/s, A = 0$. Illustrations of the four types of trajectory are shown in Fig. 1. To make the comparisons reasonable, the time duration and trajectory length are same in all 4 scenarios.

### A. Angular Diversity Comparisons

We use the entropy-based measure defined in (1) to calculate the angular diversity. Without loss of generality, we assume an arbitrary *far field* position as the true location of the emitter and then calculate the angle $\alpha$ between the current platform position and a certain point in the coordinate system, commonly the origin, at each trajectory point. By dividing the interval $(0,\ 2\pi)$ into $s = 100$ equal length subintervals, we are able to approximate the discrete probabilistic masses of the subintervals using the frequencies of occurrence from the data. Thus the discrete entropy-based angular diversity can be computed from (1).

In our experiments below, we assume two scenarios where the true emitter locates at $\mathbf{p}_e = (40000/\sqrt{2}, 40000/\sqrt{2})$ and $\mathbf{p}_e = (0, 40000)$ respectively. From (1), the angular diversity is a function of the angles $\alpha$, and therefore the angular diversity is completely determined by the shape of the trajectory, not by its relative position to the emitter. Thus the angular diversity quantities are the same in both scenarios. Angular diversities in the four trajectory cases are shown in Table I.

From Table I, we see that the spiral has the highest angular diversity among all 4 cases in the comparison. The ratios of the diversities in the other 3 cases against that in the spiral case are also shown. Since the location

TABLE I
ANGULAR DIVERSITIES FOR THE 4 TYPES OF TRAJECTORY

| Trajectory Type | Angular Diversity | Ratio Against Spiral |
|---|---|---|
| Sinusoidal | 1.9666 | 0.291 |
| Const. Acc. Turn | 5.0124 | 0.851 |
| Const. Vel. | 0.0068 | 0.102 |
| Spiral | 6.6537 | 1.000 |

estimation accuracy is inversely proportional to the angular diversity, the LBI estimator gives the best accuracy performance among four test cases by flying along the spiral shaped trajectory. On the contrary, the constant velocity scheme performs the worst because the platform acquires the least angular diversity along the trajectory.

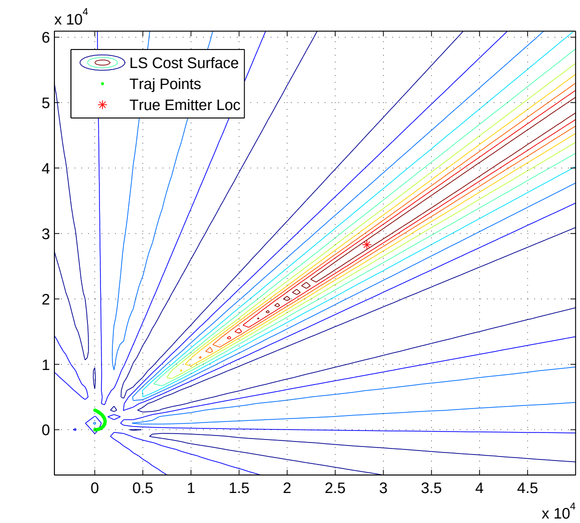
### B. Estimation Accuracy Improvements

Estimation accuracy corresponds visually to the shape and area of the Least Squares surface contours around the true emitter location which can be characterized as the area of the estimation error ellipsoid. [15] shows that the area of the ellipsoid is proportional to the CRB of the estimator which is also proportional to the determinant to the covariance matrix of the estimates. We use the determinant to the estimation covariance matrix as a single-value quantitative accuracy measure.

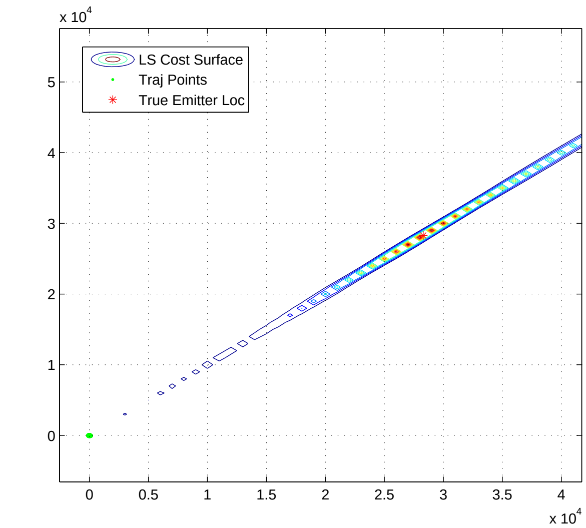
Fig. 2 shows the determinants of the covariance matrices and illustrates the contours of Least Squares cost surfaces in 4 scenarios with different trajectory patterns mentioned above. From the contour plots, we clearly see the surfaces are extremely multimodal and all have slim ridges along the target bearing direction. However, in the spiral trajectory case, the cost surface illustrates a shape peak and is much less rippled outside the neighborhood where the emitter truly resides. The Least Squares cost surface shown in Fig. 2(d) is the most desirable one when grid searching the LS cost as proposed in [4] to find a local neighborhood to apply iterative least squares algorithms on. Moreover longer the platform flies along the trajectory, smaller the area of the ridge in the spiral trajectory case gets which results in more accurate estimate.

### C. Robustness

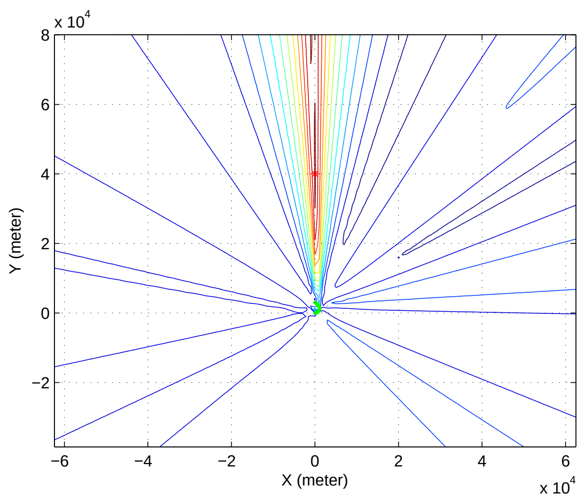
Since the angle of arrival of the emitting signal is critical to the location estimation accuracy, performances on platform trajectories which have clear moving tendencies such as the sinusoidal wiggling, constant acceleration turn and the constant velocity are sensitive to the variation of relative angle $\theta$ between the trajectory origin and the true emitter location. On the other hand, estimation accuracy performance on a spiral shaped trajectory is far



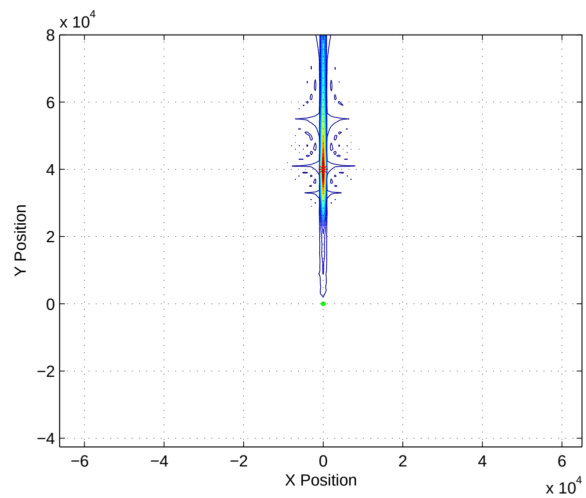

(a) Sinusoidal Wiggling (det(Cov) = $2.5531e + 5$)




(b) Constant Acceleration Turn (det(Cov) = $1.4290e + 5$)




(c) Constant Velocity (det(Cov) = $3.4267e + 5$)




(d) Spiral (det(Cov) = $1.2733e + 3$)

Fig. 2. LS Cost Surfaces for 4 Trajectory Types

more robust to the emitter position variation because spiral trajectory achieves approximate angular fairness, i.e. maximum angular diversity. The geometry and relative angle between trajectory origin and the emitter is shown in Fig. 3.

The simulation result on the robustness in the 4 trajectory cases is shown in Fig. 4 from which the robustness of the proposed spiral trajectory based location estimator is demonstrated. The estimation accuracy which is evaluated as the determinant of the covariance matrix is plotted on the logarithmic scale. Sinusoidal wiggling trajectory achieves its best performance when the emitter locates at $\theta = \pi/2, 3\pi/2$ relative to the origin while constant turn trajectory. The constant acceleration turn and the constant velocity scheme obtains their most accurate results around $\theta = pi/4, 5\pi/4$ and $\theta = k\pi/4, k = odd$ respectively. Notably the spiral trajectory dramatically outperforms the other 3 schemes throughout the entire angular axis. Moreover, the spiral trajectory based estimator shows little performance variation while the other 3 schemes suffer considerable estimation accuracy fluctuation as the relative angle $\theta$ varies. This illustrates

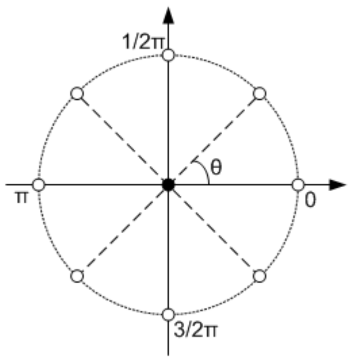



Fig. 3. Relative Angle Between the Trajectory Origin and the True Emitter Location

the robustness of the proposed trajectory pattern.

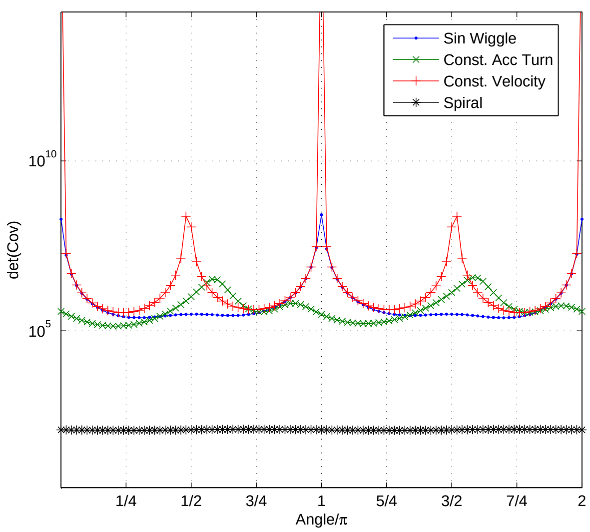



Fig. 4. Estimation accuracy sensitivity to the signal angle of arrival in the 4 cases

## REFERENCES


[1] M. L. Fowler, "Air-to-air passive location system," U.S. Patent 5,870,056, 1999.

[2] A. Kaplan, "Passive ranging method and apparatus," U.S. Patent 4,734,702, 1988.

[3] E. Jacobs and E. W. Ralston, "Ambiguity resolution in interferometry," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-17, no. 6, pp. 766–780, Nov. 1981.

[4] S. V. Czarnecki, J. A. Johnson, C. D. Gray, G. A. VerWys, and C. Gerst, "Self-resolving lbi traiangulation," U.S. Patent 5,835,060, 1998.

[5] C. M. Rose and J. J. Drude, "(aoa/lbi) emitter ranging method and apparatus," U.S. Patent 5,343,212, 1994.

[6] K. A. Struckman, "Moving transmitter correlation interferometer geolocation," U.S. Patent 7,268,728, 2007.

[7] G. B. Guard, "Method and apparatus for ambiguity resolution in rotating interferometer," U.S. Patent 5,189,429, 1993.

[8] E. M. Drogin, "Unambiguous wide baseline interferometer," U.S. Patent 4,638,321, 1987.

[9] J. R. Guerci, R. Goetz, and J. DiModica, "Passive emitter location system," U.S. Patent 5,406,291, 1995.

[10] X. P. Deng, Q. Wang, and D. X. Zhong, "Obervability of airborne passive location system with phase difference measurements," *Chinese Journal of Aeronautics*, vol. 21, pp. 149–154, 2008.

[11] K. R. Sundaram, R. K. Mallik, and U. M. S. Murthy, "Modulo conversion method for estimating the direction of arrival," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 36, no. 4, pp. 1391–1396, July 2000.

[12] K. Becker, "An efficient method of passive emitter location," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 4, pp. 1091–1104, Oct. 1992.

[13] E. W. Weisstein. Archimedean spiral. From MathWorld–A Wolfram Web Resource. [Online]. Available: http://mathworld.wolfram.com/ArchimedeanSpiral.html

[14] P. R. Mahapatra, "Emitter location independent of systematic errors in direction finders," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-16, no. 6, pp. 851–855, Nov. 1980.

[15] X. Hu and M. L. Fowler, "Sensor selection for multiple sensor emitter location systems," in *Proc. IEEE Aerospace Conference'08*, Big Sky, Nevada, Mar. 2008.