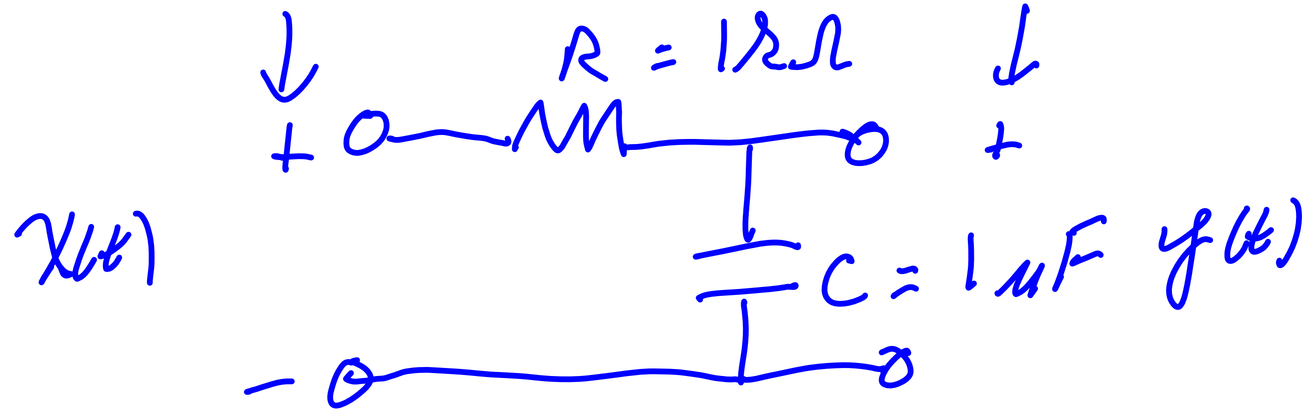


$$x(t) = 2\cos(100t + \pi/3) + 5\cos(10{,}000t)$$

$$y(t) = ??$$

Rotating phasors

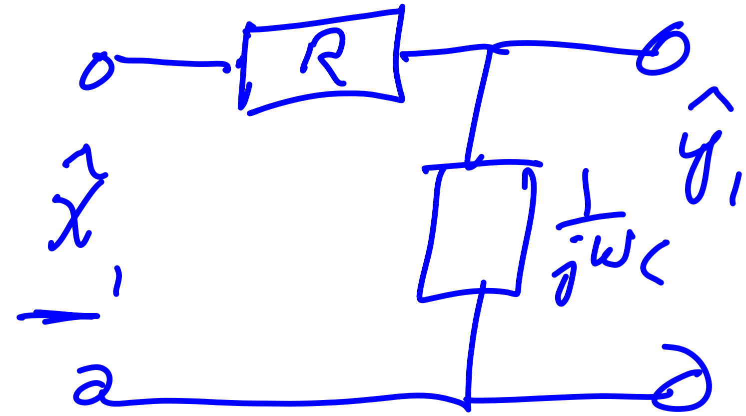
$$\hat{x}(t) = 2e^{j(100t + \pi/3)} + 5e^{j10{,}000t}$$

$$2e^{j\pi/3} e^{j100t} \rightarrow \hat{x}_1$$

$\hat{x}_2$

$\hat{x}_1$ & $\hat{x}_2$ are static phasors

$$\hat{y}_i = \hat{x}_i \underbrace{\left[\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}\right]}_{H(\omega)}$$



Frequency Response

$$H(\omega) = \frac{1}{1+j\omega RC}$$

$$\boxed{RC = 10^{-3}\ \text{sec}}$$

$$H(100) = \frac{1}{1+j(100)10^{-3}} = \frac{1}{1+j10^{-1}}$$

$$H(10{,}000) = \frac{1}{1+j(10^{4})10^{-3}} = \frac{1}{1+j10}$$

$$\hat{y}(t) = \hat{X}_1 H(\omega_1) e^{j100t} + \hat{X}_2 H(\omega_2) e^{j10{,}000t}$$

$$\omega_1 = 100 \qquad \omega_2 = 10{,}000$$

$$= \underbrace{2e^{j\pi/3}\left(\frac{1}{1+j10^{-1}}\right)}_{\text{convert to polar}} e^{j100t}$$

$$+ \underbrace{5\left(\frac{1}{1+j10}\right)}_{\text{convert to polar}} e^{j10{,}000t}$$

$$= \underbrace{A_1 e^{j\phi_1}}_{\text{output static phasor \#1}} e^{j100t} + \underbrace{A_2 e^{j\phi_2}}_{\text{output static Ph. \#2}} e^{j10{,}000t}$$

Find Output static phasor #1

$$2e^{j\pi/3}\left(\frac{1}{1+j10^{-1}}\right) = 2e^{j\pi/3}\left(\frac{1}{\sqrt{1+(10^{-1})^2}}\,e^{-j\tan^{-1}\left(\frac{10^{-1}}{1}\right)}\right)$$

$$= 2e^{j\pi/3} \times \underbrace{0.9950\,e^{-j0.0997}}_{H(100)\text{ in polar form}} = \underline{\underline{1.99\,e^{j0.9475}}}$$

Amount of attenuation @100 rad/sec

Find Output static phasor #2

$$5\left(\frac{1}{1+j10}\right) = 5\left(\frac{1}{\sqrt{1+10^2}}\,e^{-j\tan^{-1}\left(\frac{10}{1}\right)}\right)$$

$$= 5 \times \underbrace{0.0995\,e^{-j1.4711}}_{H(10{,}000)\text{ in polar form}} = \underline{\underline{0.4975\,e^{-j1.4711}}}$$

Amount of attenuation @10,000 rad/sec

So....

$$\hat{y}(t) = \underbrace{A_1 e^{j\phi_1}}\, e^{j100t} + \underbrace{A_2 e^{j\phi_2}}\, e^{j10{,}000t}$$

$$= \underbrace{1.99\, e^{j0.9475}}\, e^{j100t} + \underbrace{0.4975\, e^{-j1.4711}}\, e^{j10{,}000t}$$

Now convert back from phasors to sinusoids

Output

$$y(t) = A_1 \cos(100t + \phi_1) + A_2 \cos(10{,}000t + \phi_2)$$

$$= 1.99 \cos(100t + 0.9475) + 0.4975 \cos(10{,}000t - 1.4711)$$

Negligible change

significant attenuation

Input

$$x(t) = 2\cos(100t + \pi/3) + 5\cos(10{,}000t)$$

The "cut-off" freq. of this RC circuit is $\frac{1}{RC}$ rad/sec

For this case that is 1000 rad/sec

So the 100 rad/sec term is below the cutoff so it is "passed" w/ negligible attenuation

& the 10,000 rad/sec term is above the cutoff so it is "stopped" w/ signif. atten.

We can see how this RC circuit will affect each frequency by plotting

$$|H(\omega)| \text{ vs. } \omega$$

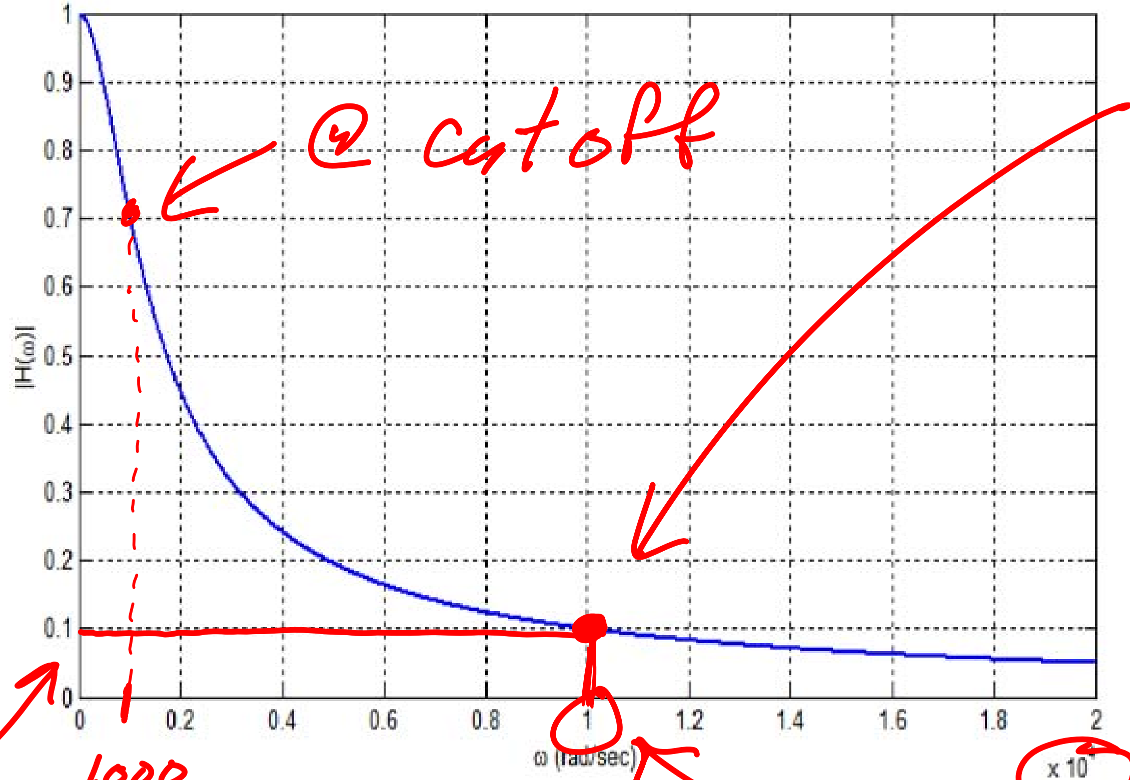
remember... this multiplies the input sinusoid's amplitude

$$|H(\omega)| = \left|\frac{1}{1 + j\omega RC}\right| = \frac{1}{|1 + j\omega RC|}$$

$$= \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

plots on next page for $\frac{1}{RC} = 1000$

@ cutoff



Attn ≈ 0.1

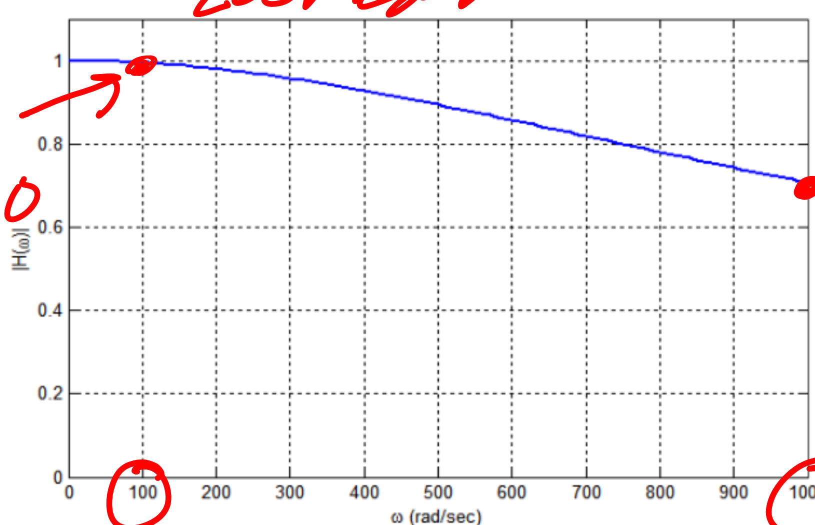
1000

$\omega = 10{,}000$

Aside: Note that @ ω = 10 x cutoff attenuation ≈ 0.1

(See Next Page)

Zoomed:



Attn ≈ 1.0

0.707

$$\boxed{\text{Cut-off @ } \omega = \frac{1}{RC} = 1000}$$

Nice Rules of thumb for RC "Lowpass" Circuit

$$|H(\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\omega_{co} \triangleq \frac{1}{RC} \quad \text{so} \ldots$$

$\boxed{\text{Rule \#1}}$

$$|H(\omega_{co})| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$$\approx 0.707$$

$\boxed{\text{Rule \#2}}$

$$|H(10\omega_{co})| = \frac{1}{\sqrt{1+\left[\left(10\frac{1}{RC}\right)\cdot RC\right]^2}} = \frac{1}{\sqrt{1+100}} = \frac{1}{\sqrt{101}}$$

$$\approx 0.1$$