25th Asilomar Conf on Signals, Systems and Computers, 1991

A Unified Formulation for Detection Using Time-Frequency and Time-Scale Methods

Mark L. Fowler and Leon H. Sibul
Applied Research Laboratory
The Pennsylvania State University
P.O. Box 30
State College, PA 16804

Abstract

A unified theory is developed for the detection of signals in nonwhite noise using time-frequency and time-scale signal transforms. This class includes the cross-Wigner distribution and certain other members of Cohen's class, the Gabor transform, and the wavelet transform.

Necessary and sufficient conditions are established such that a signal transform is applicable to the nonwhite noise detection problem. Applying this result to the reproducing kernel Hilbert space (RKHS) associated with a finite-dimensional approximation of the noise covariance leads to weighted correlator structures in the time-frequency and time-scale domains. This extends results previously available only for the white noise case.

1 Introduction

There has been much interest during the past decades in time-frequency representations such as the Wigner distribution and the Gabor transform [1], [2]. More recently, interest has turned to the wavelet transform [3], which is a time-scale representation. In particular, the application of time-frequency and time-scale methods to the problem of detecting a signal in the presence of additive noise has been considered [4] – [7], and advantages of these approaches have been noted. For example, Boashash and O'Shea [5] applied Wigner distribution techniques to the detection and classification of the firings of an underwater diesel engine in the presence of white noise and showed that the resulting detector performed better than did the classical time-domain method. They showed that the improvement is due to time-varying filtering that is inherent to the Wigner-based method. Similarly, Friedlander and Porat [6] demonstrated that the time-frequency localization provided by the Gabor transform yields the capability to detect, in the presence of white noise, partially overlapping transients with unknown decay rates, oscillation rates, and arrival times. Additionally, Tuteur [7] has shown that the wavelet transform is effective in the detection of electrocardiogram signals.

These demonstrations of the effectiveness of the time-frequency methods in overcoming some of the problems suffered by other signal detection techniques are encouraging. However, the existing time-frequency methods for detection have been formulated only for the white noise case, and therefore have limited applicability to practical detection problems. While it is true that a whitening filter could be used prior to the application of the time-frequency methods, a direct formulation for the nonwhite noise case would provide a more versatile and more widely applicable approach. For example, if a whitening filter is used, then the time-frequency correlator uses time-frequency representations of whitened versions of the signal; these representations of the whitened signals are less useful for further processing—such as signaturing and classification—than are the time-frequency representations of the actual signals. Another characteristic of the previously reported techniques is that despite the similar inner product structures of the various time-frequency and time-scale representations, the methods proposed for each transform have been developed using differing formulations. In response to these observations, we have developed a formulation that allows a wide variety of time-frequency and time-scale representations to be applied in a unified way to the detection of signals in nonwhite noise. Furthermore, this unification provides a framework within which the various time-frequency and time-scale detection methods can be compared and contrasted. Therefore, our formulation not only extends the applicability of time-frequency and time-scale methods to the nonwhite noise, but provides insight into the proper choice of a representation for a particular signal/noise environment.

In Section 2 we develop a unified transform approach to detection based on the reproducing kernel Hilbert space (RKHS) formulation of the detection problem. In particular, we give necessary and sufficient conditions for a transform to unitarilly map one RKHS onto another RKHS. In Section 3 we exploit the inherent inner product structure of each of the time-frequency and time-scale representations to show that the results of Section 2 are applicable to each of these signal transforms. The application of the wavelet transform to detection in the presence of a $1/f$ type noise is illustrated by example.

2 Unified Transform Approach to Detection

The detection problem we consider is as follows. Given a signal r received over the observation interval I, make a decision between

$$\left.\begin{array}{ll} H_1: & r(t)=\gamma(t)+n(t) \\ H_0: & r(t)=n(t) \end{array}\right\} t \in I,$$

where γ is the deterministic signal to be detected, and n is some zero-mean Gaussian noise. The detection problem is typically solved by comparing a sufficient statistic, computed from the received signal r, to some pre-determined threshold. Our results are based on the reproducing kernel Hilbert space (RKHS) formulation of the sufficient statistic. An RKHS $H(K)$ is a special Hilbert space, uniquely associated with the kernel K, that has the property that $K(\cdot,x) \in H(K)$ for all values of x and $< f, K(\cdot,x) >_{H(K)} = f(x)$ for every $f \in H(K)$. Briefly, under the assumptions that I is a bounded interval and that the noise covariance K is continuous on $I \times I$, the sufficient statistic (or likelihood ratio) is given by

$$\Lambda = < r, \gamma >_{H(K)},$$

where $H(K)$ is the RKHS that is uniquely determined by the noise covariance K [8].

The important insight provided by the RKHS formulation is that the sufficient statistic is determined completely by the inner product structure of $H(K)$. Thus, insight into the development of equivalent formulations in alternative domains (e.g., time-frequency and time-scale domains) can be gained by investigating the application of various operators to the space $H(K)$. In particular, we are interested in the question: What types of unitary mappings can be applied to $H(K)$ such that the resulting space is another RKHS? The answer to this question is provided here by developing the following theorem, which is an extension of a theorem given by Chalmers [9] that gives the relationship between the kernels of two RKHSs that are related by a bounded linear operator. Chalmers' result requires *a priori* knowledge that the two spaces are indeed RKHSs; in contrast, our result gives necessary and sufficient conditions for an operator to map an RKHS into another RKHS, and gives the relationship between the kernels.

Notation for the following theorem: $[\mathcal{T}K_1(s,t)]_{s|x}$ means "operate on $K_1(s,t)$ as a function of s and evaluate the resulting function at the point x."

Theorem 1 *Let H_1 be an RKHS with reproducing kernel K_1 defined on some set $I_1 \times I_1$, and let $\mathcal{T}: H_1 \rightarrow H_2$ be unitary. The space H_2 is an RKHS of functions defined on some set I_2 if and only if the operator $\mathcal{T}$ can be represented by*

$$(\mathcal{T}f)(x) = < f, T(\cdot,x) >_{H_1}, \qquad (1)$$

for some bivariate function T defined on $I_1 \times I_2$ that has the property that $T(\cdot,x) \in H_1$, $\forall x \in I_2$. Furthermore, the reproducing kernel K_2 for H_2 is given by

$$K_2(y,x) = \left[\mathcal{T}\overline{[\mathcal{T}K_1(s,t)]_{s|x}}\right]_{t|y}, \quad \forall x,y \in I_2. \qquad (2)$$

Outline of proof: Under the assumption that (1) defines a unitary operator, with $T(\cdot,x) \in H_1$, it follows that

$$(\mathcal{T}f)(x) = < \mathcal{T}f, \mathcal{T}T(\cdot,x) >_{H_2}.$$

This establishes that H_2 is an RKHS with reproducing kernel K_2 given by $K_2(y,x) = [\mathcal{T}T(t,x)]_{t|y}$. Conversely, under the assumption that H_2 is an RKHS, the unitary mapping $\mathcal{T}$ can be represented by

$$\begin{aligned} (\mathcal{T}f)(x) &= < \mathcal{T}f, K_2(\cdot,x) >_{H_2} \\ &= < f, \mathcal{T}^{-1}K_2(\cdot,x) >_{H_1} \\ &\triangleq < f, T(\cdot,x) >_{H_1}, \end{aligned}$$

where $T(\cdot,x) \in H_1$. Furthermore, under each of these assumptions, $T(t,x) = \overline{< K_1(\cdot,t), T(\cdot,x) >_{H_1}} = \overline{[\mathcal{T}K_1(s,t)]_{s|x}}$, which establishes (2).
□

This theorem characterizes the transforms that can be applied to the detection problem. Specifically, if $\mathcal{T}$ is a unitary operator that satisfies Theorem 1 with $H_1 = H(K)$, where K is the noise covariance, then the sufficient statistic can be expressed as

$$\Lambda = < \mathcal{T}r, \mathcal{T}\gamma >_{H(\tilde{K})}, \qquad (3)$$

where

$$\tilde{K}(y,x) = \left[\mathcal{T}\overline{[\mathcal{T}K(s,t)]_{s|x}}\right]_{t|y}, \quad \forall x,y \in I_2.$$

Unfortunately, the formulation in (3) can be difficult to apply. In order to use a specific mapping in (3), we must establish that it is a unitary mapping defined on $H(K)$ that satisfies Theorem 1. This can be difficult since the applicability of a specific mapping depends on the interplay between the mapping and the space $H(K)$; therefore, if a specific mapping works for one particular noise, it may not work for some other noise.

By considering a finite-dimensional approximation to the space $H(K)$, it is possible to apply any L^2-unitary operator to a detection problem for which the noise covariance is continuous on the bounded set $I \times I$. Specifically, the covariance K can be approximated by

$$K_N(s,t) = \sum_{n=1}^{N} \lambda_n \, \phi_n(s) \, \overline{\phi_n(t)},$$

where the $\{\lambda_n\}$ and the $\{\phi_n\}$ are the eigenvalues and the eigenfunctions, respectively, of the noise covariance K. Then we can define

$$K_N^{-1}(s,t) = \sum_{n=1}^{N} 1/\lambda_n \, \phi_n(s) \, \overline{\phi_n(t)},$$

which gives rise to an integral operator $\mathcal{K}_N^{-1}$ such that we can form an approximate sufficient statistic by

$$\begin{aligned} \Lambda_N &= <r, \gamma>_{H(K_N)} \\ &= <r, \mathcal{K}_N^{-1}\gamma>_{L^2(I)}, \end{aligned} \quad (4)$$

with $\Lambda_N \to \Lambda$ as $N \to \infty$.

By applying Theorem 1 to the finite-dimensional space $H(K_N)$ it is easy to show that it is possible to apply any L^2-unitary operator to (4) to arrive at forms for Λ_N in alternative domains. Namely, if $\mathcal{T}$ maps $L^2(I)$ unitarily onto some Hilbert space H, then

$$\Lambda_N = <\mathcal{T}r, \tilde{\mathcal{K}}_N^{-1}\mathcal{T}\gamma>_H, \quad (5)$$

where $\tilde{\mathcal{K}}_N^{-1}$ is the inverse of the integral operator that corresponds to

$$\tilde{K}_N(y,x) = \left[\mathcal{T}\overline{[\mathcal{T}K_N(s,t)]_{s|x}}\right]_{t|y}. \quad (6)$$

This formulation allows a wide variety of signal transforms to be applied to the detection problem; including the cross-Wigner distribution, the Gabor transform, and the wavelet transform.

3 Application of Time-Frequency and Time-Scale Methods

We are interested here in applying the cross-Wigner distribution, the Gabor transform, and the wavelet transform to the detection problem. It is well known that each of these transforms is, under the right conditions, a unitary operator on $L^2(\mathcal{R})$; therefore, the results of the previous section easily lead to a unified formulation for an approximation to the sufficient statistic in terms of these transforms. In this section we discuss certain aspects of these formulations in detail.

The cross-Wigner distribution (CWD) between signals f and g is defined as

$$W_{f,g}(t,\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau} f(t+\tau/2)\overline{g(t-\tau/2)}\,d\tau.$$

If g is chosen to be concentrated in time and frequency, then $W_{f,g}$ gives a linear representation of the time-frequency content of f. The CWD is a member of Cohen's class [1], which has a general member given by

$$C^{\Phi}_{f,g}(t,\omega) = 1/2\pi \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j(\xi t - \tau\omega - \xi u)}\,\Phi(\xi,\tau)\, f(u+\tau/2)\overline{g(u-\tau/2)}\,du\,d\tau\,d\xi,$$

where the choice of the kernel Φ determines a specific member of the class. Alternatively, a general member of Cohen's class can be expressed in terms of the Wigner distribution by

$$C^{\Phi}_{f,g}(t,\omega) = 1/2\pi \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(t-\tau,\omega-\xi)\,W_{f,g}(\tau,\xi)\,d\tau\,d\xi,$$

where the bivariate function ϕ is given by

$$\phi(t,\omega) = 1/2\pi \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j(\xi t - \tau\omega)}\,\Phi(\xi,\tau)\,d\tau\,d\xi. \quad (7)$$

If g is chosen as a fixed analyzing function, then the bivariate function $C^{\Phi}_{f,g}$ can be interpreted as the result of a linear operator acting on f. As seen in the previous section, if this operator is unitary on $L^2(\mathcal{R})$, then the sufficient statistic can be formed in terms of cross-Cohen representations. It is well known that the CWD can be considered as a unitary operator since it satisfies

$$1/2\pi \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_{f_1,g_1}(t,\omega)\overline{W_{f_2,g_2}(t,\omega)}\,dt\,d\omega = <f_1,f_2>_{L^2}\overline{<g_1,g_2>_{L^2}},$$

which is known as Moyal's formula [1]. However, not all members of Cohen's class satisfy Moyal's formula. Previously, Janssen [10] provided a sufficient condition for a member of Cohen's class to satisfy Moyal's formula; namely, if $|\Phi(\xi,\tau)| = 1$ for all ξ and τ, then the corresponding $C^{\Phi}_{f,g}$ satisfies Moyal's formula. (Note: Janssen's condition is necessary and sufficient only for a subclass of Cohen's class.)

The following theorem gives a *necessary and sufficient* condition for a member of Cohen's class to satisfy a form of Moyal's formula and, therefore, be applicable to the detection problem.

Theorem 2 *For a fixed g with $\|g\|_2^2 = 1/2\pi$, $C^{\Phi}_{f,g}$ satisfies*

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C^{\Phi}_{f_1,g}(t,\omega)\overline{C^{\Phi}_{f_2,g}(t,\omega)}\,dt\,d\omega = <f_1,f_2>_{L^2}$$

if and only if the mapping ϕ, defined on a closed subspace of $L^2(\mathcal{R}^2)$ by

$$(\phi F)(t,\omega) = 1/2\pi \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(t-\tau,\omega-\xi)\,F(\tau,\xi)\,d\tau\,d\xi,$$

where $\phi(t,\omega)$ is defined by (7), is unitary.

Proof: For a fixed g with $\|g\|_2^2 = 1/2\pi$, define the closed subspace $\mathbf{W_g} = \{W_{f,g} | f \in L^2(\mathcal{R})\}$. The operator $W_g : f \mapsto W_{f,g}$ unitarilly maps $L^2(\mathcal{R})$ onto $\mathbf{W_g}$. Thus, the mapping of f to $C^{\Phi}_{f,g}$ is then accomplished by applying the composition $\phi \circ W_g$ to the function f. Since the operator W_g is unitary (due to Moyal's formula), it is necessary and sufficient that the mapping ϕ be unitary.
□

The results of Section 2 now imply that any member $C^{\Phi}_{f,g}$ of Cohen's class that satisfies the above theorem may be used to approximate the sufficient statistic via (5) and (6), which reduce to

$$\Lambda_N = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C^{\Phi}_{r,g}(t,\omega)$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{K}_N^{-1}(t,\omega;t',\omega')\,\overline{C^{\Phi}_{\gamma,g}(t',\omega')}\,dt'\,d\omega'\,dt\,d\omega, \quad (8)$$

and

$$\tilde{K}_N^{-1}(t,\omega;t',\omega') = \sum_{n=1}^{N} 1/\lambda_n\, C^{\Phi}_{\phi_n,g}(t,\omega)\,\overline{C^{\Phi}_{\phi_n,g}(t',\omega')}. \quad (9)$$

This formulation results in a weighted correlation between a time-frequency representation of the received signal and a time-frequency representation of a reference signal; the weighting kernel is the inverse kernel of the reproducing kernel in the transformed RKHS, and can be interpreted as the inverse kernel of the covariance of the time-frequency representation of the noise process. This formulation is a direct extension of the unweighted time-frequency domain correlator proposed for the white noise case in [4] and [5].

The Gabor transform is another linear time-frequency representation that is closely related to the cross-Wigner distribution. Let g be an arbitrary finite-energy signal; then, the Gabor transform of a signal f, with respect to g, is given by the bivariate function $G_g f$ defined by

$$(G_g f)(\tau,\omega) = \int_{-\infty}^{\infty} f(t)\,\overline{g(t-\tau)}\,e^{-j\omega t}\,dt.$$

If g is chosen such that $\|g\|_2^2 = 1/2\pi$, then the mapping $G_g : L^2(\mathcal{R}) \to L^2(\mathcal{R}^2)$, defined by $G_g : f \mapsto G_g f$, is a unitary operator. Therefore, it is possible to use the same formulation for solving the detection problem as was used for the cross-Wigner distribution; that is, replace $C^{\Phi}_{r,g}$ and $C^{\Phi}_{\gamma,g}$ in (8) and (9) by $G_g r$ and$G_\gamma r$, respectively. In fact, there really is very little difference between these two time-frequency formulations of the sufficient statistic since

$$W_{f,g}(t,\omega) = 2e^{j\omega t}(G_{\tilde{g}} f)(2t, 2\omega),$$

where $\tilde{g}(x) = g(-x)$. Thus, the CWD is more highly concentrated in the time-frequency plane.

It is interesting to note that the Gabor transform has a discrete form that arises by sampling $(G_g f)(\tau,\omega)$ uniformly in τ and ω to give

$$(G^d f)(m,n) = \int_{-\infty}^{\infty} f(t)\,\overline{g(t-mT)}\,e^{-jn\Omega t}\,dt,$$

where the explicit dependence on g has been suppressed to enhance the readability. By proper choice of the function g and the sampling grid parameters (T,Ω), the discrete form of the Gabor transform can be made to be a unitary operator from $L^2(\mathcal{R})$ onto $\ell^2(\mathcal{Z}^2)$ [3]. The use of the discrete form results in a formulation of the sufficient statistic that is similar to that proposed in [6].

The wavelet transform of a signal f, with respect to a signal g (called the mother wavelet), is given by

$$(W_g f)(\tau,s) = \int_{-\infty}^{\infty} e^{s/2}\,\overline{g(e^s t-\tau)}\,f(t)\,dt,$$

where g is chosen such that

$$c_g \triangleq \int_0^{\infty} \frac{|G(\omega)|^2}{\omega}\,d\omega < \infty.$$

Choosing g such that $c_g = 1$ makes the wavelet transform a unitary operator [11]. Note that the variable s represents "scale" rather than "frequency." The concept of scale arises because as s goes toward positive infinity, $(W_g f)(\tau,s)$ zooms in on the small-scale details of f that are near the time point τ. This zooming property gives the wavelet transform a time resolution that adapts to the frequency range.

Because the wavelet transform is a unitary operator, it can be used for the detection of signals in nonwhite noise by applying the unified formulation developed in Section 2. Like the Gabor transform case, this application results in a formulation identical to (8) and (9), except with $C^{\Phi}_{r,g}$ and $C^{\Phi}_{\gamma,g}$ replaced by $W_g r$ and $W_g \gamma$, respectively. This application of the wavelet transform is illustrated below, where we will use the more practical discrete form of the wavelet transform that arises by sampling $(W_g f)(\tau,s)$ nonuniformly in τ and s to give

$$(W^d f)(m,n) = \int_{-\infty}^{\infty} f(t)\,S^{n/2} g(S^n t - mT)\,dt,$$

which are known as the wavelet coefficients of f. Note that the dependence of the coefficients on g has, for clarity, not been displayed on the left-hand side of this equation; also, when no confusion can arise, $(W^d f)(m,n)$ will be written as W_{mn}. It is possible to choose g and the sampling grid parameters (T,S) such that the set $\{S^{n/2} g(S^n t - mT)\}$ is an orthonormal basis of $L^2(\mathcal{R})$ and the discrete wavelet transform is a unitary operator from $L^2(\mathcal{R})$ onto $\ell^2(\mathcal{Z}^2)$, where $\mathcal{Z}$ is the set of integers [3].

4 Example: Nearly $1/f$ Noise

The discrete form of the wavelet transform was used by Wornell [12] in the analysis of a type of nonstationary process, called nearly $1/f$ noise, having measured spectra satisfying

$$\frac{k_1}{|\omega|^{\beta}} \le S(\omega) \le \frac{k_2}{|\omega|^{\beta}},$$

with $0 < k_1 \le k_2 < \infty$, and $\beta \in (0,2)$ a fixed parameter; this includes $1/f$ noise as a special case.

Consider the (real-valued, for convenience) nonstationary noise n defined [12] by the discrete wavelet expansion

$$n(t) = \sum_m \sum_n W_{mn}\, g_{mn}(t), \tag{10}$$

where W_{mn} are the random wavelet coefficients of the discrete wavelet transform, and $\{g_{mn}\}$ is a basis of orthonormal wavelets related to the mother wavelet g according to

$$g_{mn}(t) = 2^{n/2}\, g(2^n t - m). \tag{11}$$

Let the Fourier transform G of g be continuous at $\omega = 0$ and let $|G(\omega)|$ decay at least as fast as $1/\omega$. If the random sequence W_{mn} is such that for arbitrary distinct pairs n' and n, $W_{mn'}$ and W_{mn} are uncorrelated sequences, and if for each fixed n the sequence W_{mn} is white with average power $2^{-\beta n}\sigma^2$, then the noise n is a nearly $1/f$ noise [12].

If the noise n has zero-mean, then

$$E\{W_{mn}\, W_{m'n'}\} = 2^{-\beta n}\, \sigma^2\, \delta_{n,n'}\, \delta_{m,m'},$$

which shows that (10) is a Karhunen–Loève-like expansion of the noise n. Thus, its covariance can be expanded in terms of the ON wavelet basis as follows:

$$\begin{aligned} K(t,s) &= \sum_n \sum_m \sum_{n'} \sum_{m'} \left[2^{-\beta n}\, \sigma^2\, \delta_{n,n'}\, \delta_{m,m'}\right] g_{mn}(t)\, g_{m'n'}(s) \\ &= \sum_n \sum_m 2^{-\beta n}\, \sigma^2\, g_{mn}(t)\, g_{mn}(s). \end{aligned} \tag{12}$$

In practice, the wavelet transform of the received data can be computed only at a finite number of scales n and translations m. Since the effective support width of g_{mn} doubles for each successively coarser scale, the wavelet representation of a finite-duration signal requires half the number of wavelet coefficients at each successively coarser scale. Thus, if at scale N_1, wavelet coefficients are needed for $-M_1 \le m \le M_1$, then at scale $N_1 + P$, the wavelet coefficients are needed for $-2^P M_1 \le m \le 2^P M_1$. Let N_1 be the coarsest scale to be considered and let $N_1 + N$ be the finest scale, where N is some positive integer; then $N_1 \le n \le N_1 + N$. Thus, a finite-term appr the approximate sufficient statistic

$$\begin{aligned} \Lambda_{(N_1,N,M_1)} &= \sum_{n=N_1}^{N_1+N} 2^{\beta n}/\sigma^2 \sum_{m=-2^{n-N_1}M_1}^{2^{n-N_1}M_1} (W^d r)(m,n)(W^d \gamma)(m,n) \\ &\triangleq \sum_{n=N_1}^{N_1+N} (2^{\beta n}/\sigma^2)\, \Lambda_n, \end{aligned} \tag{13}$$

where Λ_n is defined to be

$$\Lambda_n = \sum_{m=-2^{n-N_1}M_1}^{2^{n-N_1}M_1} (W^d r)(m,n)\, (W^d \gamma)(m,n).$$

This has an interesting interpretation. At each scale n, the statistic Λ_n is formed by correlating $(W^d r)(m,n)$ and $(W^d \gamma)(m,n)$—as functions of m—as if making a decision in white noise. The sufficient statistic $\Lambda_{(N_1,N,M_1)}$ is then formed by fusing these "white-noise statistics" together via a linear combination, with the scale n statistic Λ_n weighted by the reciprocal of the average noise power at scale n.

5 Conclusion

We have extended a theorem on the mapping of one RKHS to another, and have used that extension to identify a class of transforms that can be used to formulate the detection problem in alternative domains. In particular, we have shown that a unified formulation for the detection problem exists that allows the use of common time-frequency and time-scale representations such as the cross-Wigner distribution, the Gabor transform, and the wavelet transform. In addition, we have described a subclass of Cohen's class of representations that can be used in this formulation. Although the formulations given here have been for the case of a deterministic signal, the random signal case can be handled by replacing the representation of γ by an appropriate estimate; this results in an estimator-correlator form for the sufficient statistic.

It should be noted that the wavelet transform and the Gabor transform belong to a class of transforms that arise from square-integrable group representations [11]. Formulations of the sufficient statistic in terms of the members of this class can be developed by applying the results of Section 2 [13]. This provides a possible means of choosing a particular alternative domain based on similarities in the group structure of a transform's representation and properties of the additive noise process. For example, the wavelet transform and $1/f$ noise each exhibit a property known as self-similarity. This type of interplay was illustrated here by showing the effectiveness of the wavelet transform for the detection of signals in the presence of $1/f$ type noises.

6 Acknowledgement

This work has been supported by the Office of Naval Research under the Fundamental Research Initiatives Program.

References

[1] L. Cohen, "Time-frequency distributions — a review," *Proc. of IEEE*, vol. 77, pp. 941–980, July 1989.

[2] M. J. Bastiaans, "Gabor's expansion of a signal into Gaussian elementary signals," *Proc. of IEEE*, vol. 68, pp. 538–539, April 1980.

[3] I. Daubechies, "The wavelet transform, time-frequency localization and signal analysis," *IEEE Trans. Inform. Theory*, vol. 36, pp. 961–1005, Sept. 1990.

[4] B. V. K. V. Kumar and C. W. Carroll, "Performance of Wigner distribution function based detection methods," *Opt. Eng.*, vol. 23, pp. 732–737, Nov./Dec. 1984.

[5] B. Boashash and P. O'Shea, "A methodology for detection and classification of some underwater acoustic signals using time-frequency analysis techniques," *IEEE Trans. Acoust., Speech, and Signal Processing*, vol. 38, pp. 1829–1841, Nov. 1990.

[6] B. Friedlander and B. Porat, "Detection of transient signals by the Gabor representation," *IEEE Trans. Acoust., Speech, and Signal Processing*, vol. 37, pp. 169–180, Feb. 1989.

[7] F. B. Tuteur, "Wavelet transformations in signal detection," in *Wavelets: Time-Frequency Methods and Phase Space*. J. M. Combes, A. Grossmann, and Ph. Tchamitchian, Eds. Berlin: Springer-Verlag, 1989, pp. 132–138.

[8] T. Kailath, "RKHS approach to detection and estimation problems—part I: deterministic signals in Gaussian noise," *IEEE Trans. Inform. Theory*, vol. 17, pp. 530–549, Sept. 1971.

[9] B. L. Chalmers, "Images of bounded linear operators on a space with kernel function," *Archiv der Math.*, vol. 26, pp. 313–316, 1975.

[10] A. J. E. M. Janssen, "On the locus and spread of psuedo-density functions in the time-frequency plane," *Philips J. Res.*, vol. 37, pp. 79–110, 1982.

[11] A. Grossmann, J. Morlet, and T. Paul, "Transforms associated to square integrable group representations, I, General results," *J. Math. Phys.*, vol. 26, pp. 2473–2479, Oct. 1985.

[12] G. W. Wornell, "A Karhunen–Loève-like expansion for $1/f$ processes via wavelets," *IEEE Trans. Inform. Theory*, vol. 36, pp. 859–861, July 1990.

[13] M. L. Fowler and L. H. Sibul, "Signal detection using group transforms," in *Proc. of IEEE ICASSP,* 1991.