EECE 301
Signals & Systems
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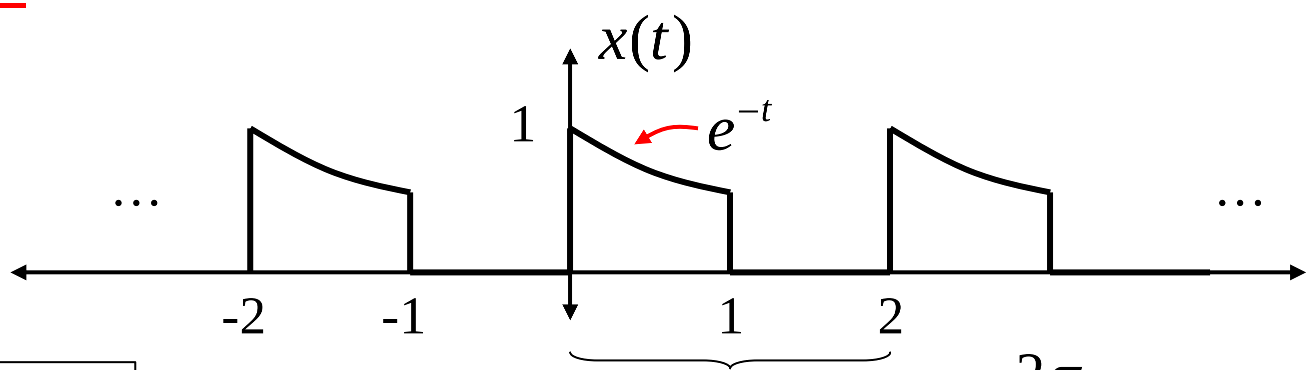
<u>Discussion #5</u>

- Fourier Series Examples

Fourier Series Examples

Example #1



choose $T = 2 \Rightarrow \omega_0 = \dfrac{2\pi}{2} = \pi \text{ rad/sec}$

$$c_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jk\omega_0 t}dt$$

$$= \frac{1}{2}\int_0^2 x(t)e^{-jk\pi t}dt$$

$$= \frac{1}{2}\left[\int_0^1 e^{-t}e^{-jk\pi t}dt + \int_1^2 0\times e^{-jk\pi t}dt\right]$$

$$= \frac{1}{2}\int_0^1 e^{-(1+jk\pi)t}dt$$

$$= \frac{1}{2}\left[\frac{-1}{1+jk\pi}e^{-(1+jk\pi)t}\right]_0^1$$

$$= \frac{-1}{2(1+jk\pi)}\left[e^{-(1+jk\pi)} - 1\right]_0^1$$

$$= \frac{1-e^{-1}e^{jk\pi}}{2(1+jk\pi)}$$

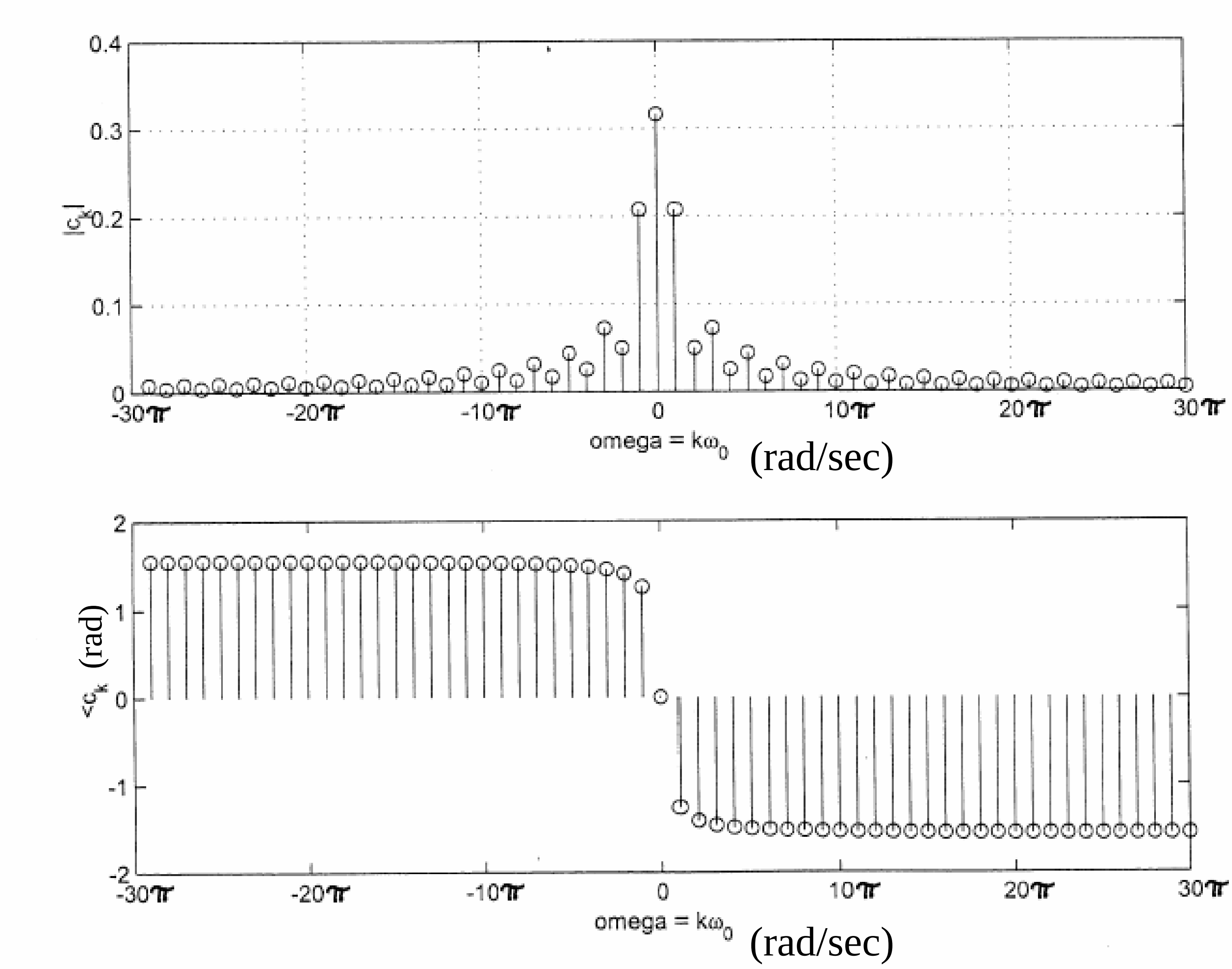
Note: $e^{-jk\pi} = \begin{cases} 1, & even\ \ k \\ -1, & odd\ \ k \end{cases}$

or equivalently $e^{-jk\pi} = (e^{-j\pi})^k = (-1)^k$

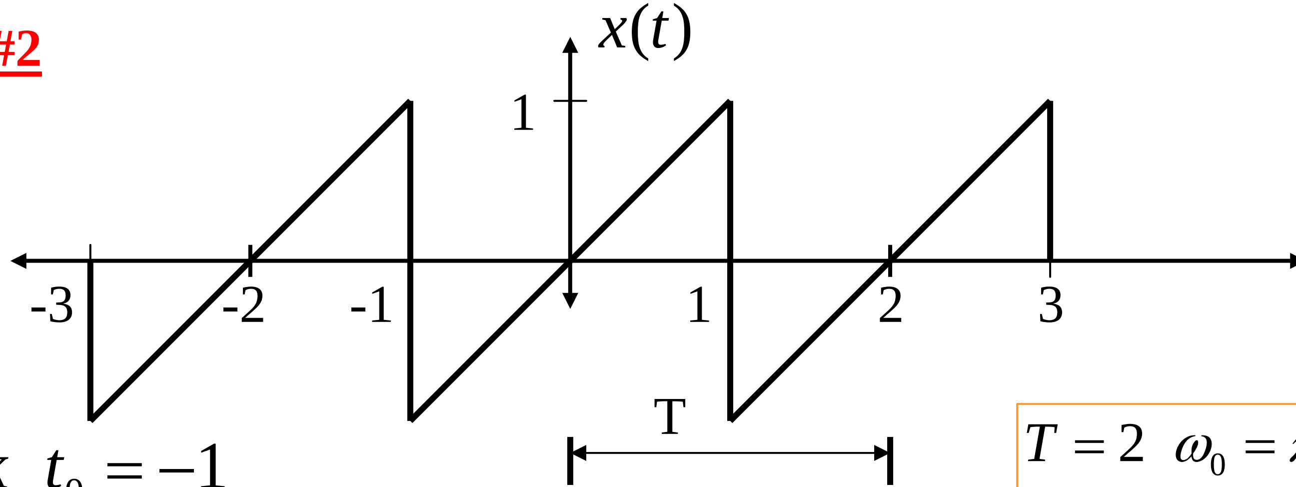
So…

$$c_k = \frac{1-e^{-1}(-1)^k}{2(1+jk\pi)}$$

Now we can use Matlab to plot $|c_k|$ & $\angle c_k$



Example #2



$pick\ \ t_0 = -1$

$T = 2 \quad \omega_0 = \pi$

$$c_k = \int_{-1}^{1} t e^{-jk\pi t} dt$$

$$= \left[\frac{e^{-jk\pi t}}{(-jk\pi)^2} (-jk\pi t - 1) \right]_{-1}^{1} \qquad k \neq 0$$

$$= \frac{-(-1)^k}{\pi^2 k^2} \left[(-jk\pi - 1) - (jk\pi - 1) \right]$$

cancel

From Integral Table

$$\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$a \neq 0$

Note:

$$e^{\pm jk\pi} = (-1)^k$$

$$(-j)^2 = j^2 = -1$$

$$c_k = \frac{+jk2\pi(-1)^k}{\pi^2 k^2} = \frac{+j2(-1)^k}{\pi k} \qquad k \neq 0$$

Also… $$c_0 = \int_{-1}^{1} t\,dt = 0$$

Easy to get the magnitude results:

$$|c_k| = \frac{2}{\pi |k|} \qquad k \neq 0$$

$$|c_0| = 0$$

Harder to get the phase results:

$$\angle c_k = \begin{cases} 0, & k = 0 \\ \dfrac{\pi}{2}, & \begin{cases} k \ \ positive\ \&\ even \\ k \ \ negative\ \&\ odd \end{cases} \\ \dfrac{-\pi}{2}, & \begin{cases} k \ \ positive\ \&\ odd \\ k \ \ negative\ \&\ even \end{cases} \end{cases}$$

<u>When k < 0:</u>

c_k is j(neg)(-1)^k

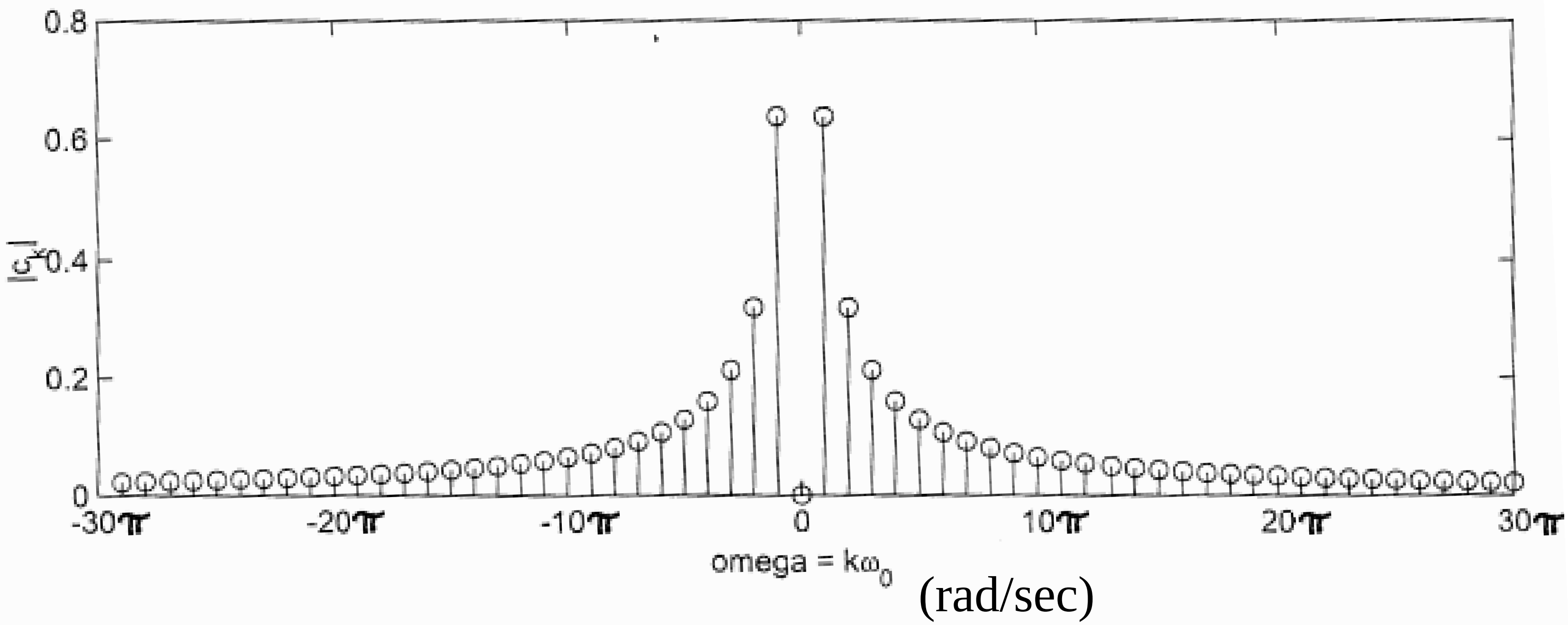
<u>When k > 0:</u>

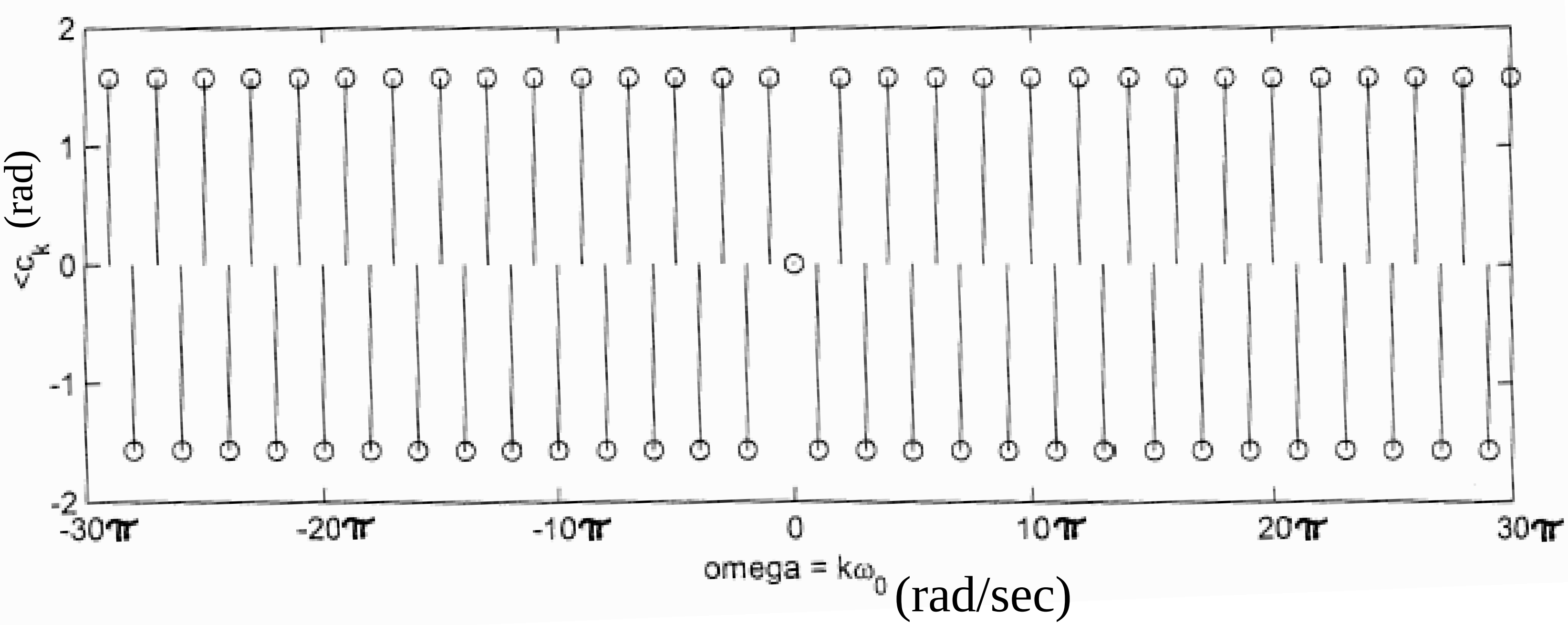
c_k is j(pos)(-1)^k

$\Rightarrow$ Since $\angle j$(neg) = -π/2

Since $\angle j$(pos) = π/2

we get this


0.8
0.6
0.4
0.2
0
|c_k|
-30π
-20π
-10π
0
10π
20π
30π
omega = kω_0
(rad/sec)


2
1
0
-1
-2
<c_k (rad)
-30π
-20π
-10π
0
10π
20π
30π
omega = kω_0
(rad/sec)

Matlab Explorations

disc_05_FS_example.m

```
function Disc_05_FS_example(type)
%
%  Computes FS-based approximation to various signal types and plays sound
%
%  Input: type = 'ramp'
%                'rand'
%                'tone1'    ... no FS used
%                'tone2'
%                'pulse'
%                'square'

Fs=60000;
Ts=1/Fs;
t=(0:60000)*Ts;
ft=2*pi*440*t;
N=length(t);

… etc. …
```