

Polar Form

(A)

$$z = r e^{j\theta}$$

If $z = a + jb$, $r = \sqrt{a^2 + b^2}$

$$r > 0$$

$\theta \in (-\pi, \pi]$ in rad

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) \quad \left(\begin{array}{l} \text{careful w/} \\ \text{calculator} \\ \text{when } a < 0 \end{array} \right)$$

Mult & Div. In Polar Form

$$(r_1 e^{j\theta_1}) (r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

$$\frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}} = \left(\frac{r_1}{r_2}\right) e^{j(\theta_1 - \theta_2)}$$

Example

$$\frac{5 + j5}{2 e^{j\pi/3}} = \frac{\sqrt{5^2 + 5^2} e^{j\pi/4}}{2 e^{j\pi/3}}$$

$$= \frac{\sqrt{2} \cdot 5}{2} e^{j(\pi/4 - \pi/3)}$$

$$= \frac{5}{\sqrt{2}} e^{-j\pi/12}$$

Some Useful Tricks

$$|z_1 z_2 z_3| = |z_1| \cdot |z_2| \cdot |z_3|$$

→ Ex: $|-2j e^{j\pi/2}| = |-2| \cdot |j| \cdot |e^{j\pi/2}|$

treat each = $2 \cdot 1 \cdot 1 = 2$
as z_1, z_2, z_3

$$\angle(z_1 z_2 z_3) = \angle z_1 + \angle z_2 + \angle z_3$$

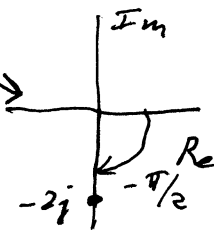
→ Ex: $\angle(-2j e^{j\pi/2}) = \angle -2j + \angle e^{j\pi/2}$

$$= -\pi/2 + \pi/2$$

$$= -\frac{5\pi}{14}$$

$$\left| \frac{z_1 z_2}{z_3 z_4} \right| = \frac{|z_1| \cdot |z_2|}{|z_3| \cdot |z_4|}$$

$$\angle\left(\frac{z_1 z_2}{z_3 z_4}\right) = \angle z_1 + \angle z_2 - \angle z_3 - \angle z_4$$



$$(r e^{j\theta})^n = r^n e^{jn\theta}$$

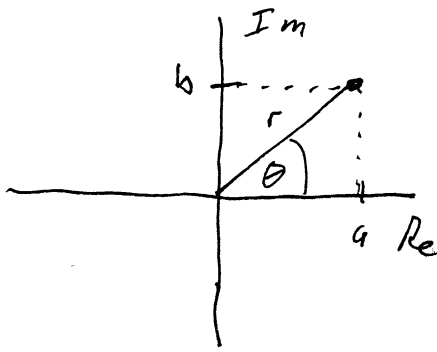
$$(r e^{j\theta})^{1/n} = r^{1/n} e^{j\theta/n}$$

$$\angle(\text{Pos. Real}) = 0$$

$$\angle(\text{Neg. Real}) = \pm\pi$$



Conversion



$$r = \sqrt{a^2 + b^2}$$

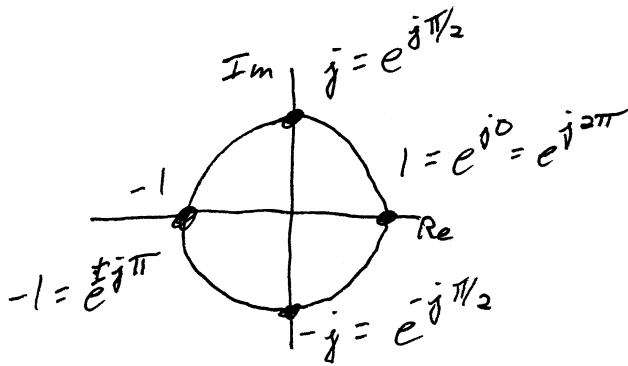
$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

careful if $a < 0$

$$a = r \cos \theta$$

$$b = r \sin \theta$$

But... Don't just blindly apply these!
 Look for simple shortcuts!



Example

Find $\angle(e^{j\pi} - 1)$

$\uparrow = -1$

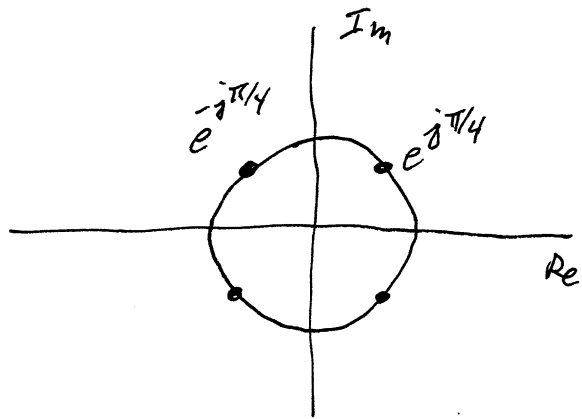
$$\Rightarrow = \angle -2 = \pm \pi$$

$$e^{j\pi/4} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$e^{-j\pi/4} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$e^{+j3\pi/4} = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$e^{-j3\pi/4} = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$



Example: $3e^{j\pi/4} = 3/\sqrt{2} + j3/\sqrt{2}$