BINGHAMTON UNIVERSITY
State University of New York

EECE 301
Signals & Systems

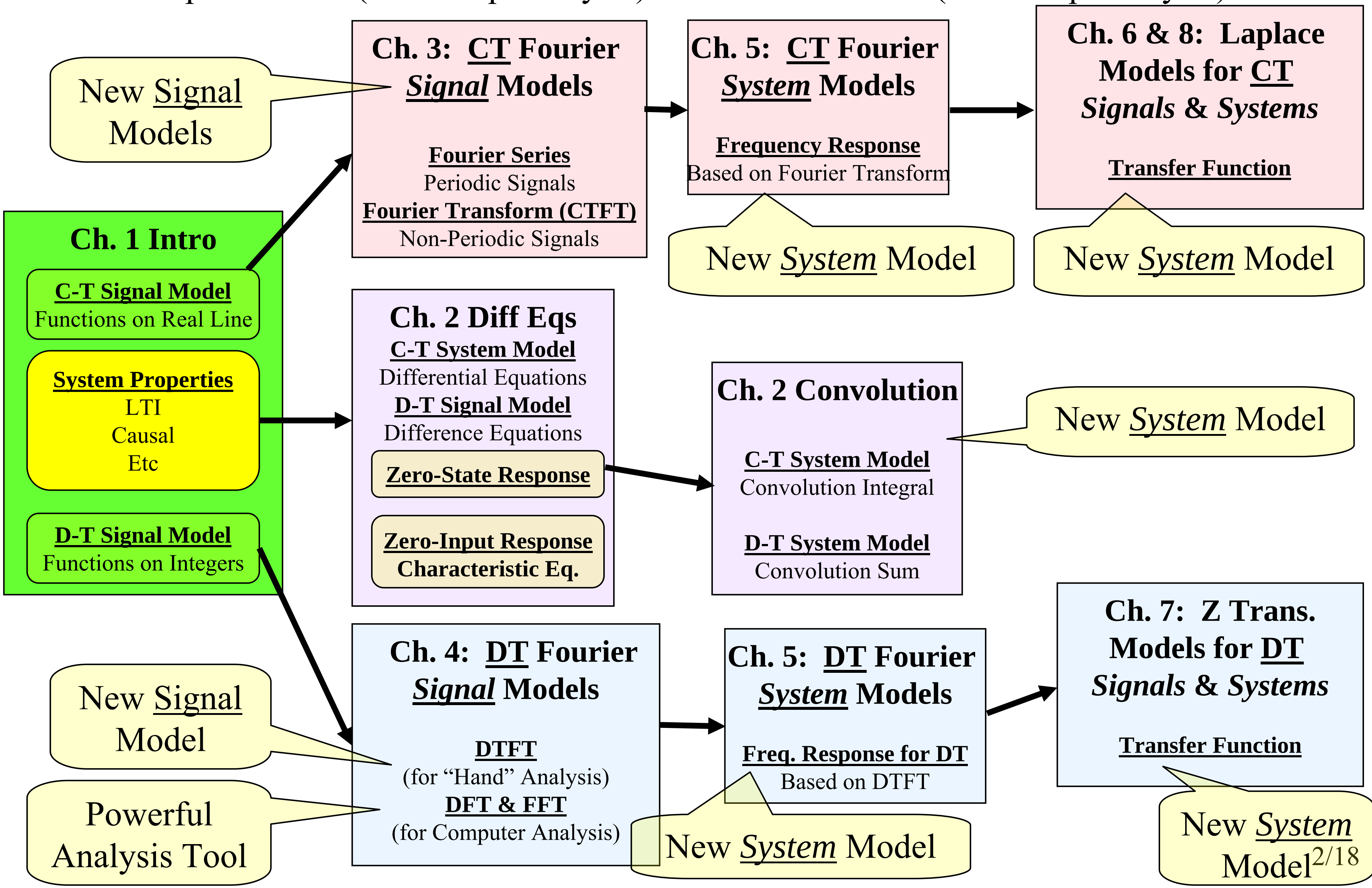
Prof. Mark Fowler

Note Set #4

- Systems and Some Examples
- Reading Assignment: Sections 1.3 & 1.4 of Kamen and Heck

Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).



Systems

- Physically… a system is something that "takes in" one or more input signals and "produces" one or more output signals…
  – Maybe it is a circuit
  – Maybe it is a mechanical thing
  – Maybe it is... ????

Aircraft: -Input: position of control stick

-Output: position of aircraft

Stereo Amplifier: -Input: voltage from CD player

-Output: voltage to speakers

"RF" means "Radio Frequency"

Cell Phone: -Input: RF signal into antenna

-Output: voltage to speaker

Guitar "Effects Box": - Input: voltage from guitar pickup

- Output: voltage (send to amps or another effect)

System Models

- EEs usually think about systems through a variety of related models
- We can represent a physical circuit through a schematic diagram.
- We can represent the schematic as block diagram with a mathematical model…
  - The math model gives a way to quantitatively relate a given mathematical representation of an input signal into a mathematical representation of the output signal

Physical View

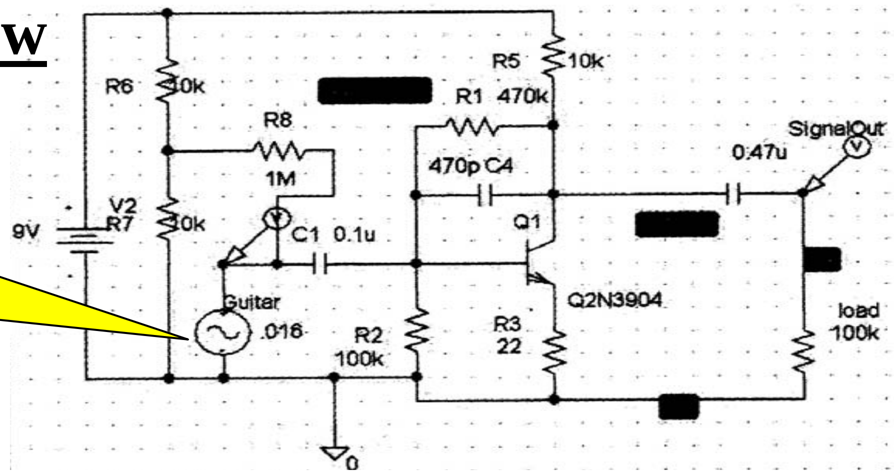
Apply input signal here as a voltage (or a current)

Get output signal here as a voltage (or a current)

Image from **llg.cubic.org/tools/sonyrm/**

Schematic View

Apply guitar signal here as a voltage



Output signal is the voltage across here

From *Pedal Power Column* by Robert Keeley, in Musician's Hotline Magazine

System View

Math Function for Input

$x(t)$ → Math Model of System → $y(t)$

Math Function for Output

Math Models for Systems

- Many physical systems are modeled w/ **Differential Eqs**
  - Because physics shows that electrical (& mechanical!) components often have "V-I Rules" that depend on derivatives

$$\boxed{a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_1 \frac{dx(t)}{dt} + b_0 x(t)}$$

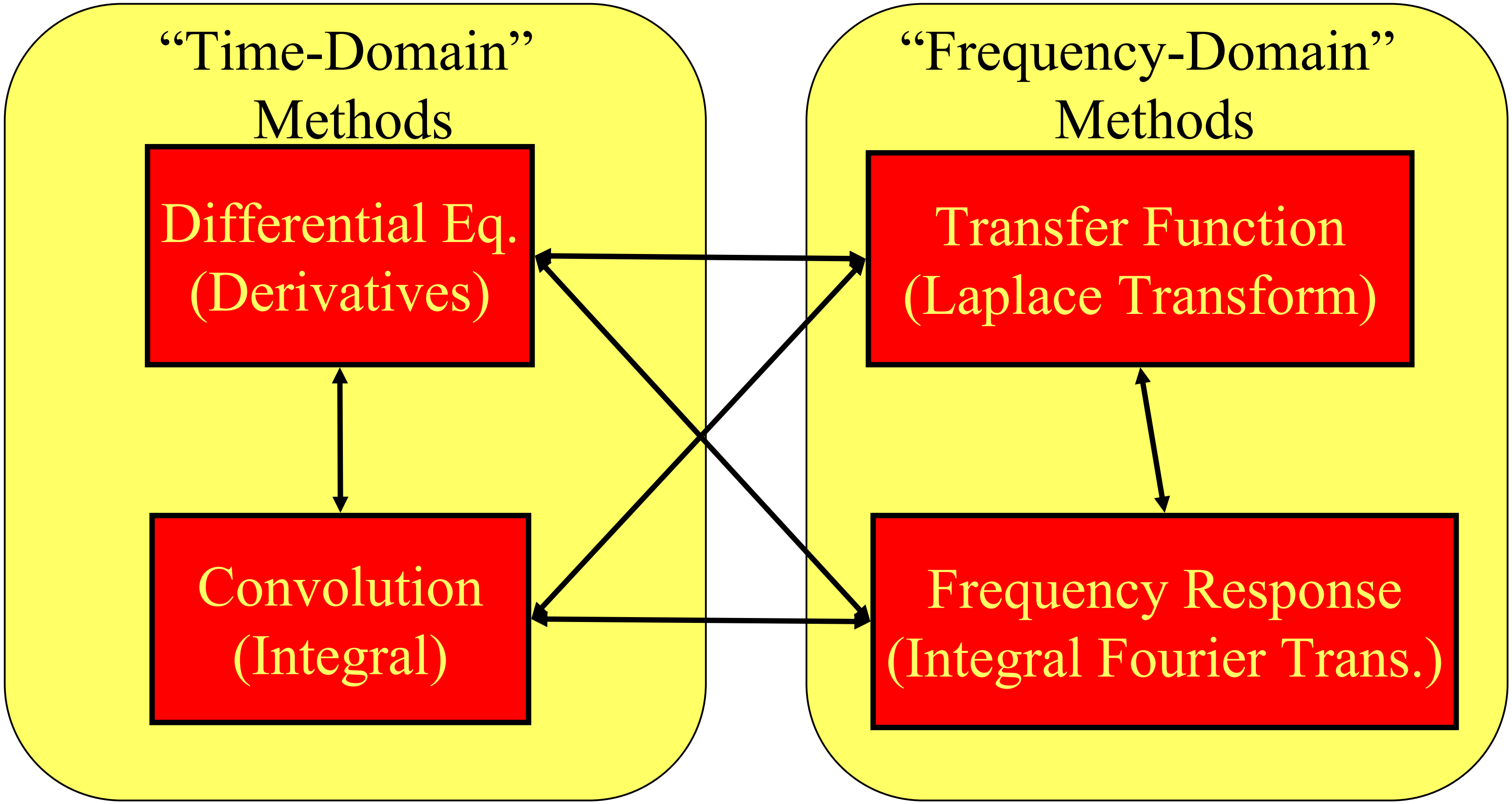
Given: Input $x(t)$
Find: Ouput $y(t)$

This is what it means to "solve" a differential equation!!

- However, engineers use **Other Math Models** to help solve and analyze differential eqs
  - The concept of **"Frequency Response"** and the related concept of **"Transfer Function"** are the most widely used such math models
    - > **"Fourier Transform"** is the math tool underlying Frequency Response
  - Another helpful math model is called **"Convolution"**

Relationships Between System Models

- These 4 models all are equivalent
- ….but one or another may be easier to apply to a given problem



1.4 Examples of Systems

1.4.1 Example System: RC Circuit (C-T System)

A simple C-T system

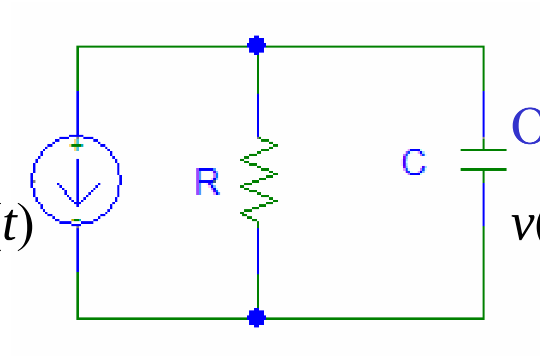
You've seen in Circuits Class that *R*, *L*, *C* circuits are modeled by Differential Equations:

- From Physical Circuit… get schematic
- From Schematic write circuit equations… get Differential Equation
- Solve Differential Equation for specific input… get specific output

“Schematic View”:

Input

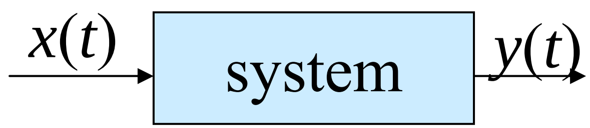
$x(t) = i(t)$



Output

$v(t) = y(t)$

“System View”:



Circuits class showed how to model this physical system mathematically:

$$\boxed{\frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{C} x(t)}$$

Given input $x(t)$, the **output** $y(t)$ is the **solution** to the differential equation.

Recall “RC time constant”

- Consider that the input "starts at $t = t_0$":

(i.e. $x(t) = 0 \quad \text{for} \quad t < t_0$)

- Let $y(t_0)$ be the output voltage when the input is first applied (initial condition)
- Then, the solution of the differential equation gives the output as:

$$y(t) = \underbrace{y(t_0)e^{-(1/RC)(t-t_0)}}_{\substack{\text{Part due to Initial Condition} \\ \text{("Zero Input Response")}}} + \underbrace{\int_{t_0}^{t} \frac{1}{C} e^{-(1/RC)(t-\lambda)} x(\lambda) d\lambda}_{\substack{\text{Part due to Input} \\ \text{("Zero State Response")}}}$$

Part due to Initial Condition
(**"Zero Input Response"**)

Part due to Input
(**"Zero State Response"**)

Recall: This part is the solution to the "Homogeneous Differential Equation"

1. Set input $x(t) = 0$
2. Find characteristic polynomial (Here it is $\lambda + 1/RC$)
3. Find all roots of characteristic polynomial: λ_i (Here there is only one)
4. Form homogeneous solution from linear combination of the $\exp\{\lambda_i(t-t_o)\}$
5. Find constants that satisfy the initial conditions (Here it is $y(t_0)$)

In this course we focus on finding the <u>zero-state</u> response (I.C.'s = 0)

$$y_{zs}(t) = \int_{t_0}^{t} \underbrace{\frac{1}{C} e^{-\frac{1}{RC}(t-\lambda)}}_{\triangleq h(t-\lambda)} x(\lambda) d\lambda$$

$$\boxed{y_{zs}(t) = \int_{t_0}^{t} h(t-\lambda) x(\lambda) d\lambda}$$

General form for so-called "linear, constant-coefficient differential equations"

Ch. 3 will look at this general form…
It's called "convolution"

Big picture:

Nature is filled with "Derivative Rules"

- **Capacitor and Inductor i-v Relationships**
- **Force, Mass and Acceleration Relationships**
- **Etc.**

That leads to Differential Equations

$\Rightarrow$There are a lot of practical C-T systems that can be modeled by differential equations.

Other Examples of C-T Systems

-Car on level surface

-Mass-Spring-Damper System

-Simple Pendulum

D-T System Example

Recall: We are mostly interested in D-T systems that arise in computer processing of signals collected by sensors.

However, we illustrate with a common financial system that is D-T. This provides a simple example from a familiar scenario.

Let $x[n]$, $n = 1, 2, 3, \ldots$ be a sequence of monthly loan payments

Input

D-T signal because you are not continuously paying!

Let $y[n]$ be the balance after the n^{th} month's payment.

Output

Initial condition: $y[0]$ = amount of loan

Let I be the annual interest rate… so $I/12$ = monthly rate

Now, after 1 month the New Balance is:

$$y[1] = \underbrace{y[0]}_{\text{Old Balance}} + \underbrace{\frac{I}{12}y[0]}_{\text{Increase Due to Interest}} \underbrace{-\,x[1]}_{\text{Reduction Due to Payment}} = \left(1+\frac{I}{12}\right)y[0] - x[1]$$

<u>In general</u>: $$\underbrace{y[n]}_{\text{Balance after } n \text{ Months}} = \left(1+\frac{I}{12}\right)\underbrace{y[n-1]}_{\text{Balance after } n\text{-1 Months}} - x[n]$$

<u>Can Re-Write as</u>: $$\underbrace{y[n] - \left(1+\frac{I}{12}\right)y[n-1]}_{\text{“difference” } \Rightarrow \text{ “Difference Equation”}} = -x[n]$$

Difference equations are easily computed recursively on a computer:

Pg. 35 from Textbook's 2nd edition

```
% Loan Balance program
% Program computes loan balance y[n]
y0 = input ('Amount of loan ');
I = input ('Yearly Interest rate ');
c = input ('Monthly loan payment ');     % x[n] = c
y = [];          % defines y as an empty vector
y(1) = (1 + (I/12))*y0 - c;
for n=2:360
   y(n) = (1 + (I/12))*y(n-1) - c;
   if y(n) < 0, break, end
end
```

Figure 1.31 MATLAB program for computing loan balance.

TABLE 1.1 LOAN BALANCE WITH $200 MONTHLY PAYMENTS

$x[n] = 200\,u[n]$

n	$y[n]$	n	$y[n]$
1	$5859.99	19	$3086.47
2	5718.59	20	2917.33
3	5575.78	21	2746.51
4	5431.54	22	2573.97
5	5285.85	23	2399.71
6	5138.71	24	2223.71
7	4990.1	25	2045.95
8	4840	26	1866.41
9	4688.4	27	1685.07
10	4535.29	28	1501.92
11	4380.64	29	1316.94
12	4224.44	30	1130.11
13	4066.69	31	941.41
14	3907.36	32	750.83
15	3746.43	33	558.33
16	3583.89	34	363.92
17	3419.73	35	167.56
18	3253.93	36	−30.77

TABLE 1.2 LOAN BALANCE WITH $300 MONTHLY PAYMENTS

$x[n] = 300\,u[n]$

n	$y[n]$	n	$y[n]$
1	$5759.99	13	$2685.76
2	5517.59	14	2412.61
3	5272.77	15	2136.74
4	5025.5	16	1858.11
5	4775.75	17	1576.69
6	4523.51	18	1292.46
7	4268.75	19	1005.38
8	4011.43	20	715.43
9	3751.55	21	422.59
10	3489.06	22	126.81
11	3223.95	23	−171.92
12	2956.19		

The book show~~s (see Eq. 1.43)~~ed that the solution for the loan balance has an explicit form (“closed form”):

$$y[n] = \underbrace{(1+\frac{I}{12})^n y[0]}_{\text{Due to I.C. (Zero-Input Response)}} - \underbrace{\sum_{i=1}^{n}(1+\frac{I}{12})^{n-i} x[i]}_{\text{Due to Input (Zero-State Response)}}, \quad n = 1,2,3,\ldots$$

Can be found using “characteristic polynomial” methods similar to those used for Differential Equations

$$y_{ZS}[n] = \sum_{i=0}^{n} h[n-i]x[i]$$

Compare to C-T:

$$y_{ZS}(t) = \int_{t_0}^{t} h(t-\lambda)x(\lambda)d\lambda$$

“Input-Output” Relationships

The textbook shows another example of a DT system (sect. 1.4.3) but doesn't discuss it as a Difference Equation.

Instead it expresses the example system as:

$$y[n] = \tfrac{1}{3}\big(x[n] + x[n-1] + x[n-2]\big)$$

Called a "Moving Average"

Notice that a Difference Eq gives an implicit relationship between input and output (i.e., you need to "solve" it to find the output)…

But <u>this</u> example shows an explicit relationship (writes the output as a direct function of the input)

Note that we can write the example as $y[n] = \sum_{i=0}^{2} \tfrac{1}{3} x[n-i]$

which looks a lot like what we saw for the Difference Eq example:

$$y_{ZS}[n] = \sum_{i=0}^{n} h[n-i]x[i]$$

BIG PICTURE

- Physical (nature!) systems are modeled by <u>differential equations</u>
(C-T Systems)

- D-T systems are modeled by <u>difference equations</u>

- Both C-T & D-T systems (at least a large subset) are solved by:
- **- Characteristic polynomial methods for ZI Response &**
- **- Integral/Summation In-Out relationship for ZS Response**