BINGHAMTON UNIVERSITY

State University of New York

EECE 301
Signals & Systems
Prof. Mark Fowler

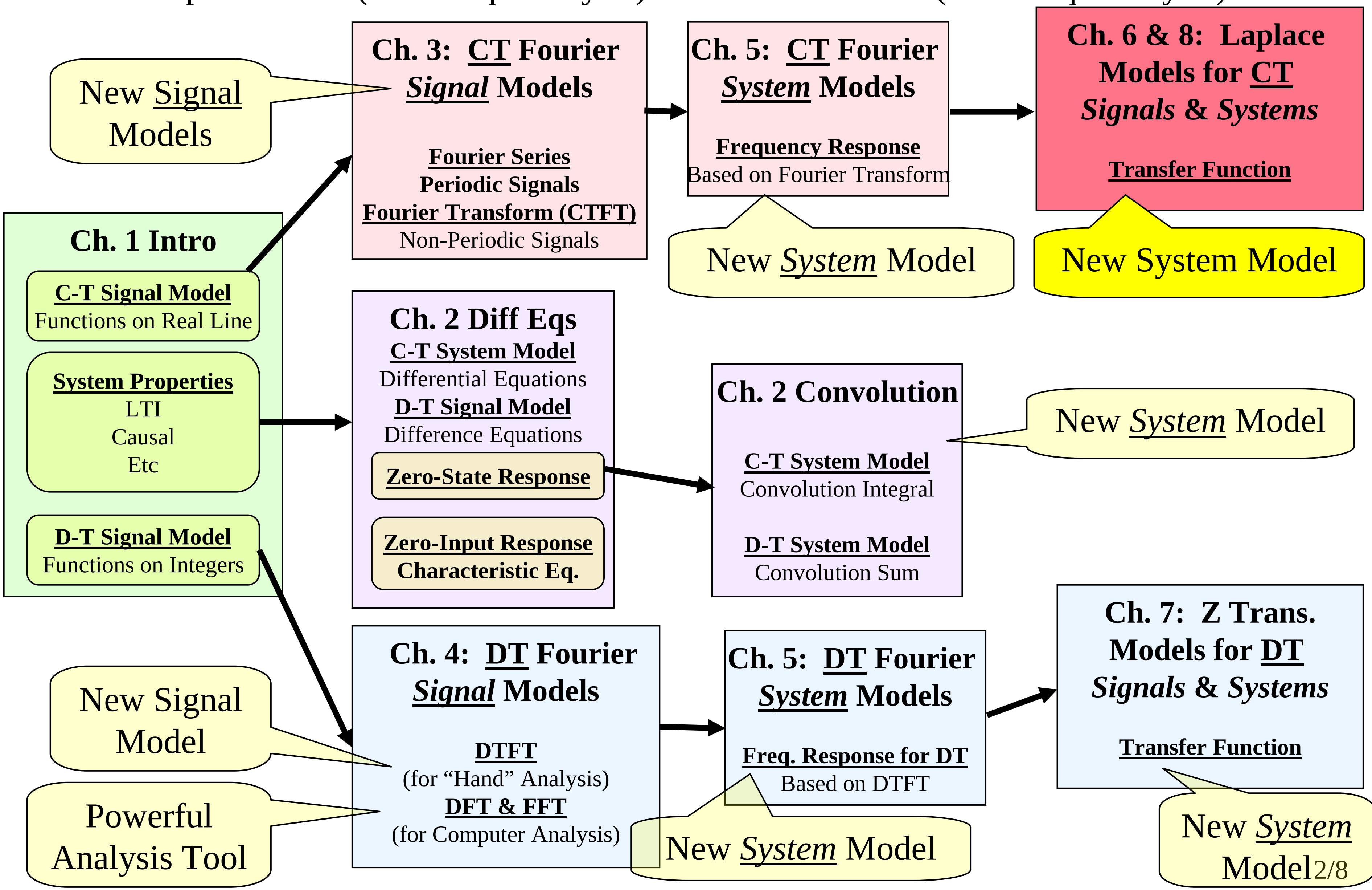
Note Set #28a

- C-T Systems: Laplace Transform… Partial Fractions
- Reading Assignment: “Notes 28a Reading on Partial Fractions”

(On my website)

Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).



There is a set of notes on-line called "…. Reading Notes on Partial Fraction Expansion"

You should read these on your own…

They lead you through several examples of using partial fractions to take a ratio of two high-order polynomial and expand it into a sum of ratios of simple polynomials.

You should be able to:

- Perform a simple partial fraction by hand (e.g., during an exam)
- Use matlab to compute more complicated partial fractions

Here I'll cover the main ideas…

Partial Fraction Expansion and Inverse LT

When trying to find the inverse Laplace transform it is helpful to be able to break a complicated ratio of two polynomials into forms that are on the Laplace Transform table.

An Example:

$$\mathcal{L}^{-1}\left\{\left[\frac{1/RC}{(s+1/RC)s}\right]\right\} = \mathcal{L}^{-1}\left\{\left[\frac{1}{s} - \frac{1}{s+1/RC}\right]\right\}$$

Factor LT into simpler forms

$$= \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}}_{=u(t)} - \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s+1/RC}\right\}}_{=e^{-(t/RC)}u(t)}$$

Linearity of LT

Here the factoring could be done "by inspection"… in general, this factoring can be done using the method of "partial fraction expansion" (PFE)

Motivation… Going the "Other Way"

This has poles at s = -3 and s =-5

Suppose we had this:

$$Y(s) = \boxed{\frac{2}{(s+3)} + \frac{4}{(s+5)}} + 2$$

A term like this is called a "Direct Term"

We could find the common denominator and combine:

$$Y(s) = \frac{2(s+5)}{(s+3)(s+5)} + \frac{4(s+3)}{(s+3)(s+5)} + 2\frac{(s+3)(s+5)}{(s+3)(s+5)}$$

$$= \frac{2s^2 + 22s + 47}{s^2 + 8s + 15}$$

We want to go from here… to here…

We’ll illustrate with an example like you should be able to do on an exam:

<u>Ex. #1</u>: <u>No Direct Terms</u>, <u>No Repeated Roots</u>, <u>No Complex Roots</u>

$$Y(s) = \frac{3s-1}{s^2+3s+2}$$

If the highest power in the numerator is less than the highest power in the denominator then there will be no direct terms.

By using the quadratic formula to find the roots of the denominator we can verify that there are no repeated or complex roots.

The roots are: s = -2 and s = -1… so we can write: $$Y(s) = \frac{3s-1}{(s+1)(s+2)}$$

If there are no repeated roots and no direct terms we can always write it as

$$Y(s) = \frac{r_1}{(s+1)} + \frac{r_2}{(s+2)}$$

The numbers r_1 and r_2 are called the “***<u>residues</u>***”… we need to find them!

Now we exploit what we know:

$$\frac{3s-1}{(s+1)(s+2)} = \frac{r_1}{(s+1)} + \frac{r_2}{(s+2)}$$

Multiply each side by (s+1) gives:

$$\frac{(3s-1)(s+1)}{(s+1)(s+2)} = \frac{r_1(s+1)}{(s+1)} + \frac{r_2(s+1)}{(s+2)}$$

Canceling (s+1) where we can gives:

$$\frac{(3s-1)}{(s+2)} = r_1 + \frac{r_2(s+1)}{(s+2)}$$

Setting s = –1 gives:

$$\frac{(-3-1)}{(-1+2)} = r_1 \quad \Rightarrow \quad r_1 = -4$$

All of this is summarized by this

$$\boxed{r_1 = Y(s)(s+1)\big|_{s=-1}}$$

Similarly… we find the other residue using: $\boxed{r_2 = Y(s)(s+2)\big|_{s=-2} = 7}$

Then we have: $$Y(s) = \frac{-4}{(s+1)} + \frac{7}{(s+2)}$$

Each of these terms is on the LT Table, so we get

$$y(t) = \mathcal{L}^{-1}\left\{\frac{-4}{(s+1)}\right\} + \mathcal{L}^{-1}\left\{\frac{7}{(s+2)}\right\}$$

$$= -4e^{-t}u(t) + 7e^{-2t}u(t)$$

See the set of notes on my website called "…. Reading Notes on Partial Fraction Expansion" for details on how to use MATLAB to find Partial Fraction Expansions for the more complicated cases.