Non-MSE Data Compression for Emitter Location for Radar Pulse Trains

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ABSTRACT

This paper ties together and extends several recent results we have presented. We previously showed: (i) the usefulness of non-MSE distortion criteria in data compression for time-difference-of-arrival (TDOA) emitter location (SPIE 2001 & 2002), and (ii) the ability to exploit redundancy between radar pulses in a joint TDOA/FDOA (frequency-difference-of-arrival) location scheme (SPIE 2001 & 2002). In (ii) we showed how to compress radar signals by gating around the detected pulses and then putting the pulses into the rows of a matrix which is then compressed through use of the SVD; this approach employed a purely MSE distortion criterion. An open question in this approach was: Is it possible to eliminate some of the pulses from the pulse matrix to increase the compression ratio without significantly sacrificing location accuracy?

We resolve this question by applying our proposed non-MSE to the FDOA accuracy and finding the optimal set of pulses to remove from the pulse matrix. The removal of pulses is shown to have negligible impact on the FDOA accuracy but does degrade the TDOA accuracy from that achievable using the SVD-based compression without pulse elimination. However, we demonstrate that the SVD method includes an inherent de-noising effect (common in SVD-based signal processing) that provides an improvement in TDOA accuracy over the case of no compression processing; thus, the overall impact on TDOA/FDOA accuracy is negligible while providing compression ratios on the order of 100:1 for typical radar signals.

Keywords: data compression, singular value decomposition, emitter location, time-difference-of-arrival, TDOA, frequency-difference-of-arrival, FDOA

1. INTRODUCTION

A common way to locate electromagnetic emitters is to measure the time-difference-of-arrival (TDOA) and the frequency-difference-of-arrival (FDOA) between pairs of signals received at geographically separated platforms.[1,2,3] The measurement of TDOA/FDOA between these signals is done by coherently cross-correlating the signal pairs.[2,3] This requires that the signal samples of the two signals are available at a common platform, which is accomplished by transferring the signal samples over a data link from one platform to the other.

An important aspect of this processing that was not widely addressed in the literature until recently is that the available data link rate often is insufficient to accomplish the transfer within the time requirement unless some form of lossy data compression is employed. To mitigate this, we have proposed data compression methods[4–7], which can be grouped into two main categories of approach:

- exploiting redundancy between pulses when the emitter to be located is a radar and
- the more general approach of exploiting the relative importance of specific time-frequency components of general signals (i.e. communication or radar signals) through the use of a non-Mean-Square-Error (non-MSE) distortion measure.

This paper ties together and extends several recent results we have presented in these two areas. We previously showed: (i) the usefulness of non-MSE distortion criteria in data compression for time-difference-of-arrival (TDOA) emitter location[4,5] and (ii) the ability to exploit redundancy between radar pulses in a joint TDOA/FDOA (frequency-difference-of-arrival) location scheme[6,7]. In (ii) we showed how to compress radar signals by gating around the detected pulses and then putting the pulses into the rows of a matrix which is then compressed through use of the SVD; this approach employed a purely MSE distortion criterion. An open question in this approach was:

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Is it possible to eliminate some of the pulses from the pulse matrix to increase the compression ratio without significantly sacrificing location accuracy? We resolve this question by applying our proposed non-MSE to the FDOA accuracy and finding the optimal set of pulses to remove from the pulse matrix. The removal of pulses is shown to have negligible impact on the FDOA accuracy but does degrade the TDOA accuracy from that achievable using the SVD-based compression without pulse elimination. However, we demonstrate that the SVD method includes an inherent de-noising effect (common in SVD-based signal processing) that provides an improvement in TDOA accuracy over the case of no compression processing; thus, the overall impact on TDOA/FDOA accuracy is negligible while providing compression ratios on the order of 100:1 for typical radar signals.

The two signals to be correlated are the complex envelopes of the received RF signals. The two noisy received signals to be processed are modeled as

$$\hat{s}(k) = s(k) + n(k)$$

$$\hat{d}(k) = d(k) + v(k)$$

where $s(k)$ and $d(k)$ are the complex baseband signals of interest and $n(k)$ and $v(k)$ are complex white Gaussian noises. The signal $d(k)$ is a delayed and doppler shifted version of $s(k)$. The signal-to-noise ratios (SNR) for these two signals are denoted *SNR* and *DNR*, respectively[‡]. To cross correlate these two signals, one of them (assumed to be $\hat{s}(k)$ here) is compressed, transferred to the other platform, and then decompressed before cross-correlation, as shown in Figure 1. Signal $\hat{s}_c(k)$ has SNR of SNR_c after lossy compression/decompression.

In Section 2 we review our use of SVD for compression of radar pulse trains. In Section 3 we first review our earlier developments on distortion criteria that are not based on a pure mean-square error (MSE) measure and then develop results for pulse elimination based on our non-MSE distortion measures. In Section 4 we present simulation results and in Section 5 we provide concluding remarks.

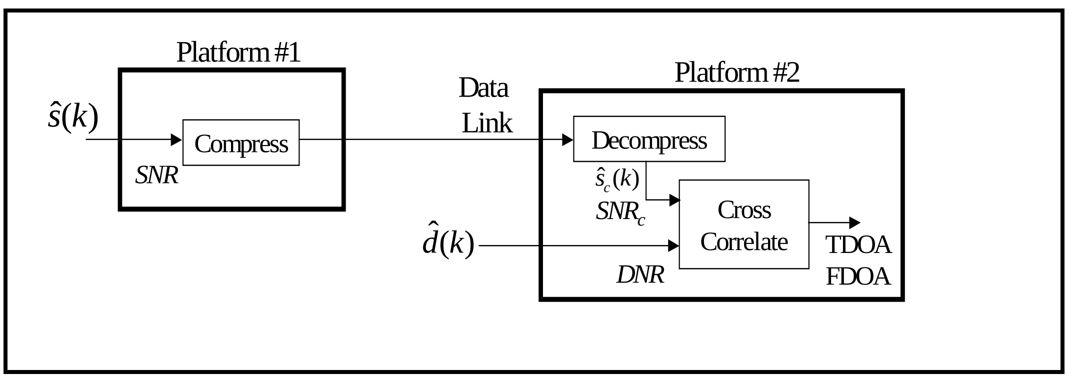

Figure 1: System Configuration for Compression

2. SVD-BASED DATA COMPRESSION FOR RADAR PULSE TRAINS

This section gives an overview of the previously proposed SVD-based method[6,7]. Before compression processing, Platform #1 receives and digitizes the radar waveform. It is assumed that the radar waveform contains a sequence of nearly-evenly spaced pulses and that the SNR is large enough at the compression platform to allow detection/gating of the pulses. Once the signal has been digitized, it undergoes data compression as described in Figure 2. It is clear that the pulses in the waveform, and their relative positions, contain the information required, and the "dead spaces" between the pulses contain no useful information. So, naturally, the first step in the data compression is to remove the unwanted samples between pulses by using pulse gating; all compression ratios stated here take the gated signal as the original, non-compressed signal (i.e., the reduction due to gating out the "dead spaces" is not included as part of the compression ratio).

[‡] SNR (non-italic) represents an acronym for signal-to-noise ratio; *SNR* (italic) represents the SNR for $\hat{s}(k)$.

Pulse gating[7] and pulse matrix formation are illustrated in Figure 3. By design, a typical radar-interception receiver detects the beginning and end of each pulse and measures the times at which these events occur. These time measurements will be called the measured leading edge (LE) and measured trailing edge (TE); for each detected pulse the number of samples that lie between TE and LE (inclusive) is the measured pulse width. In order to minimize the amount of side information that must be sent, the pulse gating method optimally chooses the parameters *G*, *W* and *T*.[7] After the Pulse Matrix is formed, the pulses in the rows are aligned using either fractional delay FIR filters or DFT-based processing in order to obtain an Aligned Pulse Matrix that has rank of nearly one[6]. The amount of alignment imparted to each pulse is sent as side information in the sequence $\Delta_1, \Delta_2, \Delta_3, \ldots$ as shown in Figure 2. If desired, the Aligned Pulse Matrix can be reshaped by putting multiple pulses per row – proper reshaping has been shown to maximize the compression ratio[6]; in this paper, though, we focus on the single-pulse-per-row case, although the results are applicable to the more general case. Because *W* is typically larger than the true pulse width, after alignment any excess columns outside the pulse width can be trimmed.

Let the size of this trimmed Aligned Pulse Matrix $\mathbf{P}$ be $p \times n$: there are p pulse rows and each pulse row has n samples. The resulting nearly rank-one matrix $\mathbf{P}$ is decomposed using the SVD, which is used create $\tilde{\mathbf{P}} = \mathbf{u}_1 \mathbf{v}_1^H$, an exactly rank-one approximation to the Aligned Pulse Matrix, where $\mathbf{u}_1$ and $\mathbf{v}_1$ are the left and right singular vectors (respectively) of corresponding to the largest singular value.[6] The information that is needed to reconstruct the signal is:

1. The $n \times 1$ right singular vector (RSV) $\mathbf{v}_1$ (i.e., the prototype pulse)
2. The $p \times 1$ left singular vector (LSV) $\mathbf{u}_1$ (i.e., the reconstruction magnitudes and phases)
3. The p-1 time shifts ($\Delta_1, \Delta_2, \Delta_3, \ldots, \Delta_{p-1}$)
4. The gating parameters *G*, *W*, and *T* (integers).

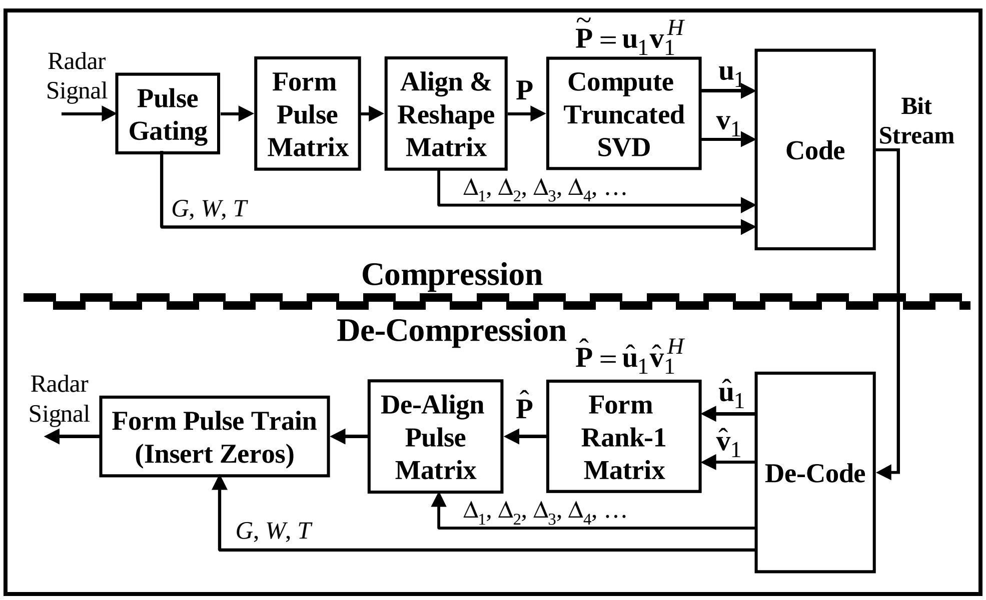

Figure 2: Overview of SVD-Based Compression

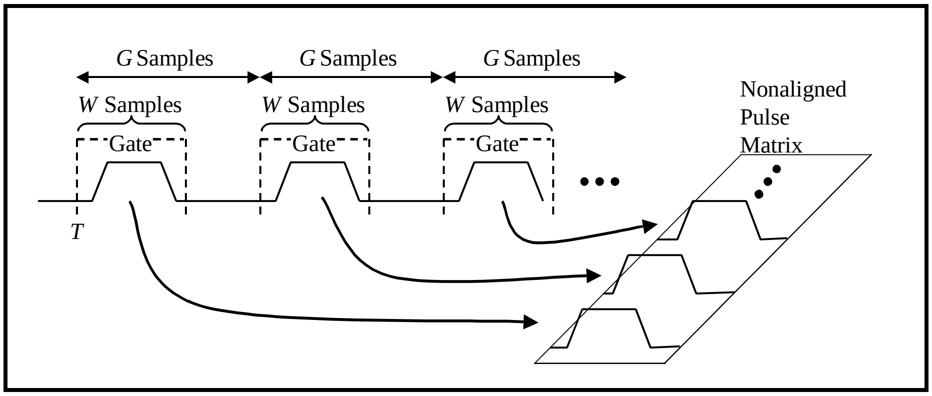

Figure 3: Pulse Gating and Pulse Matrix Formation

Effective methods for coding the singular vectors $\mathbf{u}_1$, $\mathbf{v}_1$ and the alignments Δ_1, Δ_2, Δ_3, … have been determined.[6] The compression ratio achieved by this single-pulse-per-row method is[6,7]

$$CR = \frac{16pn + 2p + 16}{22p + 9n + 24}.$$

From the denominator in the equation we note that the compression ratio is more sensitive to the value of p than to n, and that is the factor that motivates the exploration of putting multiple pulses per row[6]. However, if we can eliminate l rows from the pulse matrix $\mathbf{P}$ prior to the SVD processing, then only $p - l$ rows remain and the compression ratio becomes

$$CR = \frac{16pn + 2p + 16}{22(p - l) + 9n + 24},$$

which clearly will increase as l increases (i.e., as more rows are eliminated). The focus of this paper is to optimally choose which rows (i.e., pulses) to eliminate from the pulse matrix to allow increased compression ratio without significantly degrading the location accuracy. To answer this we look to the other compression approach we have proposed that uses non-MSE distortion criteria created specially for emitter location applications.[4,5]

3. NON-MSE DISTORTION CRITERIA AND PULSE ELIMINATION

To choose an appropriate distortion criteria for TDOA/FDOA applications it is important to understand the impact of compression on the TDOA/FDOA accuracies rather than its impact on the signal fidelity (e.g., MSE) as is commonly done in compression algorithms. We use the Cramer-Rao bounds (CRB) for TDOA/FDOA estimates[3] to gain insight into what signal characteristics impact TDOA/FDOA accuracies. From the structure of the CRBs one can determine what parts of the signal are most important for estimating TDOA/FDOA and use that as a guide for establishing distortion measures. In particular, we use this approach here to determine the proper way to eliminate pulses.

To ease our analysis we make the following assumptions on the complex baseband signal: (i) it consists of a train of pulses that have approximately equal pulse amplitude, (ii) it has been collected over the interval $t \in [-T/2, T/2]$, (iii) its temporal centroid[3] is at $t = 0$, (iv) its spectrum is inside the band $f \in [-W/2, W/2]$, and its spectral centroid[3] is at $f = 0$. All but the first assumption are easily assured in practice through proper "front-end" processing. While the first assumption rarely holds in practice it simplifies the analysis enough to allow derivation of theoretical results that can be compared to simulation results to validate the processing. The processing itself is applicable even when the first assumption does not hold.

1. Background On Non-MSE Distortion Criteria

After cross-correlation the output SNR is given by

$$SNR_o = \frac{WT}{\frac{1}{SNR} + \frac{1}{DNR} + \frac{1}{SNR \times DNR}}$$
$$\stackrel{\Delta}{=} WT \times SNR_{eff}$$

where WT is the time-bandwidth product (or coherent processing gain), with W being the noise bandwidth of the receiver and T being the duration of the received signal and SNR_{eff} is a so-called effective SNR[3]. The accuracies of the TDOA/FDOA estimates are governed by the Cramer-Rao bound (CRB) for TDOA/FDOA given by[3]

$$\sigma_{TDOA} \geq \frac{1}{2\pi\, B_{rms} \sqrt{2SNR_o}}$$

and

$$\sigma_{FDOA} \geq \frac{1}{2\pi\, D_{rms} \sqrt{2SNR_o}}, \tag{1}$$

where B_{rms} is the signal's RMS bandwidth in Hz given by

$$B_{rms}^2 = \frac{\int_{-W/2}^{W/2} f^2 |S(f)|^2 df}{\int_{-W/2}^{W/2} |S(f)|^2 df},$$

with $S(f)$ being the Fourier transform of the signal $s(k)$ and D_{rms} is the signal's RMS duration in seconds given by

$$D_{rms}^2 = \frac{\int_{-T/2}^{T/2} t^2 |s(t)|^2 dt}{\int_{-T/2}^{T/2} |s(t)|^2 dt},$$

where, without loss of generality, it is assumed that the signal is centered at $t = 0$ and the time interval over which the signal exists is symmetric around $t = 0$. The denominators in the equations for the RMS widths can be considered as normalizing factors on $|S(f)|^2$ and $|s(t)|^2$, respectively, so that these equations have the form identical to the equation for variance; thus, the root-mean-squared (RMS) terminology.

In TDOA/FDOA applications it is crucial that the compression methods minimize the impact on the TDOA/FDOA estimation error rather than stressing minimization of MSE as is common in many compression techniques. We have argued[4] that compression algorithms suitable for TDOA/FDOA applications should strive to maximize the denominators of the CRBs. Our work to date[4,5] has focused on choosing frequency components within a transform coding setting that will maximize the denominator of the TDOA CRB. In this paper we are interested in addressing the issue of eliminating pulses, which will have the largest effect on the RMS duration, and therefore will have the largest effect on FDOA accuracy.

2. Pulse Elimination Method

For our work here we assume that $SNR \approx DNR$ and $SNR >> 1$; future work will address other cases. Using these SNR assumptions gives

$$SNR_0 \equiv \frac{WT \times SNR \times SNR}{2 \times SNR + 1} \approx WT \times \frac{SNR}{2}$$

and then using this and

$$SNR \propto \frac{\int_{-T/2}^{T/2} |s(t)|^2 dt}{N_o W}$$

in the FDOA part of (1) gives

$$\sigma_{FDOA} \geq \frac{C_1}{\sqrt{\int_{-T/2}^{T/2} t^2 |s(t)|^2 dt}} , \tag{2}$$

where C_1 is some constant. In (2) we see that the pulses that are near the ends of the collected signal interval are more important than those in the center of the data, since the pulses near $t = 0$ have little contribution to the integral in (2). This insight motivates the following processing steps:

1. Select the number of pulses, η, is to be deleted.
2. Select: Compute $\Omega_i = \sum (n\Delta t)^2 |s_i(n\Delta t)|^2$ for each pulse.
3. Order: Order the Ω_i in the descendent order, $\Omega_i \geq \Omega_j$, $i > j$
4. Delete: Delete the smallest η number of Ω_i .
5. SVD: Use our previously proposed SVD compression method[6,7] to compression the rest of undeleted pulses.
6. Estimation: Use the ambiguity function to estimate the TDOA/FDOA[3].

Note that Step 2 accounts for the different pulse amplitudes and therefore the equal amplitude assumption is made only for the analysis of the algorithm.

3. Analysis of Pulse Elimination

While (2) describes the impact of pulse elimination on the accuracy of the FDOA estimate, we'd like a more explicit result. Under the equal-amplitude pulse assumption we can write the pulse train of N pulses as

$$s(t) = \sum_{i=1}^{N} s_o(t - t_i) e^{j\phi_i} , \tag{3}$$

where $s_o(t)$ is the prototype pulse, t_i is the time position of the i^{th} pulse, and ϕ_i is the phase of the i^{th} pulse. Using this in (2) we get

$$\sigma_{FDOA} \geq \frac{C_1}{\sqrt{\int_{-T/2}^{T/2} t^2 |s(t)|^2 dt}} \approx \frac{C_1}{\sqrt{\sum_{i=1}^{N} t_i^2} \times \sqrt{\int_0^{\Delta} |s_o(t)|^2 dt}} = \frac{C_{FDOA}}{\sqrt{\sum_{i=1}^{N} t_i^2}} . \tag{4}$$

This shows that under the equal pulse amplitude assumption, pulses should be eliminated starting with the center pulse and working bi-directionally outward.

We'd also like a result that allows us to understand the effect that pulse elimination has on the accuracy of the TDOA estimate. Similar to (2), when $SNR \approx DNR$ and $SNR >> 1$ we get

$$\sigma_{TDOA} \geq \frac{C_2}{\sqrt{\int_{-W/2}^{W/2} f^2 |S(f)|^2 df}} . \tag{5}$$

From (3) and properties of the Fourier transform we have that

$$S(f) = \sum_{i=1}^{N} S_o(f) e^{j(2\pi f t_i - \phi_i)}$$

and then

$$\begin{aligned} |S(f)| &= \left| \sum_{i=1}^{N} S_o(f) e^{j(2\pi f t_i - \phi_i)} \right| \\ &\leq \sum_{i=1}^{N} |S_o(f)| = N|S_o(f)|, \end{aligned}$$

where we have used the inequality $|X+Y| \leq |X| + |Y|$. Using this result in (5) then gives

$$\sigma_{TDOA} \geq \frac{C_2}{N\sqrt{\int_{-W/2}^{W/2} f^2 |S_o(f)|^2 df}}. \tag{6}$$

This shows that eliminating *l* of the *N* pulses would increase the bound in (6) to

$$\sigma_{TDOA} \geq \frac{C_2}{(N-l)\sqrt{\int_{-W/2}^{W/2} f^2 |S_o(f)|^2 df}} = \frac{C_{TDOA}}{(N-l)}. \tag{7}$$

Although there is no guarantee that the cross-correlation processing should achieve the bound in (7) it still gives some motivation to expect that the TDOA estimation error's standard deviation is likely to vary with *l* as $1/(N-l)$. Note that the effect on TDOA shown in (7) does not depend on which pulses are eliminated; the effect in (7) is simply due to the loss of processing gain due to elimination of pulses.

4. SIMULATION RESULTS

To illustrate the capabilities of SVD-based compression with pulse elimination we present some simulation results using a simulated radar signal (complex baseband) chosen to satisfy the assumptions made above. The signal is a train of pulses having linear FM modulation within each pulse. The simulated signal had the following parameters: 80 pulses, pulse width (PW) of 40μsec, a pulse repetition interval (PRI) of 70μsec, a maximum FM frequency deviation of ±2 MHz, and a sampling rate of 4.92346 MHz[§]. Monte Carlo simulations were performed to evaluate the standard deviation of the TDOA/FDOA estimation errors, where 400 runs were done for each evaluation.

The first set of simulation runs were performed to establish the baseline performance of the SVD-based compression method[6,7] without pulse elimination. The results are shown in Figure 4 where the estimation accuracy is assessed as a function of *SNR* (with *SNR* = *DNR*) both with and without SVD-based compression. Surprisingly, even though the compression ratio is 88.8:1, the compressed signal actually yields *better* TDOA/FDOA accuracy at the lower SNR values (and equivalent accuracy at the higher SNR values). This is due to the fact that the SVD provides and inherent de-noising property[6,8]; however, the de-noise seems to have less effect on the FDOA accuracy – we are currently working to better understand the nature of the SVD-based de-noising improvements.

[§] Notes: (i) The PRI was set low for convenience to reduce the total number of samples used in the processing; this ensures that the time necessary to run simulations is not unreasonably long; (ii) The sampling rate was chosen so the sampling interval was incommensurate with the PRI to ensure that the sampled pulses were not perfectly aligned.

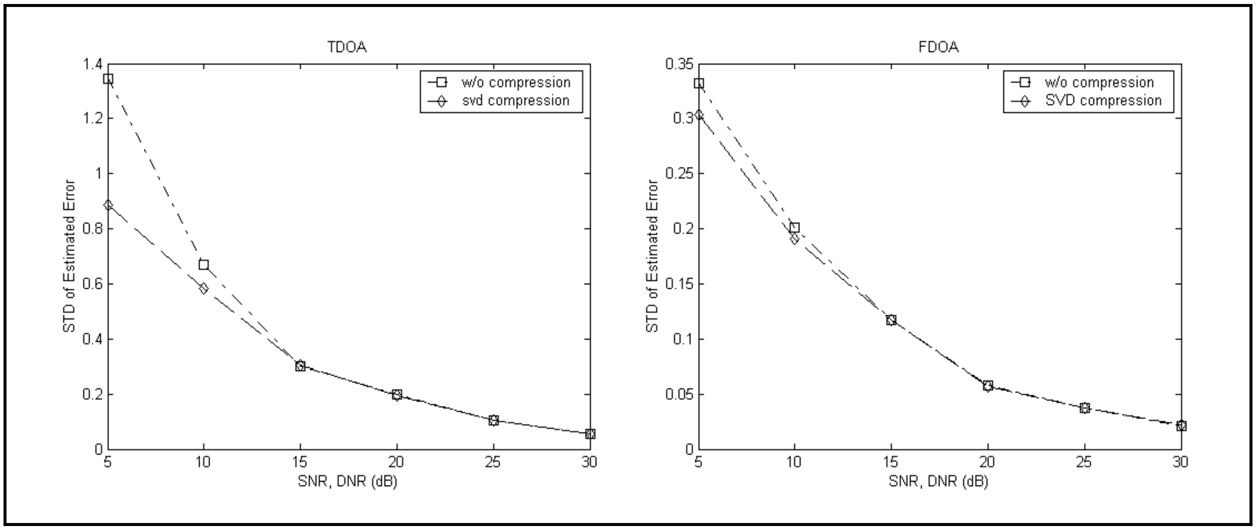

Figure 4: Simulation results showing standard deviation of TDOA/FDOA estimation error for a simulated radar signal when compressed 89:1 using SVD-based compression of the full pulse matrix (i.e., without pulse elimination); in each plot the SNRs of the two cross-correlated signals are set equal to each other as they are varied over a range. For comparison, results are shown for the case of no compression.

As we noted above, if pulses are eliminated according to the guidance of (2) then we expect negligible degradation in the FDOA accuracy up to a point; however, we would expect the TDOA estimation error standard deviation to increase according to (7). Fortunately, the results in Figure 4 indicate that, due to the de-noising, we have TDOA accuracy to "spare"; this, then is a perfect setting for applying pulse elimination. To first see how many pulses can be eliminated we ran simulations at the point $SNR = DNR = 10$ dB and evaluated the error standard deviation after SVD compression with pulse elimination, as shown in Figure 5, where the standard deviation values are presented relative to the value achieved without compression. For this case we see that we can eliminate half of the 80 pulses without suffering degradation in the FDOA accuracy, but there is some degradation of the TDOA when half of the pulses are eliminated. It should be noted that the shapes of the curves using "partial" compression roughly match the shapes predicted by (4) and (7).

Finally, for the case of eliminating half of the 80 pulses, we present results in Figure 6 that show how the TDOA/FDOA accuracy varies with SNR. Through elimination of half the pulses the compression ratio increases from 89:1 to 105:1 but the TDOA/FDOA accuracy is, remarkably, still roughly the same as when no compression is used.

5. CONCLUSIONS

We have fused two different approaches we have been independently pursuing for data compression for emitter location processing. The first method uses the SVD to exploit the pulse-to-pulse redundancy in a radar pulse train. The second uses new non-MSE distortion measures to guide the development of data compression methods that remove location-irrelevant parts of the signal. In the earlier investigations of the SVD methods it was recognized that elimination of pulses from the pulse matrix is desirable since it increases the compression ratio that can be achieved. However, questions remained regarding how to choose which pulses should be removed and what impact that removal would have on the TDOA/FDOA accuracy. In this paper we showed that the non-MSE approach provides answers to those questions. Through simulations we have shown that this method has great potential: achieving compression ratios of around 100:1 with virtually no degradation in the TDOA/FDOA accuracy.

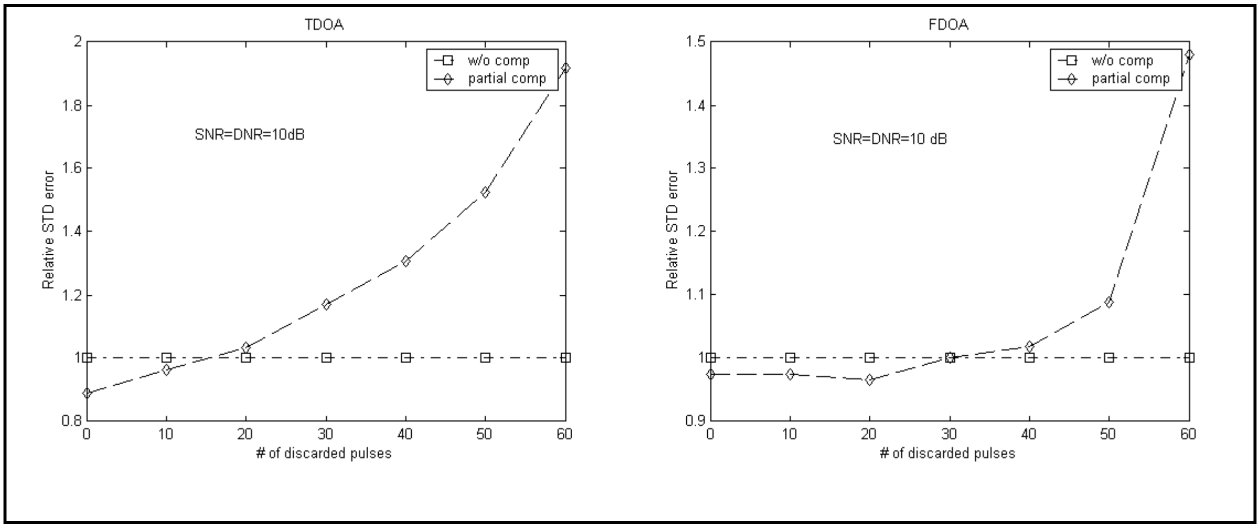

Figure 5: Simulation results showing the effect that discarding pulses has on the TDOA/FDOA estimation error standard deviation. The term "partial comp" means that SVD compression was done on a partial pulse matrix, after pulse elimination. For comparision, results are shown for the case without compression.

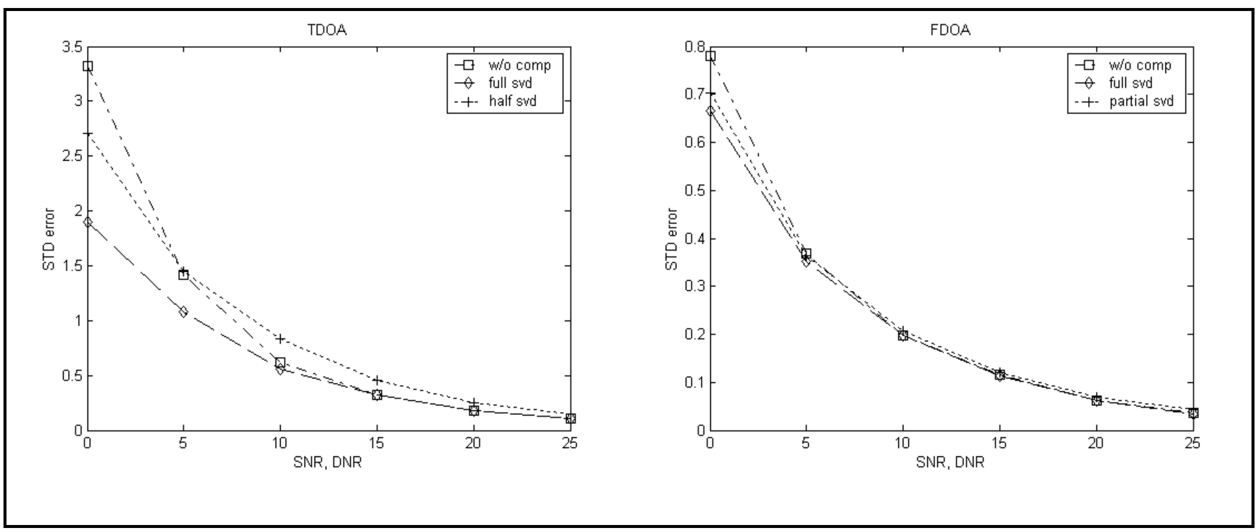

Figure 6: Simulation results showing standard deviation of TDOA/FDOA estimation error for a simulated radar signal when compressed using SVD-based compression. The three curves compare the cases (i) without compression, (ii) full SVD-based compression (i.e., no pulse elimination) with compression ratio of 89:1, and (iii) SVD-based compression where half of the pulses are eliminated to give a compression ratio of 105:1.

6. REFERENCES

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