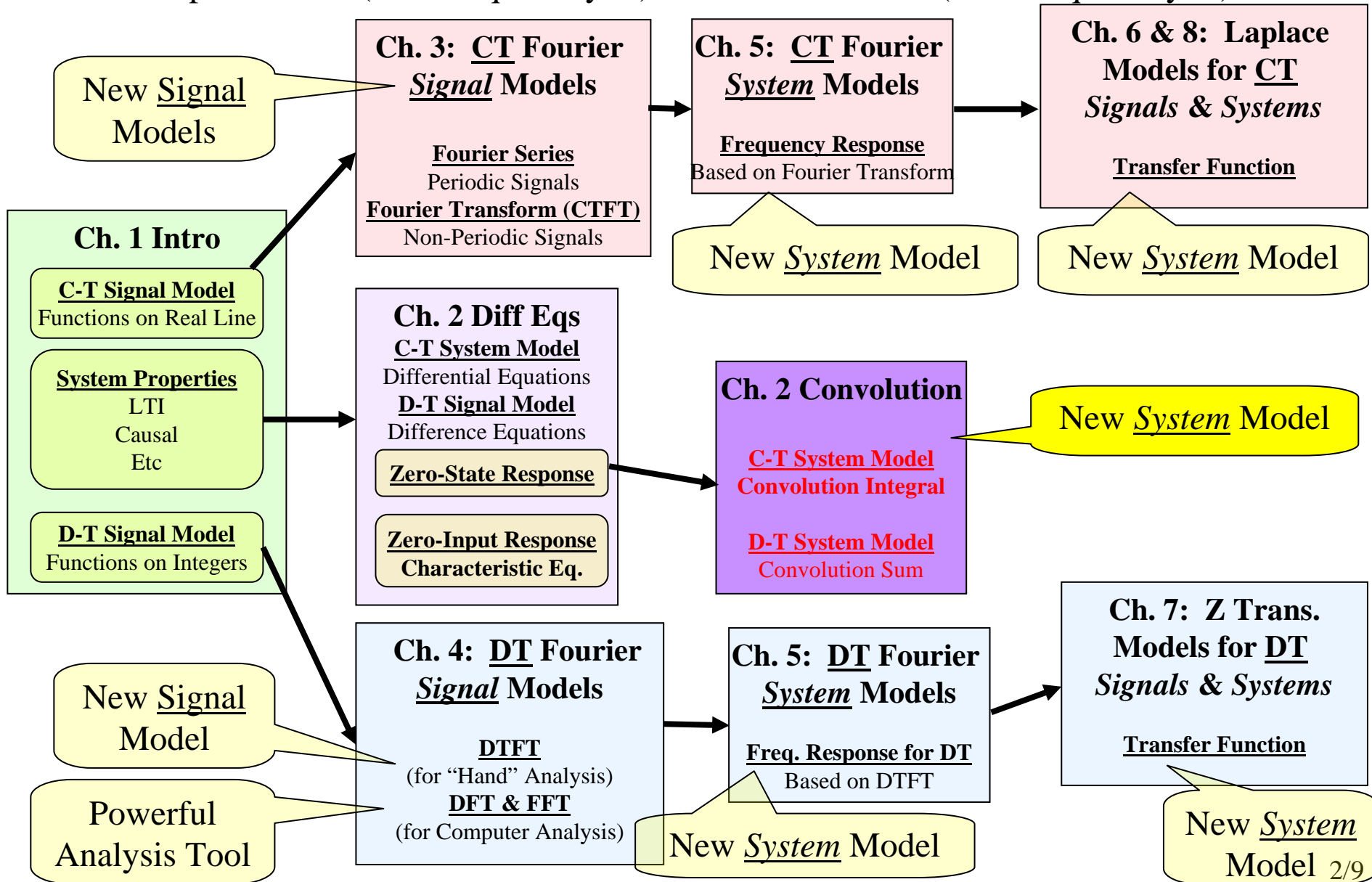# EECE 301
Signals & Systems

Prof. Mark Fowler

Note Set 11a

Example: Using D-T Convolution to Simulate the Effect of an RC Circuit on a Guitar Signal

Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).

New Signal Models

Ch. 3: CT Fourier _Signal_ Models

Fourier Series
Periodic Signals
Fourier Transform (CTFT)
Non-Periodic Signals

Ch. 5: CT Fourier _System_ Models

Frequency Response
Based on Fourier Transform

New System Model

Ch. 6 & 8: Laplace Models for CT _Signals & Systems_

Transfer Function

New System Model

Ch. 1 Intro

C-T Signal Model
Functions on Real Line

System Properties
LTI
Causal
Etc

D-T Signal Model
Functions on Integers

Ch. 2 Diff Eqs
C-T System Model
Differential Equations
D-T Signal Model
Difference Equations
Zero-State Response

Zero-Input Response
Characteristic Eq.

Ch. 2 Convolution

C-T System Model
Convolution Integral

D-T System Model
Convolution Sum

New System Model

New Signal Model

Powerful Analysis Tool

Ch. 4: DT Fourier _Signal_ Models

DTFT
(for "Hand" Analysis)
DFT & FFT
(for Computer Analysis)

Ch. 5: DT Fourier _System_ Models

Freq. Response for DT
Based on DTFT

New System Model

Ch. 7: Z Trans. Models for DT _Signals & Systems_

Transfer Function

New System Model 2/9

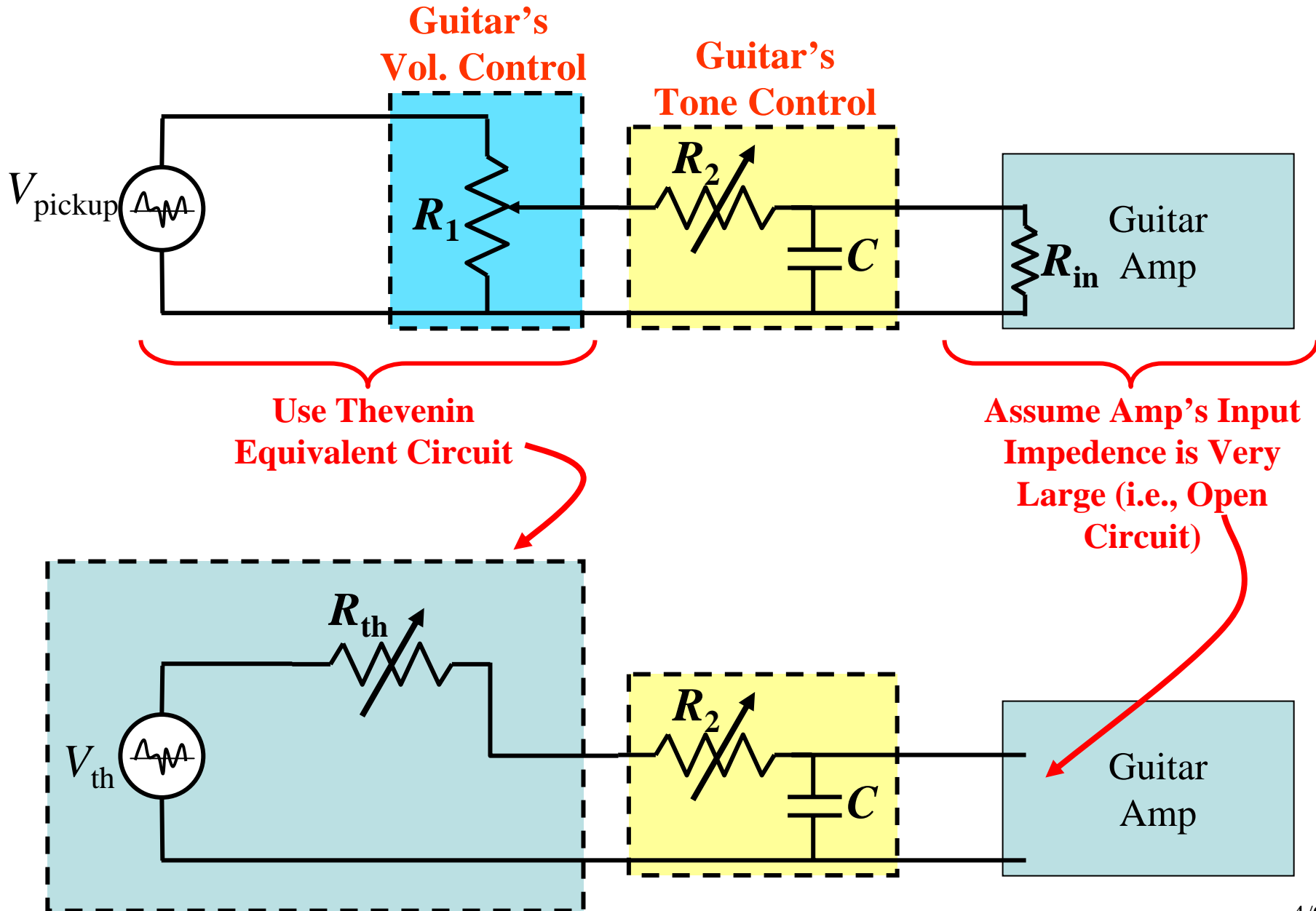# Example: D-T Processing to Simulate an RC Filter

The m-file RC_filt.m is available for download from the course website.

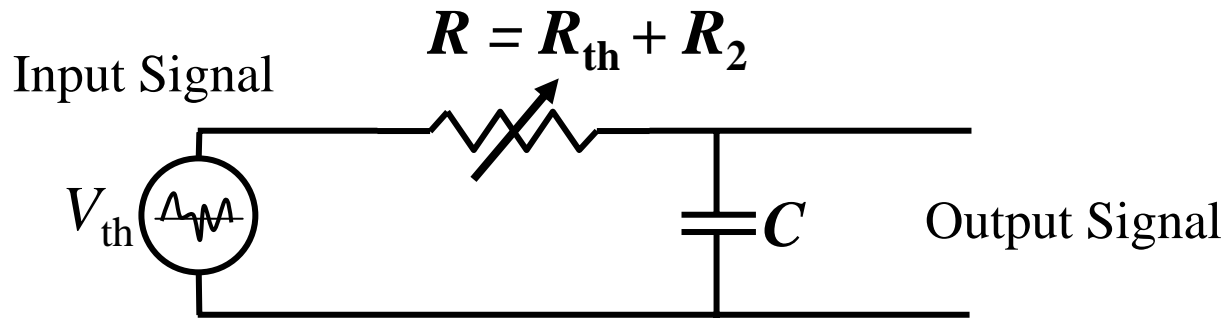The wav file guitar1.wav is also available there.

To run it all you need to do is:
1. Download these files to a folder on your computer
   a. Make sure Matlab's current directory is the folder where you have put the files (change it if needed)
2. At the Matlab command line type: RC_filter
3. The m-file will run
   a. Follow the prompts given in the Matlab Command Window to play the original signal and the signal after going through the RC Circuit
   b. Finally, the m-file will plot the impulse response $h(t)$ for the RC Circuit

Physical Set-Up of Guitar and RC Circuit

Guitar's Vol. Control

Guitar's Tone Control

V_{pickup}

R_1

R_2

C

R_{in} Guitar Amp

Use Thevenin Equivalent Circuit

Assume Amp's Input Impedence is Very Large (i.e., Open Circuit)

R_{th}

V_{th}

R_2

C

Guitar Amp

Circuit Model for Physical Set-Up

Input Signal

$R = R_{th} + R_2$

V_{th}

C

Output Signal

We know that this RC circuit has an impulse response given by: $h(t) = \frac{1}{RC} e^{-t/RC}$

$R = 100 \text{ k}\Omega$

$C = 5 \text{ nF}$
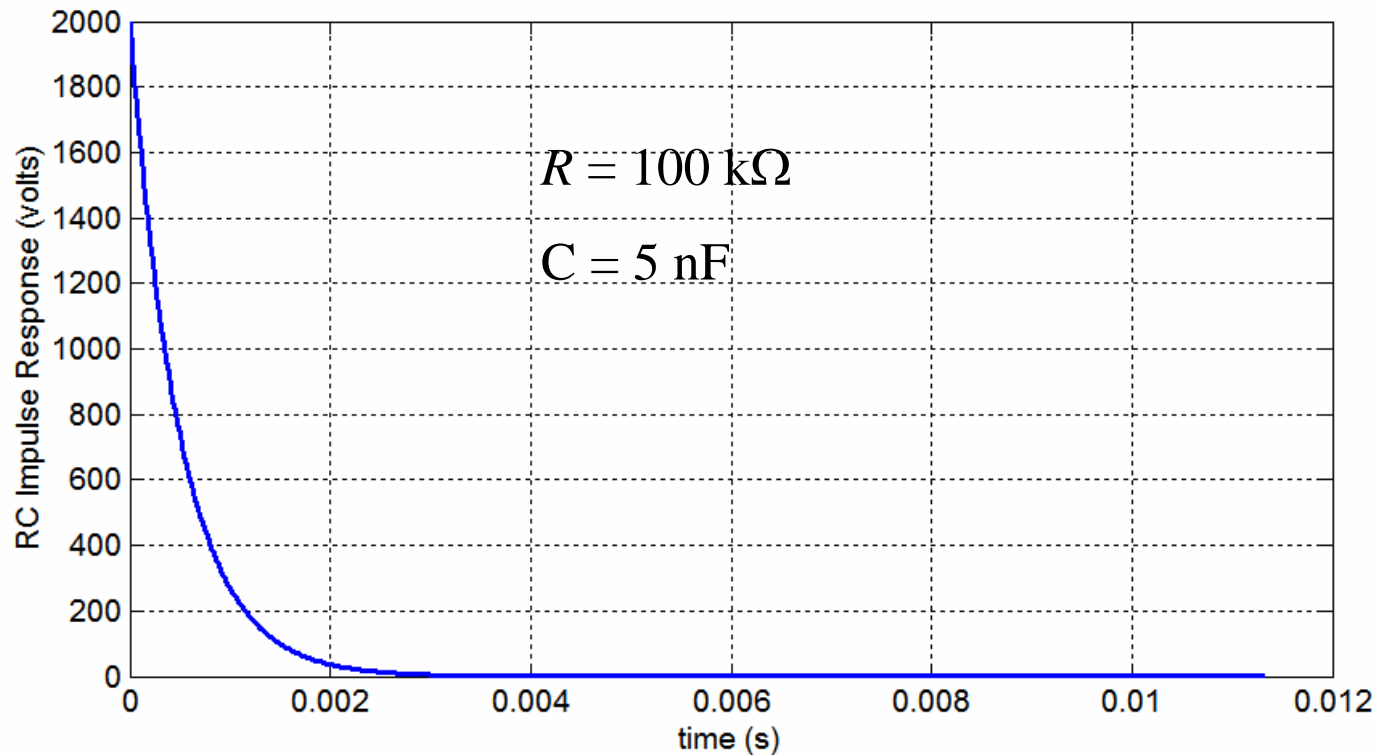
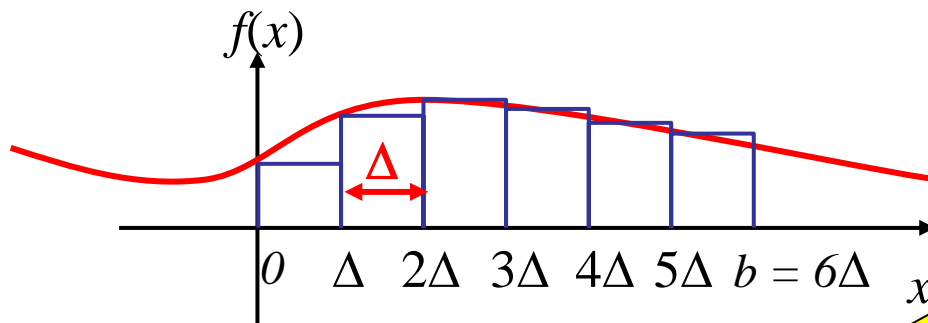# Modeling the C-T Circuit Using an *Approximate* D-T System

We can use D-T convolution to numerically approximate a C-T convolution.

Recall…

- **C-T Convolution involves an Integral**

- **D-T Convolution involves a Summation**

Recall from Calc I…

Integration was _defined_ as the limit of a series of Summations

So… to approximately compute the integral we only need **Samples** of the function!

$$\int_0^b f(x)dx \approx \sum_{i=0}^{5} f(i\Delta)\Delta$$

Obviously… to be reasonably accurate we need Δ quite small

So apply this idea
to C-T convolution:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$\approx \sum_{i=-\infty}^{\infty} x(iT)h(t-iT)T$$

Now… we'd be happy to get closely spaced samples of the output… so replace t by nT

$$y(nT) \approx \sum_{i=-\infty}^{\infty} x(iT)h(nT-iT)T = \sum_{i=-\infty}^{\infty} x[i]h[n-i]T = x[n] * (Th[n])$$

D-T
Convolution!!

This result says that if
   • $h(t)$ is the C-T system's impulse response
   • T is the spacing between time samples in the D-T simulation

Then the equivalent D-T system has an impulse response given by:

$$h[n] = Th(nT)$$

So… for our case we have:

$$h[n] = \frac{T}{RC}e^{-nT/RC}$$

Simulating the Circuit in Matlab

```
C=5e-9;
R=100e3;   %%% sets RC = 0.5 ms
RC=R*C;   %% compute RC time constant for use later

[x,Fs]=wavread('guitar1.wav');   %  read in wave file of guitar recording

del_t=1/Fs;   % Fs is sampling rate read in by wavread.m so invert it to get time spacing
t=(0:500)*del_t;   %% create 501 point time sample vector
h=(1/RC)*exp(-t/RC);   %% compute impulse response of this RC circuit at the 501 time
points

%% use del_t*h(nTs) to make D-T conv approximate C-T conv
%%  See Eq. (3.64) in Sect 3-5 of book

y=conv(x,del_t*h);  %% this D-T convolution approximates the C-T convolution that
              %% the RC circuit does
```

Now run RC_filt.m to hear the effect of this filter on the guitar signal. You should note that the guitar now sounds muffled… that is because the filter has removed some of the high frequency content.

You can experiment by trying different values for R to get different RC values… the so-called "cut-off" frequency is given by $f_c = 1/2\pi RC$ (Hz)

Big Picture

A physical RC circuit *does* convolution for itself

But here we used D-T convolution to approximate C-T convolution…

Then used MATLAB to numerically compute the approximating D-T convolution to numerically analyze the circuit.

These kinds of numerical analyses are wide-spread in real-world design processes!!!