THE IMPACT OF SIGNAL MODEL DATA COMPRESSION FOR TDOA/FDOA ESTIMATION

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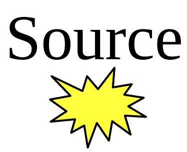
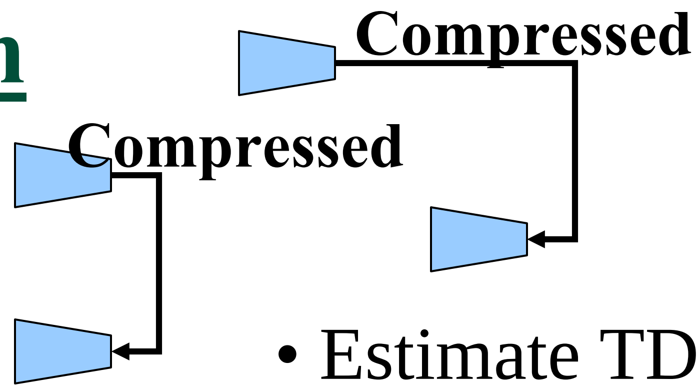
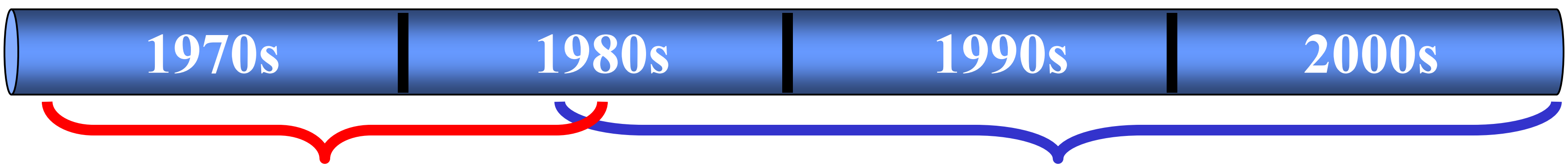
The Problem





- Estimate TDOA/FDOA for Pairs
- Use TDOA/FDOA to Locate



Sonar-Driven Research

Hann, Tretter, Knapp, Carter, Schultheis, Weinstein, Etc.

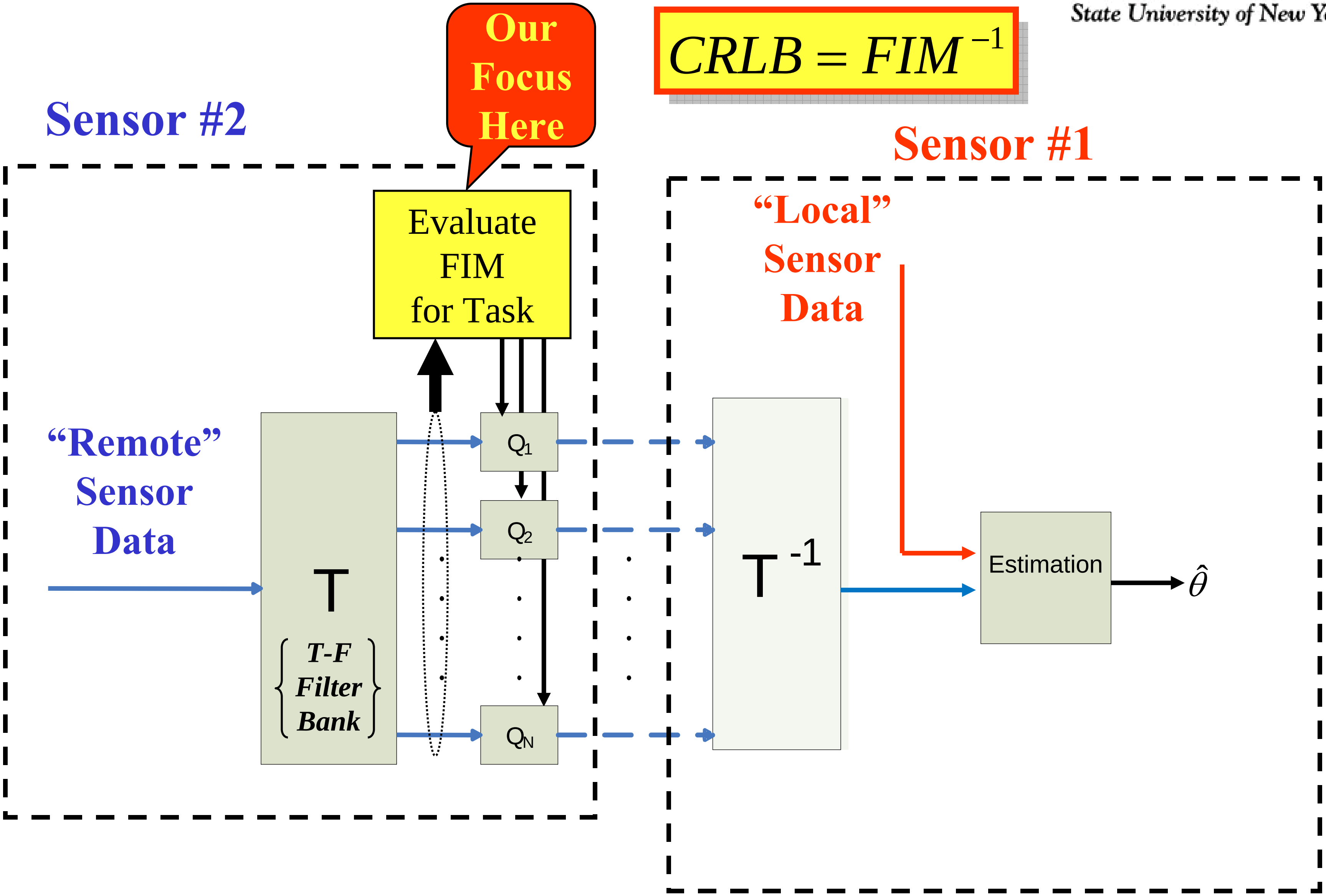
Radar/Comm-Driven Research

Stein, Chestnut, Berger, Blahut, Torrieri, Etc.

Question: How much of the Sonar TDOA/FDOA estimation work can be carried over to the Radar/Comm arena??

Answer: Not as much as many Radar/Comm researchers/practitioners think!

Compression Framework



Signals: Sonar vs. Radar/Comm

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Two sampled passively-received complex-valued baseband signals:

$$r_1[n] = s(nT - \tau_1)e^{j\nu_1 nT} + w_1[n]$$

$$r_2[n] = s(nT - \tau_2)e^{j\nu_2 nT} + w_2[n]$$

Noise Model

- Zero-mean WSS processes
- Gaussian
- Independent of each other

This much is the same for each case…

At least when the narrowband approximation can be used… which we assume here so we can focus on the impact of differences in the *statistical* model.

Models: Sonar vs. Radar/Comm

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- **Passive Sonar**
  - Signal = Sound from Boat
  - Erratic signal behavior
  - **Model as Random Process**
    - Zero-mean WSS
    - Gaussian
    - Independent of Noise
  - Expected values taken over signal + noise ensemble
    - Estimation accuracy is average over all possible noises and signals

- **Passive Radar/Comm**
  - Signal = Pulse Train
  - Structured signal behavior
  - **Model as Deterministic**
    - Specific pulse shape
    - Pulse width & spacing
  - Expected values taken over only noise ensemble
    - Estimation accuracy is average over all possible noises for one specific signal

PDFs: Sonar vs. Radar/Comm

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- For both cases the received data vector… is Gaussian.
- But how TDOA/FDOA is embedded is very different.

This is the key… it impacts significant differences in:

- **Fisher Info Matrix (FIM) / Cramer-Rao Bound (CRB)**
- **ML Estimator Structure**

Passive Sonar PDF:

TDOA/FDOA in Covariance

$$p_{ac}(\mathbf{r};\boldsymbol{\theta}) = \frac{1}{\det\left(\pi \mathbf{C}_{\boldsymbol{\theta}}\right)} \exp\left\{-\mathbf{r}^H \mathbf{C}_{\boldsymbol{\theta}}^{-1} \mathbf{r}\right\}$$

Passive Radar/Comm PDF

TDOA/FDOA in Mean

$$p_{em}(\mathbf{r};\boldsymbol{\theta}) = \frac{1}{\det\left(\pi \mathbf{C}\right)} \exp\left\{-(\mathbf{r}-\mathbf{s}_{\boldsymbol{\theta}})^H \mathbf{C}^{-1} (\mathbf{r}-\mathbf{s}_{\boldsymbol{\theta}})\right\}$$

FIM/CRB: Sonar vs. Radar/Comm

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- For general Gaussian case the elements of the FIM:

$$[J_{gg}]_{ij} = 2\,\mathrm{Re}\left(\left[\frac{\partial \boldsymbol{\mu}_{\boldsymbol{\theta}}}{\partial \theta_i}\right]^H \mathbf{C}_{\boldsymbol{\theta}}^{-1}\left[\frac{\partial \boldsymbol{\mu}_{\boldsymbol{\theta}}}{\partial \theta_j}\right]\right) + \mathrm{tr}\left(\mathbf{C}_{\boldsymbol{\theta}}^{-1}\frac{\partial\, \mathbf{C}_{\boldsymbol{\theta}}}{\partial\, \theta_i}\mathbf{C}_{\boldsymbol{\theta}}^{-1}\frac{\partial\, \mathbf{C}_{\boldsymbol{\theta}}}{\partial\, \theta_j}\right)$$

- Leads to VERY different forms for the two cases:

Passive Sonar FIM:

$$[J_{sonar}]_{ij} = \mathrm{tr}\left(\mathbf{C}_{\boldsymbol{\theta}}^{-1}\frac{\partial\, \mathbf{C}_{\boldsymbol{\theta}}}{\partial\, \theta_i}\mathbf{C}_{\boldsymbol{\theta}}^{-1}\frac{\partial\, \mathbf{C}_{\boldsymbol{\theta}}}{\partial\, \theta_j}\right)$$

Difficult to assess… usually use "Whittle's Theorem"
Depends on Covariance Sensitivity to Parameter

Passive Radar/Comm FIM:

$$[J_{radar}]_{ij} = 2\,\mathrm{Re}\left(\left[\frac{\partial \mathbf{s}_{\boldsymbol{\theta}}}{\partial \theta_i}\right]^H \mathbf{C}^{-1}\left[\frac{\partial \mathbf{s}_{\boldsymbol{\theta}}}{\partial \theta_j}\right]\right)$$

Easy to numerically assess…
Depends on Signal Sensitivity to Parameter

Impact of FIM: Sonar vs. Radar

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- **Because the forms are different… any sonar-case result is unlikely to carry over to radar-case:**

- **Passive Sonar**
  - TDOA and FDOA Estimates are **Uncorrelated**
    - Holds under mild assumption of large BT

- **Passive Radar/Comm**
  - TDOA and FDOA Estimates are **Correlated**

$$[\mathbf{J}_{em}]_{12} = 2\,\mathrm{Re}\left\{\frac{1}{\sigma_1^2}\sum_n -jnTs^*(nT-\tau_1)s'(nT-\tau_1) + \frac{1}{\sigma_2^2}\sum_n -jnTs^*(nT-\tau_2)s'(nT-\tau_2)\right\}$$

- **This has an impact on data compression…**

Compression: Sonar vs. Radar/Comm

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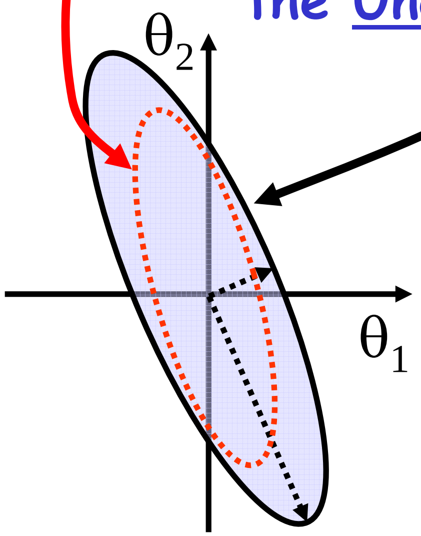
- Doing data compression for radar case we need to account for the non-zero off-diagonal FIM elements

Fowler/Chen ICASSP 2005

The "Correlation Issue"

Compressed FIM ellipse is inside the Uncompressed FIM ellipse



When estimates are highly correlated the compression can't perturb the correlation very much!!!

Fowler/Chen Sbmt. to T-AES

Theorem: *For the transform coding framework outlined above, the post-compression FIM has an information ellipse that lies inside the original FIM ellipse .*

R-D Viewpoints

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Three Views of "Source"

Probabilistic	Signal Class	Specific Input
Theoretical Bounds, Etc. (**Classical** R-D Theory)	Compression that exploits typical behavior	Compression optimized to exploit specific behavior ("**Operational**" R-D Theory)

- **Coding Scheme (*framework*)**: design based on typical features
- **Coding Parameters (*bit allocation*)**: chosen on input-by-input basis to optimize to a particular input

Operational R-D methods don't "care" what other possible realizations might occur "next time" – the only thing that matters is what does the data actually collected look like… ➔ Deterministic Signal Model!!!

Comments

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The two signal models lead to important differences in the results for the FIM.

We have argued… regardless of the type of signal expected, when using the FIM as a distortion measure in an operational rate-distortion sense the signal should be viewed as deterministic.