Blind Channel Identification for the Emitter Location Problem:
A Least Square Approach

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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "Blind Channel Identification for the Emitter Location Problem: A Least Square Approach" by **Cheung C. Chau** in partial fulfillment of the requirements for the degree of **Master of Science**.

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Abstract

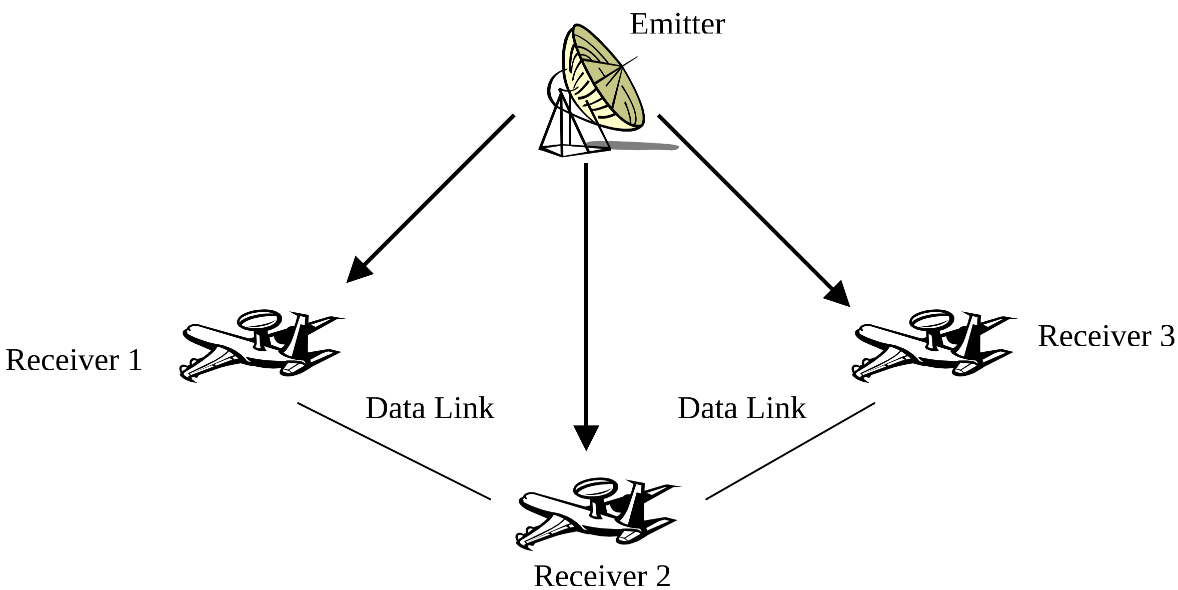
The location of radar emitters has been an important research topic for the military, for law enforcement and rescue operations. The characteristics of an emitted radar signal once received by several receivers can be exploited to locate the emitter. Two of these characteristics are: (1) the differences in Doppler shift in the signal's frequency and (2) the difference in the time that the emitted signal arrived at the sensors. These measurements are known as Frequency Difference of Arrival (FDOA) and Time Difference of Arrival (TDOA). While TDOA and FDOA information is sufficient to solve for the location of a radar emitter, the multipath effect obscures the accuracy of the location. A multipath transmission occurs when a transmitted signal arrives at the receiver by two or more paths of different delay and amplitude attenuation. For example, the signal can be received by direct path between emitter and receiver and also by reflections from other objects, such as mountains, buildings and so on. In this case, the emitted signal will arrive at the receiver in the form of a direct path signal plus various reflections with various delays. The undesired multipath effect should be corrected before the use of TDOA and FDOA. The problem can be solved by first identifying the channel and then from the channel estimate, single out the direct path signal. Unfortunately, a training sequence based channel identification algorithm cannot be used in this case since no training signal is available in trying to locate the emitter. As a result, there is a strong need for a special kind of channel identification algorithm, known as blind channel identification, that do not require the transmission of a training sequence. There are many existing blind channel identifications that are based on channel outputs and knowledge of

the probabilistic model of channel input. In the situation of emitter's location, the input statistical model may not be known, or there may not be sufficient data to obtain an accurate estimate of the statistics. In this paper, a deterministic Least Square approach is studied and computer simulations are used to demonstrate the potential of the proposed algorithm.

Chapter 1

Background

In the emitter location problem, the time-difference-of-arrival (TDOA) and the frequency-difference-of-arrival (FDOA) are often used to locate the electromagnetic emitters. The following diagram describes a system with one ground-based emitter at an unknown remote location and three receivers mounted on aircraft. The three receivers receive the emitted pulse trains to estimate the emitter location.



The TDOA is the time difference of arrival of the transmitted signal between two different receivers. For example, the pulse arrives at Receiver 1 at t_1 seconds and the same pulse arrives at receiver at t_2 seconds, then the Time-Difference-Of-Arrival (TDOA) becomes t_1- t_2. Also, the distance between the emitter and the receiver should be $R_1(t_1)$ and $R_2(t_2)$ respectively. Note that the distance between the emitter and the receiver is dependent on time. Since the emitter is assumed to be fixed, the times t_1 and t_2 become directly proportional to R_1 and R_2. That is

$$\begin{aligned} t_1 &= kR_1 \\ t_2 &= kR_2 \end{aligned} \tag{1.1}$$

which implies that

$$TDOA = t_1 - t_2 = k*(R_1 - R_2). \tag{1.2}$$

From this equation, the emitter should be located on the locus of a hyperbola, where the TDOA is a constant. In other words, the emitter should lie on the hyperboloid surface where receiver 1 and receiver 2 are the foci. Since the location of the emitter is 3-D, the TDOA between receiver 1, receiver 2 and receiver 3 cannot provide us the exact location of the radar emitter. One can either add another receiver to the system or exploit the FDOA of the system to locate the emitter. Due to the Doppler shift in the system, the center frequency of the received signal is different from the center frequency of the transmitted signal. For example, if the center frequency at receiver 1 and receiver 2 are f_1 and f_2 respectively, then FDOA of the system is defined as

$$FDOA = f_1 - f_2 \tag{1.3}$$

Once the TDOA and the FDOA estimates are calculated, the emitter location can be found. The concept of ambiguity function is often used to estimate the TDOA and FDOA of such system.

The ambiguity function is a time-frequency correlation function and is useful in the emitter location problem. The cross-ambiguity function (an ambiguity function between two different received signals) represents the energy in two different received signals as a function of time delay and Doppler shift. Let $s_1(t)$ and $s_2(t)$ be the two signals from two different receivers, then the cross-ambiguity function is defined as

$$C_{12}(\omega, \tau) = \int_{-\infty}^{\infty} s_1(t) s_2^{\ *}(t-\tau) e^{-j\omega t} dt \ , \tag{1.4}$$

where ω and τ represent various time delay and Doppler shift. If the two receivers are also fixed or FDOA = 0, then the cross-ambiguity function becomes

$$C_{12}(\tau) = \int_{-\infty}^{\infty} s_1(t) s_2^{\ *}(t-\tau) dt \ , \tag{1.5}$$

which is a function of time delay τ alone. This is the only case (FDOA = 0) will be considered here.

A multipath transmission takes place when a transmitted signal arrives at the receiver by two or more paths of different delays. For example, in this system, the signal can be received by direct path between the emitter and the receiver and also by reflections from other objects, such as mountains, buildings and so on. In this case, the received signal can be represented as the sum of several signals with different attenuation and different time delay. That is,

$$x(t) = \sum_{k=1}^{M} h_k s(t - \tau_k) \ , \tag{1.6}$$

where h_k represents the different attenuations and τ_k represents the different time delays of the original signal $s(t)$. Again, the received signal consists of multiple versions of the original signal.

The presence of multipath raises practical difficulties in the use of the ambiguity function to estimate the TDOA and the FDOA between receivers. Since the direct path signal is obscured by the reflections, we can no longer use the TDOA and the FDOA of the receiving signal to estimate the exact location of the emitter. For example, substitute equation (1.6) into (1.5), the cross-ambiguity function become

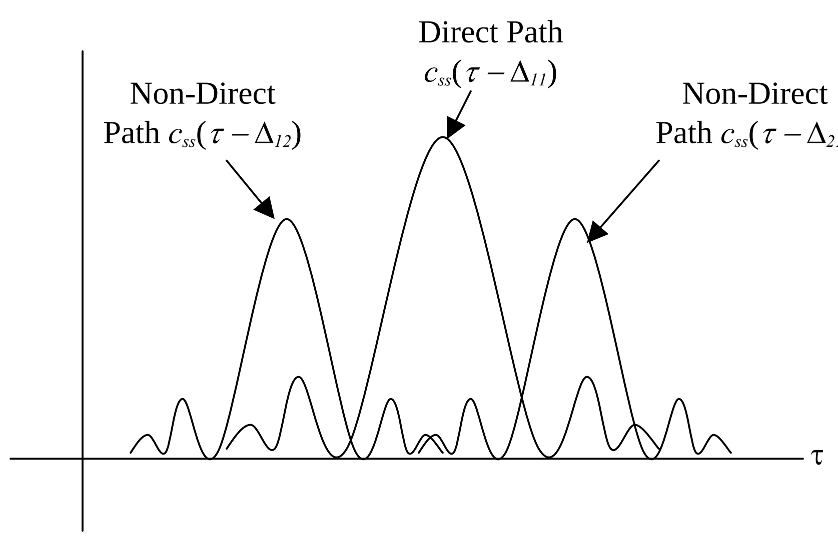
$$C_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t) x_2^*(t-\tau) dt$$

$$= \int_{-\infty}^{\infty} \sum_{k=1}^{M_1} h_{1k} s(t-\tau_k) \sum_{l=1}^{M_2} h_{2l} s(t-\tau_l-\tau) dt$$

$$= \sum_{k=1}^{M_1} \sum_{l=1}^{M_2} h_{1k} h_{2l} \int_{-\infty}^{\infty} s(t-\tau_k) s(t-\tau_l-\tau) dt$$

$$= \sum_{k=1}^{M_1} \sum_{l=1}^{M_2} h_{1k} h_{2l} c_{ss}(\tau-\Delta_{lk}) dt \,, \qquad (1.7)$$

where $\Delta_{lk} = \tau_l - \tau_k$ and $c_{ss}(\tau)$ is the autocorrelation function of the emitted signal $s(t)$. Also, note that equation (1.7) describes the case where FDOA is equal to zero. The following diagram illustrates how the $c_{ss}(\tau-\Delta_{lk})$ terms interfere with each other



By identifying the channel impulse response, the non-direct paths can be equalized or eliminated.

Traditionally, a training sequence can be used to identify the channel impulse response and further recover the direct path signal. The effects of multipath can be modeled as a channel input passing through a finite impulse response channel:

$$x[n] = \sum_{i=0}^{L} h[i]x[n-i] \tag{1.8}$$

where L is the order of the channel. The direct path signal can be recovered by eliminating all the channel coefficients corresponded to the non-direct path component in equation (1.7). However, in our case, there will be no training sequence available to the receivers for channel identification. In this situation, blind channel identification, a more sophisticated identification algorithm, is needed.

Chapter 2

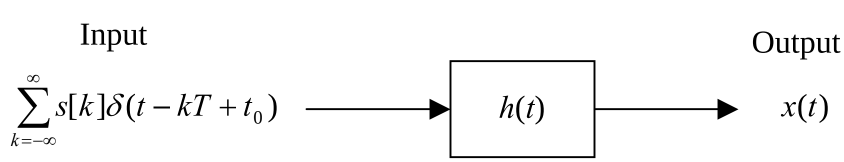
Theory

Blind Channel Equalization has become an important research topic in digital communication systems since the first published work by Y. Sato [1] in 1975. Channel equalizers and identifiers have played an important role in digital communication systems for decades. There do exist many different methods for implementing an optimal and efficient channel equalizers. Since many digital communication systems are often constrained by limited bandwidth, it is desirable to construct the channel equalizers without consuming excessive channel bandwidth. By eliminating training data in the more traditional approach of equalization, blind channel equalizations presents a bandwidth efficient solution to these equalization and identification problems. Also, there is practical need for many digital communication receivers (for example in HDTV and radar emitter location problem) to equalize unknown channels without the knowledge of the training sequences.

Blind equalization is a process during which one can recover an unknown input data sequence from the system output without the knowledge of the channel. One can immediately realize that this is a more challenging problem than the traditional equalization approach (with the training sequence). The problem of channel identification is almost identical to that of channel equalization. Given the input and output signal of a communication system, the parameters of this communication system can be uniquely identified if the input signal is continuously exciting. After the channel is identified, an optimal channel equalizer can be built accordingly.

Digital communication requires that digital signals be transmitted over some specific medium (channel) between the transmitter and the receiver. Analog media such as radio channel and telephone cables usually impart various types of distortion to the transmitted signal. Limited channel bandwidth, multipath and fading are the major types of these channel distortion. For most channels, channel equalizer can be used to remove or compensate these unwanted distortions. Traditionally, one can estimate and/or compensate the channel by sending a sequence of known training data through the channel [2]. This approach is considered impractical in situation like electronic warfare. Blind channel equalization presents a more realistic solution to channel equalization in these situations. This paper is dedicated to introduce how to apply the Least-Square approach to blind channel identification and equalization in the emitter location problem.

Major signals in digital communication system have a broad spectrum with a significant amount of low-frequency content. Transmission of these signals therefore requires an ideal analog channel. Typically, however, the channel is dispersive in that its impulse response deviates from the ideal. The problem of channel equalization can be addressed [6] using the system configuration shown below



where $\{s[k]\}$ is a sequence of baseband–equivalent complex valued random input in which the element $s[k]$ belongs to a complex constellation of QAM symbols. Linear time-invariant (LTI) channel $h(t)$ describes all the inter-connections between the transmitter and the receiver (including matched filter) at baseband. $x(t)$ is the LTI channel output (or the matched filter output) and can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} s[k]h(t - kT + t_0) + w(t) \tag{2.1}$$

Note that $w(t)$ is the AWGN added to the system and t_0 represents the arbitrary delay. The matched filter output is then sampled and written as

$$x[k] \equiv x(kT) \quad and \quad h[i] \equiv h(iT + t_0)\,. \tag{2.2}$$

The baud rate sampled input and output relationship can be written in discrete form as

$$x[k] = \sum_{i=-\infty}^{\infty} h[i]s[k-i] + w[k]\,. \tag{2.3}$$

The sampled channel is distortionless and ideal if

$$h[i] = h[v]\delta[i - v] \tag{2.4}$$

where v represents certain amount of time delay. The ideal channel output would be

$$x[k] = h[v]s[k - v]\,. \tag{2.5}$$

When the channel is dispersive, $h[i]$ is nonzero at more than one instance. The result of signal transmission over such a channel is that the channel output $x[k]$ depends on multiple symbols in $\{s[k]\}$. This interference, referred to as inter-symbol interference (ISI), is a major source of errors in the process of reconstructing our input sequence $\{s[k]\}$. As we can see from above mathematical model, ISI caused by multipath in bandlimited (frequency selective) time dispersive channels distorts the transmitted signal. ISI is one of the major obstacles to reliable high-speed signal transmission over a dispersive channel. Channel equalization is referred to the signal processing that removes or reduces ISI.

Channel equalization can be partitioned into two broad categories [5]: Maximum-Likelihood Sequence Estimation (MLSE) and equalization with filters. One can further partition the latter category into different subcategories. The filters can be either linear or non-linear. They can be grouped according to their ability to adapt in a time-varying environment. Also, they can be described corresponding to the sampling rate at the matched filter output, which may be either symbol spaced or fractionally spaced.

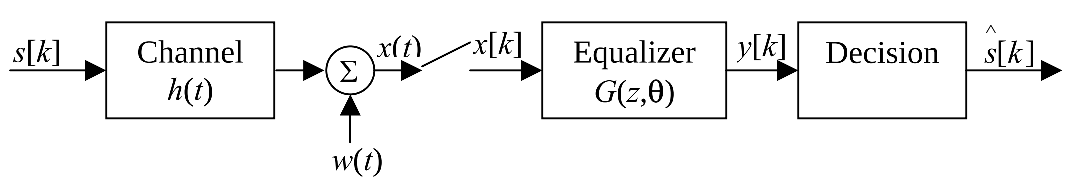
Let's first discuss how to accomplish channel equalization with a linear filter. These approaches rely on finding a near-ideal filter, which will reverse channel distortion. In most communication systems, the baud-rate sampled matched filter output $x[k]$ can be written as

$$x[k] = \sum_{i=-\infty}^{\infty} h[i]s[k-i] + w[k] \qquad (2.6)$$

Again, when the channel is NOT distortionless, its impulse response $h[k]$ can make the output $x[k]$ depend on more than one element of $\{s[k]\}$. Limited channel bandwidth, multi-path, and channel fading are three of the major causes of ISI in digital communication systems. The task of equalization with linear filters can be translated to the problem of channel identification. The channel in z-transform notation is defined as

$$H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k} \ . \qquad (2.7)$$

After the discrete channel $H(z)$ is identified, the equalizer $G_{mmse}(z,\theta)$ can be built (see fig below) according to some special criteria such as minimum mean square error (MMSE) criterion and zero-forcing criterion.



The purpose for this equalizer is to generate $y[k]$ such that after the decision device

$$\hat{s}[k] = s[k - v] \ . \tag{2.8}$$

This expression describes the desired channel-equalizer response to be a v-sample delay of the original signal with zero ISI. In MMSE, parameter vector θ of the equalizer $G(z,\theta)$ is selected in order to minimize the mean square error criterion between $s[k\text{-}v]$ and y[k] and equalizer can be written in z-transform [3] as

$$G_{mmse}(z,\boldsymbol{\theta}) = \frac{H^*(z^{-1})z^{-v}}{H(z)H^*(z^{-1}) + S_w(f)} \ , \tag{2.9}$$

where * denotes the complex conjugate and $S_w(f)$ represents the power spectrum density of $w[k]$. On the other hand, if the zero-forcing criterion is used, then the equalizer transfer function [3] becomes

$$G_{zf}(z,\boldsymbol{\theta}) = \frac{z^{-v}}{H(z)} \ . \tag{2.10}$$

Note that $H(z)$ is needed in either case to construct the equalizers. Also, from equation (3.10), one should conclude that zero-forcing equalizer tends to perform badly when $H(z)$ has zeros near the unit circle and when the signal to noise ratio is low.

In the linear equalizer describes above, the equalizer samples are assumed to be spaced at the reciprocal of the baud rate. This spacing is optimum if the matched filter before the equalizer is matched to the channel distorted transmitted pulse. Unfortunately, in most practical systems, the receiver filter is matched to the transmitted signal pulse instead and the sampling time is optimized for this suboptimum filter. This approach makes the equalizer very sensitive to the choice of sampling time. In contrast to the symbol rate equalizer, a fractionally spaced equalizer can be shown [3] to be an effective way to mitigate the timing difficulty. Another advantage of the FSE is that signal transmission may begin with an arbitrary sampling phase. The detailed proof of the above statements is beyond the scope of this paper. However, the application of fractionally spaced equalizer (FSE) is essential to the study of blind channel equalization using only second-order statistics. So this paper will introduce the framework of the FSE in this section.

A fractionally spaced equalizer (FSE) is obtained by sampling the channel output at a rate faster than the symbol rate. Let p be an integer and Δ be the new sampling rate, where $\Delta = T/p$. Now rewrite the channel output as

$$x(k\Delta) = \sum_{n=0}^{\infty} s[n]h(k\Delta - np\Delta + t_0) + w(k\Delta)\ . \tag{2.11}$$

Now divide the oversampled channel output $x(k\Delta)$ into p linearly-independent subchannels.

$$x_i[k] \equiv x[(kp+i)\Delta] = x(kT + i\Delta), \qquad i = 1, \ldots, p. \tag{2.12}$$

Also, express each subchannel transfer function as

$$H_i(z) = \sum_{k=0}^{\infty} h_i[k] z^{-k} \tag{2.13}$$

and

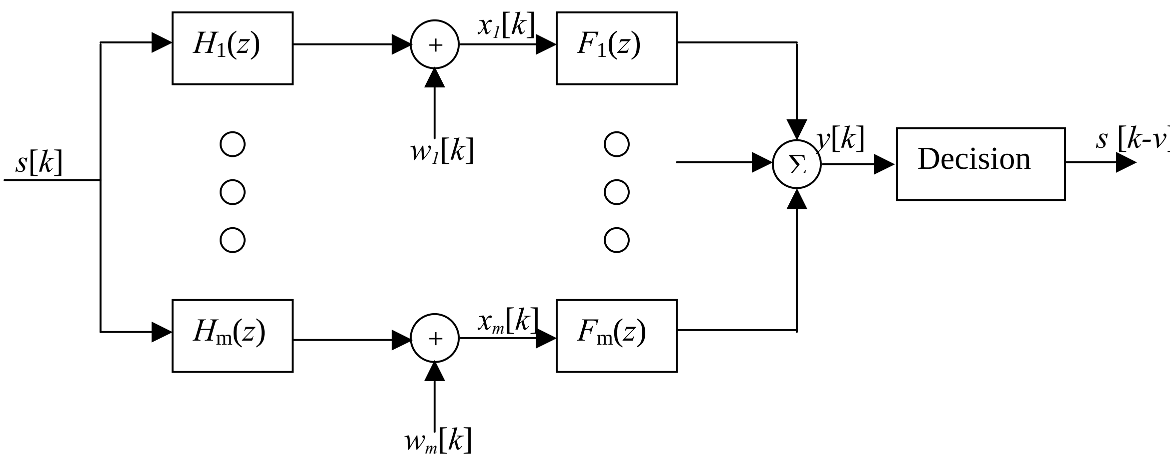
$$h_i[k] \equiv h(kT + i\Delta + t_0) \tag{2.14}$$

$$w_i[k] \equiv w(kT + i\Delta) \tag{2.15}$$

are the impulse response and noise for each subchannel respectively. The p subchannel outputs become

$$x_i[k] = \sum_{n=0}^{\infty} h_i[n]s[k-n] + w_i[k], \quad i = 1,\ldots, p. \tag{2.16}$$

Thus, the p different subchannels can be interpreted [6] as p discrete FIR channels with a common input $s[k]$ as shown below



From the figure above, for each subchannel output $x_i[k]$, there is a corresponding equalizer filter $F_i(z)$. In fact, the FSE equalizer in this case is a vector of filters

$$F_i(z) = \sum_{k=0}^{\infty} f_i[k]z^{-k}, \quad i = 1,\ldots, p. \tag{2.17}$$

Then, the output of these p different filters are summed to form the equalizer output

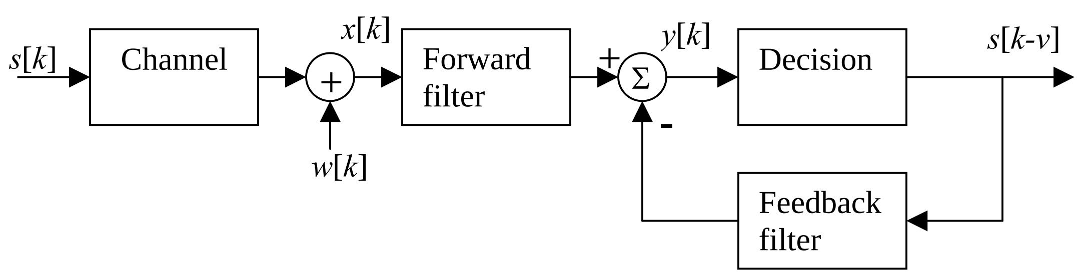
$$y[k] = \sum_{i=1}^{p}\sum_{n=0}^{m} f_i[n]x_i[k-n]. \tag{2.18}$$

As mention above, with the training sequence, FSE has the advantage of suppressing timing errors. Also, FSE can achieve zero-forcing criterion without amplifying the channel noise. That is

$$\sum_{i=1}^{p} F_i(z)H_i(z) = z^{-v} . \tag{2.19}$$

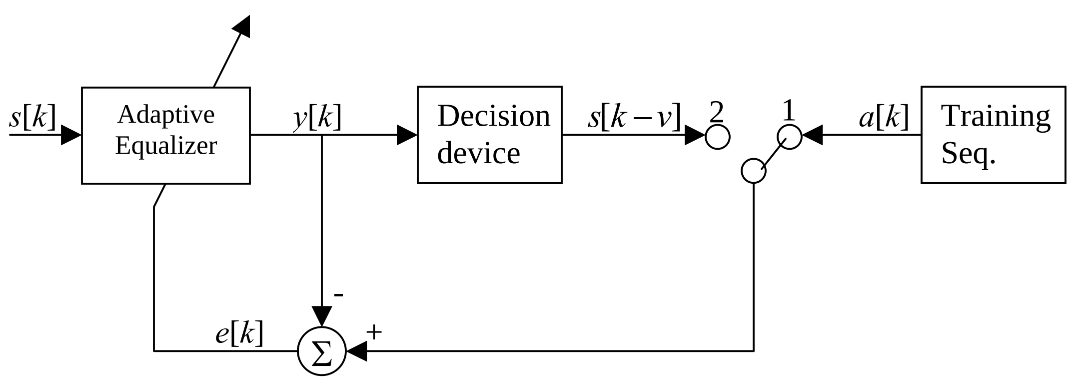
Also, this multiple channel view leads itself nicely and naturally to the emitter location problem. Since the emitter signal is intercepted by multiple receivers, in that case, each subchannel in the FSE is the physical channel between the emitter and the receiver intercepting the signal.

When the channel distortion is too severe for a linear equalizer to handle, nonlinear equalizers are used. Linear equalizer does not perform well on channels having spectral nulls. Such channels are often encountered in wireless communication systems. Sometime linear equalizer put too much gain in the vicinity of the spectral null, thereby enhancing the noise present in those frequencies. A decision feedback equalizer (DFE) is a nonlinear equalizer that uses previous detector decisions to remove the ISI on pulses that are currently being demodulated [3]. Its structure is shown below



The basic theory behind DFE is that if the symbols previously detected are assumed to be correct, then the ISI produced by these symbols can be eliminated at the output of the forward filter by subtracting past symbol values with appropriate weighting in the feedback filter. The forward and feedback filter can be attuned at the same time to satisfy certain criterion such as MMSE. As shown in [6], forward filter in this structure can be either symbol-spaced equalizer or FSE. The presence of the feedback filter makes DFE a nonlinear device. In fact, the major drawback of DFE is error propagation through this feedback filter. In some extreme situation, the error propagation will lead to a system breakdown.

In the development of the equalization methods above, we implicitly assumed that either the impulse response or the frequency response of the channel is time-invariant. However, in many cases, the equalizer has to deal with a time-varying channel. In these cases, the equalizer is designed to be adaptive to the time-varying channel. The adaptation may be initialized by minimizing the error between the desired (from the training sequence) outputs and the actual equalizer outputs. Then, the adaptation continues by minimizing the error between the decision device output and the equalizer outputs. The following diagram illustrates this criterion



From the diagram, the error is defined as

$$e[k] = a[k] - y[k] \qquad or \qquad e[k] = s[k - v] - y[k] \tag{2.20}$$

depends on the position of the connection (position 1 or position 2). The error $e[k]$ is used to estimate the direction in which the filter should be adjusted [2]. There are two different modes of operation in the adaptive equalizer: training mode (position 1) and decision-directed mode (position 2). During the training mode, a known training sequence is compared to the actual equalizer output to form the error signal. Then, by adjusting the equalizer's parameters, the mean square symbol error is minimized. After the equalizer's parameters reaches its optimum values, the equalizer switches to its decision-directed mode. In the decision-directed mode, the decision device output $s[k-v]$ can be used in place of training sequence to form the error signal and continue to track slow channel variations.

The use of a linear filter as an equalizer relies purely on the ability of the linear filter to reverse the channel distortion. The major drawback is that the sampled discrete channel must satisfy some strong condition in order to be compensated by linear equalizer. For baud-rate sampled channel outputs, the channel must have a stable inverse, namely, it must be of minimum phase. For fractionally sampled channel output, all subchannels must not share any common non-minimum phase zeros in order for a stable channel inverse to exist.

Also, these channel equalization approaches are followed by a symbol-by-symbol memoryless decision device, which does not take into consideration the fact that post-equalization noise is no longer white. Thus, a symbol-by-symbol decision ignores the noise correlation, and performance loss is often encountered in feed-forward and feedback equalizer. A better but more computational complex method is the use of

Maximum-Likelihood Sequence Estimation (MLSE). MLSE is what should be discussed next.

Assume that the channel has finite impulse response of order L. Then,

$$x[k] = \sum_{i=0}^{L} h[i]s[k-i] + w[k]\,, \tag{2.21}$$

where $w[k]$ is white Gaussian noise. Given that the channel impulse response is known or has been estimated, the input sequence can be estimated by minimizing

$$\sum_{k=L}^{\infty} \left| x[k] - \sum_{i=0}^{L} h[i]s[k-i] \right|^2 \tag{2.22}$$

Note that by minimizing the above expression, we are implicitly maximizing the likelihood function. If $s[k]$ is restricted to M different symbols, then the Viterbi algorithm can be implemented to find the most likely input sequence from M^L different possible input sequences (or states). Thus, the M^L states represent all possible combinations of the following L-tuples

$$(s[k-1], s[k-2], \cdots, s[k-L])\,. \tag{2.23}$$

The trellis diagram is determined by the number of states (number of possible different input sequences) while the metrics of the Viterbi algorithm depends on the estimated channel $h[k]$. Thus, estimation of the channel impulse response is crucial for the implementation of the MLSE. Since it provides the minimum probability of a symbol error under AWGN, the MLSE is optimum. However, It is a nonlinear channel equalizer and becomes more complicated when number of states M^L getting large.

Chapter 3

Blind Channel Equalization

As was shown from the previous section, knowledge of channel impulse response is essential in designing our equalizer filter and MLSE metrics. Identification of unknown channel impulse response can be achieved with a training sequence. In some cases, sending a training sequence is considered impractical (see introduction). Also, the use of training sequence can reduce bandwidth efficiency.

A better way to solve this problem comes from the use of blind channel equalization. There are two ways to approach this blind equalization problem. The first one is to use blind channel identification to define the channel impulse response. Once the channel impulse response is known, an appropriate equalizer or MLSE metrics can be constructed to remove the ISI. The second approach is to eliminate the step of blind channel identification and directly derive the equalizer from input statistics and the channel output. The latter approach will be introduced, however, the focus of this paper is on the first approach.

Let’s first discuss the direct blind channel equalization. Since the training sequence is missing, the receiver does not have access to the desired equalizer output ($a[k]$ in the previous diagram). Obviously, in blind channel equalization, there is a need to define a non-MSE type cost function to be minimized. The design of the blind equalizer thus becomes a matter of defining a special mean cost function $E\{\Psi(\cdot)\}$, which implicitly involves higher order statistics of the channel output. By exploiting these higher order statistics, the channel transfer function phase can be recovered. The reason for missing transfer function phase response is that the second order statistics, namely, the power spectrum density function provides information only on the channel magnitude response. Consider the following linear input-output relationship of a single-input-single-output (SISO) channel

$$S_x(\omega) = \left|H(\omega)\right|^2 S_s(\omega) + S_w(\omega)\ . \tag{3.1}$$

From the above expression, one can clearly conclude that the phase information of the channel $H(\omega)$ cannot be retrieved from the power spectrum of the channel input $S_s(\omega)$, channel output $S_x(\omega)$ and the channel noise $S_w(\omega)$. The following few paragraphs should summarize several method of direct blind adaptive equalization.

By removing the training mode from the adaptive equalizer, the decision direct mode alone becomes the simplest form of adaptive blind channel equalization. It minimizes the MSE between the equalizer output and the decision output (see Chapter 3 about adaptive equalizer). The performance of this equalization approach is highly dependent on the equalizer's initial condition. That is, local convergence is likely if there are significant differences between the initial parameter and the actual parameter of the channel impulse response.

In 1975, Sato [1] introduced the first truly blind equalizer for multilevel PAM signals. The equalizer is actually identical to the decision direct mode of an adaptive equalizer when the PAM input is binary. In this algorithm, the special error function is defined as

$$\psi(x) = \Psi'(x) = x - R * \mathrm{sgn}(x), \qquad where\ \ R \equiv \frac{E\left|s[k]\right|^2}{E\left|s[k]\right|}\ . \tag{3.2}$$

In 1980, Benveniste et al. [8] generalized the Sato's error function into

$$\psi_b(y[k]) = \psi_a(y[k]) - R_b * \operatorname{sgn}(y[k]), \qquad where \;\; R_b \equiv \frac{E\{\psi_a(s[k]) * s[k]\}}{E|s[k]|}. \tag{3.3}$$

The generalization uses an odd function $\psi_a(x)$, whose second derivative should satisfy

$$\psi_a^{''}(x) \geq 0, \qquad for \qquad \forall x \geq 0. \tag{3.4}$$

Then the so-called “stop-and-go” algorithm was introduced by Picchi and Prati [9] in 1987. Picchi figures that the sign of the error signal $\psi(y[k])$ often determines the convergence characteristics of these adaptive algorithms. So, this “stop-and-go” algorithm allows adaptation “to go” only when the several derivative functions agree in sign for current output $y[k]$. When the derivative functions sign differ for current output $y[k]$, parameter adaptation is stopped and maintain their current values.

One of the most popular blind channel equalization method known as Constant Modulus Algorithm (CMA) was presented in [10]. By integrating the Sato error function $\psi(x)$. The Sato algorithm has equivalent cost function

$$\Psi(y[k]) = \frac{1}{2}(|y[k]| - R)^2. \tag{3.5}$$

Godard generalized this expression in his paper [10] and specified a new class of error function as

$$\Psi_q(y[k]) = \frac{1}{2q}(|y[k]|^q - R_q)^2, \qquad\qquad q = 1, 2, \cdots \tag{3.6}$$

where

$$R_q \equiv \frac{E|s[k]|^{2q}}{E|s[k]|^q}. \tag{3.7}$$

Using stochastic gradient descent approach, the Godard algorithm can adaptively adjusts the equalizer’s parameter to its optimum value in most cases. For further study of these adaptive blind methods and its convergence analysis, the reader should consult a book like [6]. Although adaptive SISO blind channel equalization can give satisfactory result under appropriate initialization, they have two major weaknesses. First, local convergence is more than likely in many adaptive blind equalization schemes. Also, the convergence of these schemes is somewhat slow and may require a large amount of data samples. In a fast time-varying environment, the second requirement is utterly undesirable. Recently, new and better schemes have been proposed to overcome these two major drawbacks. By exploiting the spectral diversities of oversampled channel output or the use of multiple sensors, the blind channel identification can be achieved with only the second order statistics [11][12][13].

Chapter 4

The Least Square Approach

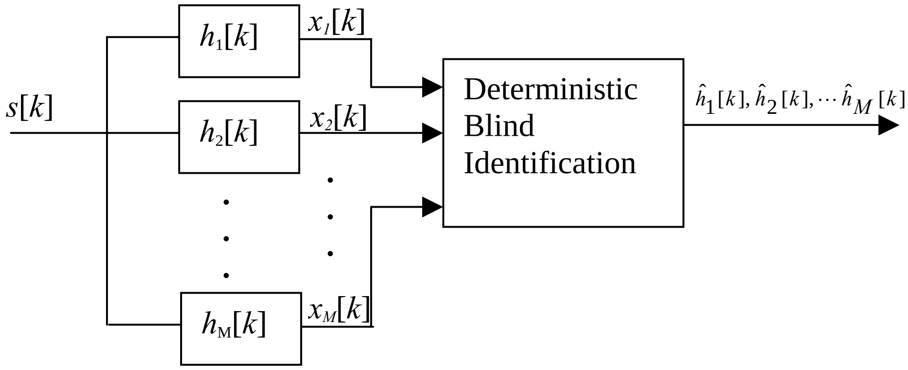
As mentioned in the previous section, the second-order stationary statistics do not contain sufficient information on a nonminimum phase system. Gardner [13] proved that the second-order statistics (SOS) of cyclostationary signals provide enough information for the phase recovery. There are many different papers that have been written on blind channel identification based on SOS, i.e., TXK method [11], Subspace method [12] and Gardner [13]. However, this paper will focus on a new innovative approach, namely, the Least-Square approach to this blind channel identification problem [14].

In the papers [11] [12] [13], the authors assumed certain statistical model for the channel input. However, in many cases, the statistics may not be available or impossible to obtain. For example, there may not be enough data samples to estimate an accurate statistical model in a fast fading environment. It can be proved that, under certain conditions, it is possible to identify multichannel FIR system without even having knowledge of statistical model of the channel input [14].

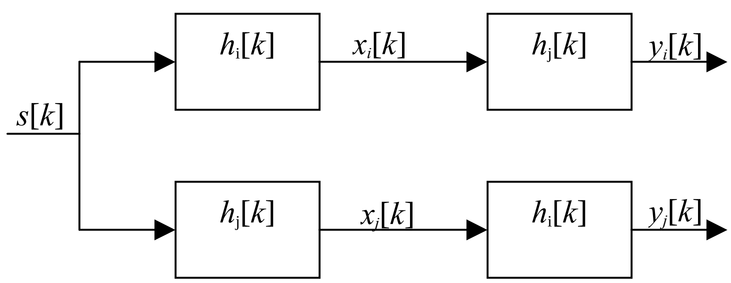
The basic idea behind this innovative approach is the concept of subchannel matching. Before going into the detail of subchannel matching. Let's first restates the problem of blind channel identification

$$x_i[k] = \sum_{j=0}^{L} h_i[j]s[k-j], \qquad i = 1 \cdots M , \tag{4.1}$$

where M is the number channels and L is the maximum order the M channels. The blind channel identification problem can be stated as follows: Given the observations of channel output $\{x_i[k],\ i = 1, \ldots, M;\ k = L, \ldots, N\}$, determine the channel $\{h_i[k]\}_{i=1}^{M}$ and further recover the input signals $\{s[k]\}$. The following schematic illustrate the concept of this deterministic blind identification approach.



Clearly, this is an indirect blind channel equalization problem. Note that the concept of multichannel should not be restricted to multiple physical receivers or sensors. Oversampled channel output as in FSE can also be modeled as single-input multi-output system. At first, this identification problem does not seem to have a solution. By observing each individual channel, obviously, it is impossible to determine $h_i[k]$ and $s[k]$ uniquely without any training signal. Here is where the concept of subchannel matching comes in. Consider any two pairs of signals in our SIMO system



Assume the system is noiseless, and

$$x_i[k] = h_i[k] \otimes s[k] \tag{4.2a}$$

$$x_j[k] = h_j[k] \otimes s[k]\,. \tag{4.2b}$$

Then

$$\begin{aligned} y_i[k] = h_j[k] \otimes x_i[k] &= h_j[k] \otimes \left(h_i[k] \otimes s[k]\right) \\ &= h_i[k] \otimes \left(h_j[k] \otimes s[k]\right) \\ &= y_j[k] \end{aligned} \tag{4.3}$$

where $\otimes$ stands for convolution. For SIMO systems with M subchannels, there will be $M(M\text{-}1)/2$ such output pairs. The use of such subchannel matching is the fundamental idea behind the new blind channel identification.

Subchannel matching between the i^{th} and the j^{th} outputs in matrix equation requires that

$$\begin{bmatrix} \mathbf{X}_i(L) & -\mathbf{X}_j(L) \end{bmatrix} \begin{bmatrix} \mathbf{h}_j \\ \mathbf{h}_i \end{bmatrix} = \mathbf{0}\,, \tag{4.4}$$

where $\mathbf{h}_m \equiv \left[h_m[L], \quad \cdots \quad , h_m[0]\right]^T$ and

$$\mathbf{X}_m(L) = \begin{bmatrix} x_m[L] & x_m[L+1] & \cdots & x_m[2L] \\ x_m[L+1] & x_m[L+2] & \cdots & x_m[2L+1] \\ \vdots & \vdots & \ddots & \vdots \\ x_m[N-L] & x_m[N-L+1] & \cdots & x_m[N] \end{bmatrix}, \tag{4.5}$$

where $\mathbf{X}_m(L)$ is a (N-2L+1) by (L+1) matrix. Now, combine all the channel coefficients into a single vector

$$\mathbf{h} \equiv [\mathbf{h}_1^T, \mathbf{h}_2^T, \cdots, \mathbf{h}_M^T]^T\,, \tag{4.6}$$

where $\mathbf{h}$ is a $M(L+1)$ by 1 vector. Also, combine all the subchannel matches with respect to i^{th} subchannel and express in matrix form

$$\mathbf{X}^i(L) = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{X}_{i+1}(L) & -\mathbf{X}_i(L) & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \underbrace{\mathbf{0} \quad \cdots \quad \mathbf{0}}_{i-1 \ blocks} & & & \underbrace{\mathbf{X}_M(L) \quad \mathbf{0} \quad \cdots \quad -\mathbf{X}_i(L)}_{M-i+1 \ blocks} & & & \end{bmatrix}. \tag{4.7}$$

For example, for $M = 5$,

$$\mathbf{X}^1(L) = \begin{bmatrix} \mathbf{X}_2(L) & -\mathbf{X}_1(L) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{X}_3(L) & \mathbf{0} & -\mathbf{X}_1(L) & \mathbf{0} & \mathbf{0} \\ \mathbf{X}_4(L) & \mathbf{0} & \mathbf{0} & -\mathbf{X}_1(L) & \mathbf{0} \\ \mathbf{X}_5(L) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{X}_1(L) \end{bmatrix} \tag{4.8}$$

and

$$\mathbf{X}^2(L) = \begin{bmatrix} \mathbf{0} & \mathbf{X}_3(L) & -\mathbf{X}_2(L) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_4(L) & \mathbf{0} & -\mathbf{X}_2(L) & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_5(L) & \mathbf{0} & \mathbf{0} & -\mathbf{X}_2(L) \end{bmatrix} \tag{4.9}$$

and so on. In general, one can put all these M signal matrices into a matrix $\mathbf{X}(L)$, where

$$\mathbf{X}(L) = \left. \begin{bmatrix} \mathbf{X}^1(L) \\ \vdots \\ \underbrace{\mathbf{X}^{M-1}(L)}_{M \ blocks} \end{bmatrix} \right\} \frac{M(M-1)}{2} \ blocks \tag{4.10}$$

and $\mathbf{X}(L)$ is $(N-2L+1)\dfrac{M(M-1)}{2}$ *by* $(L+1)M$ matrix. Now, one can incorporate all $M(M\text{-}1)/2$ pairwise matching equations to estimate the unknown channel impulse response by solving the matrix equation

$$\mathbf{X}(L)\mathbf{h} = \mathbf{0}\,. \tag{4.11}$$

Apparently, in the noiseless case, vector $\mathbf{h}$ is in the null space of the above matrix equation. In order to use the Least-Square approach, rewrite this expression in the equivalent scalar form

$$\|\mathbf{X}(L)\mathbf{h}\|^2 = \mathbf{h}^H \mathbf{X}(L)^H \mathbf{X}(L)\mathbf{h} = 0\,. \tag{4.12}$$

Now, define matrix **A** such that

$$\mathbf{A} = \mathbf{X}(L)^H \mathbf{X}(L)\,. \tag{4.13}$$

Then, the condition in equation (5.12) can be rewritten as

$$\mathbf{h}^H \mathbf{A}\mathbf{h} = 0\,. \tag{4.14}$$

By solving equation (4.14) with singular value decomposition (SVD), the channel impulse response can be identified up to a constant ambiguity. That is, the eigenvector of **A** correspond to the zero eigenvalue of **A** is the best estimate of **h** with a constant ambiguity. To prove this, let λ_i and $\mathbf{u}_i$ be the i^{th} eigenvalue and eigenvector of matrix **A** respectively, that is

$$\mathbf{A}\mathbf{u}_i = \lambda_i \mathbf{u}_i\,. \tag{4.15}$$

By choosing the eigenvector $\mathbf{u}_i$ for which $\lambda_i = 0$, and substitute $\mathbf{u}_i$ for **h** in equation (4.14)

$$\begin{aligned}
\mathbf{h}^H \mathbf{A}\mathbf{h} &= \mathbf{u}_i^H \mathbf{A}\mathbf{u}_i && \textit{by } (4.15) \\
&= \mathbf{u}_i^H \lambda_i \mathbf{u}_i \\
&= \lambda_i \underbrace{\mathbf{u}_i^H \mathbf{u}_i}_{a\ scalar} && \textit{where } \lambda_i = 0 \\
&= 0
\end{aligned}$$

This shows that $\mathbf{u}_i$ is the best estimate of **h** and is unique if matrix **A** has only one zero eigenvalue.

As mentioned before, this least square method does not assume any statistical model for the channel input. This advantage leads to a simpler implementation scheme. In the case where noise is present, the linear equations become non-homogenous. However, one can still estimate **h** by solving the following least squares problem:

$$\min_{\hat{\mathbf{h}}} \left\| \mathbf{X}(L)\, \hat{\mathbf{h}} \right\|^2 \tag{4.16}$$

or equivalently,

$$\min_{\hat{\mathbf{h}}} \hat{\mathbf{h}}^H \mathbf{A} \hat{\mathbf{h}} . \tag{4.17}$$

The estimated channel impulse response $\hat{\mathbf{h}}$ should be equal to the eigenvector of **A** that corresponds to the smallest eigenvalue of matrix **A**.

Although the proposed channel identification algorithm does not assume any particular statistical model for the channel input, however, the input characteristics are certainly not to be neglected. An obvious degenerate case is that the channel input contain only zero. In that case, channel output will provide no information about the channel impulse response. Now, consider the identifiability conditions of SIMO blind identification [14]. First, the channel impulse response can be identified uniquely if and only if the data matrix $\mathbf{X}(L)$ is of rank $M(L+1)$-1. Second, the blind identification solution is unique if the subchannel impulse response $h_i[k]$ and $h_j[k]$ do not share any common zero [6]. That is, if $h_i[k]$ and $h_j[k]$ are the coefficients for two different polynomials, then the two polynomials should not share any common roots. These two conditions give the sufficient conditions for blind identification of SIMO system. Signals, such as white noise, are rich in frequency content and therefore the sufficient conditions can be matched by increasing the number of data samples. However, the degeneration can easily occur when the data sequence is extremely short.

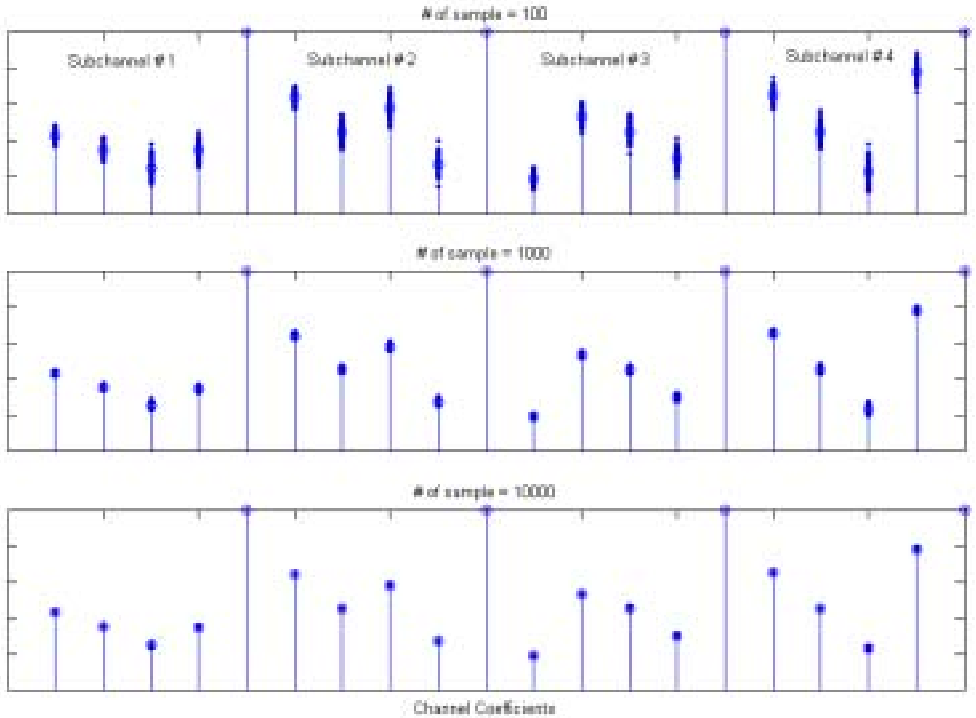
Chapter 5

Computer simulations

Computer simulations were conducted to evaluate the performance of the proposed algorithm. A set of MatLab programs was written to perform these computer simulations.

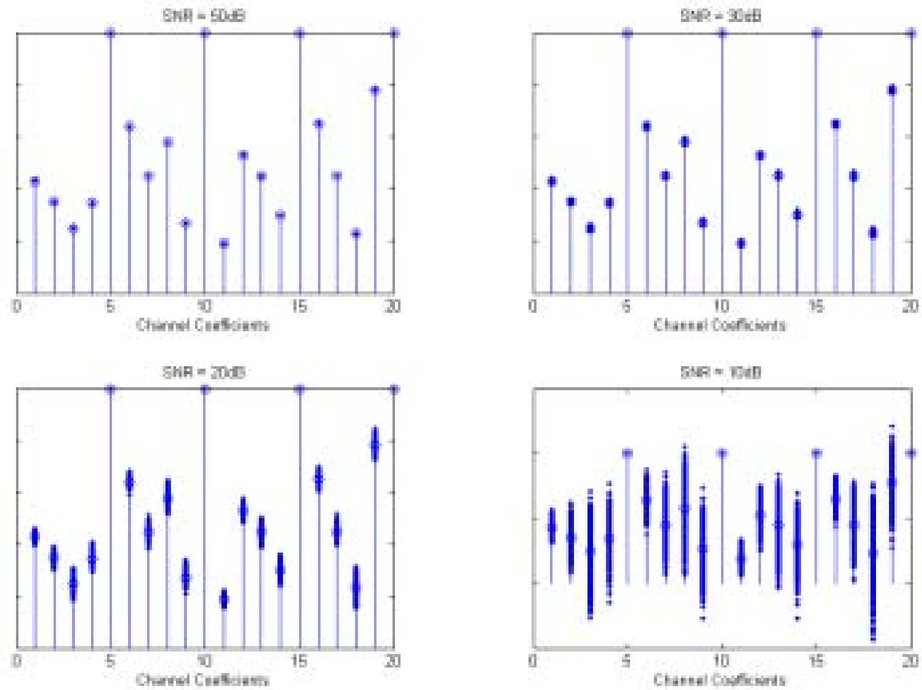
5.1 Computer simulations for Random Signal as Channel Input

In the first simulation study, the number of channels is fixed at $M = 4$ and the signal-to-noise (SNR) is fixed at 25 dB. In the figure next page, the results were obtained by using three different numbers of samples, varying from 100 to 10000. In the figure, an open circle at the end of a stem represents the true channel impulse response and each little dot represents the estimation results for a given simulation run. For each coefficient estimated, there is a grouping of the little dots around the true value. Each grouping contains 1000 points, which represents 1000 simulation runs. Note that as the number of samples increase the estimation error decreases resulting in tighter groupings around the true channel impulse response (the circle).



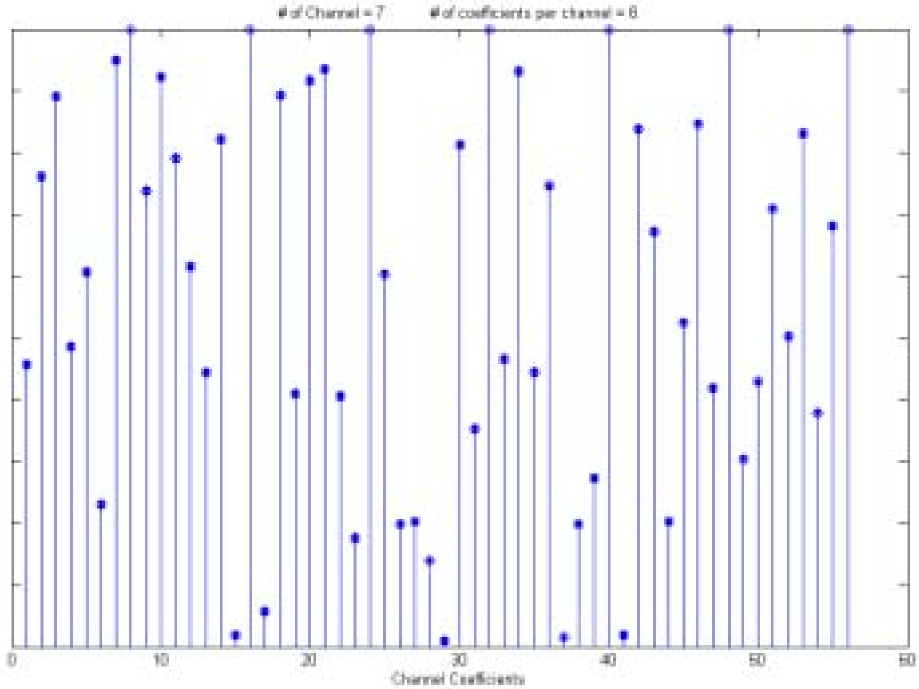
# of sample = 100
Subchannel # 1
Subchannel # 2
Subchannel # 3
Subchannel # 4
# of sample = 1000
# of sample = 10000
Channel Coefficients

In the second simulation study, the number of samples used equals 1000 and the number of channels is fixed at 4. By varying the SNR from 50 to 10 dB, the following diagram was plotted.

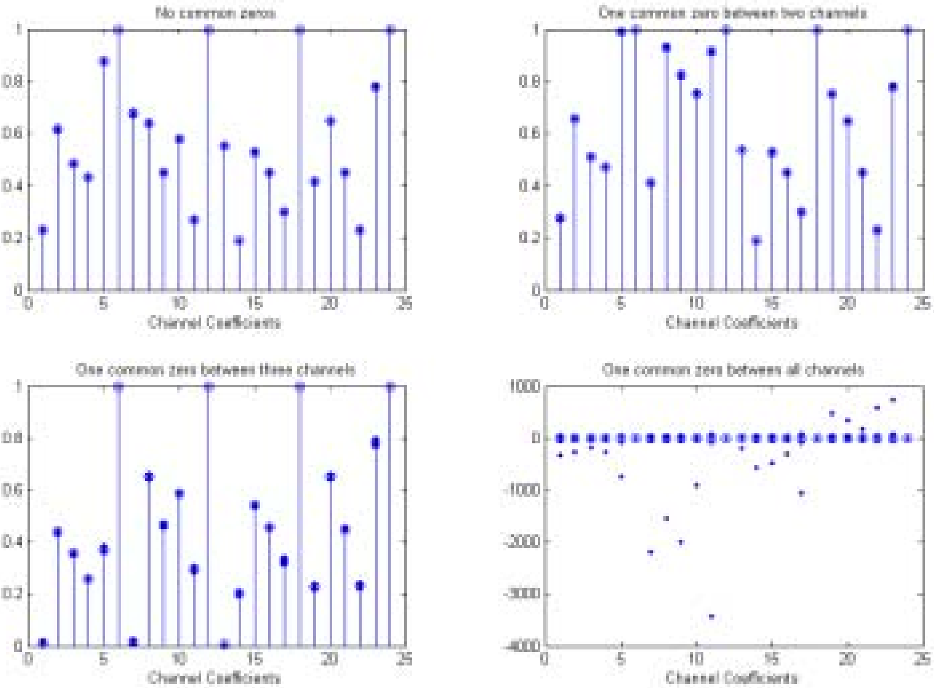


As the SNR varied from 50dB to 10dB, the accuracy of the channel identification degraded and the algorithm ceased to provide reliable results between 20dB and 10dB.

In the third simulation study, the result shows that the number of channels and the number of coefficient can be arbitrary. With the SNR equals 40dB and the number of samples used equals 1000, the following diagram was plotted.



From the diagram, one can see that the channel coefficients were successfully estimated even when the number of subchannel is increased to 7 and the channel length is increased to 8. Now, switching back to the 4-subchannel setting, the next computer simulation shows how the common zeros degrade the estimation performance. With SNR again equal to 40 dB and the number of samples used fixed at 1000, the following diagrams with different number common zeros were obtained.



The common zero weakens the performance of the algorithm only when the common zero is presented in every channel. Now, the next computer simulation should show that the order of each subchannel could be different from each other without affecting the identification capability. In this simulation, we have four different subchannels and the order of each subchannel is the following:

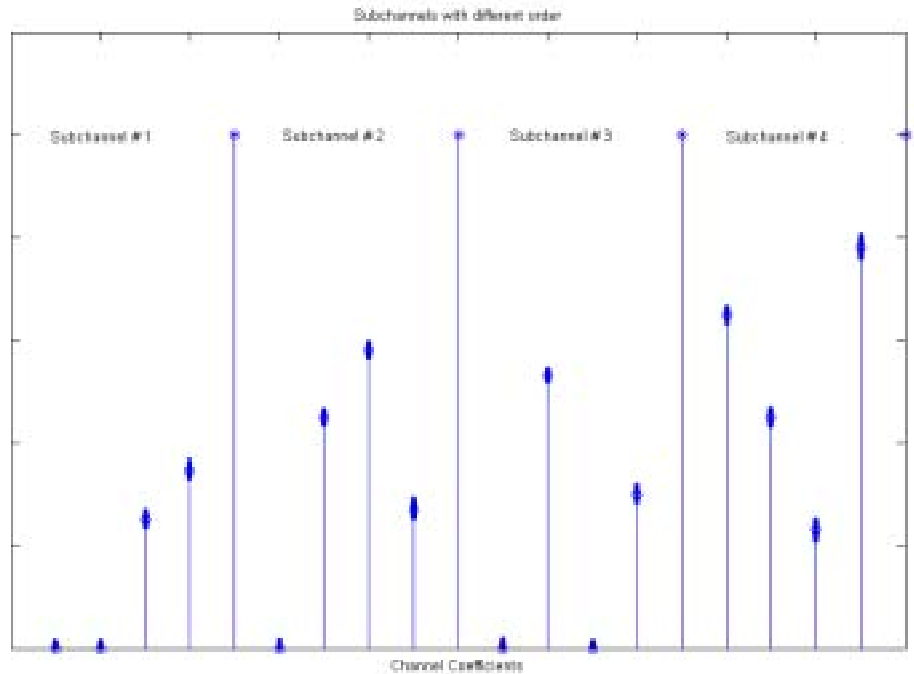
Subchannel 1: $L = 2$

Subchannel 2: $L = 3$

Subchannel 3: $L = 2$

Subchannel 4: $L = 4$,

where L is the order of each subchannel. With SNR fixed at 25 dB and number of samples used equals 1000, the following diagram illustrated that the different order of each subchannel did not affect the identification capability.


Subchannels with different order
Subchannel #1
Subchannel #2
Subchannel #3
Subchannel #4
Channel Coefficients

5.2 Computer simulations for FM Signal as Channel Input

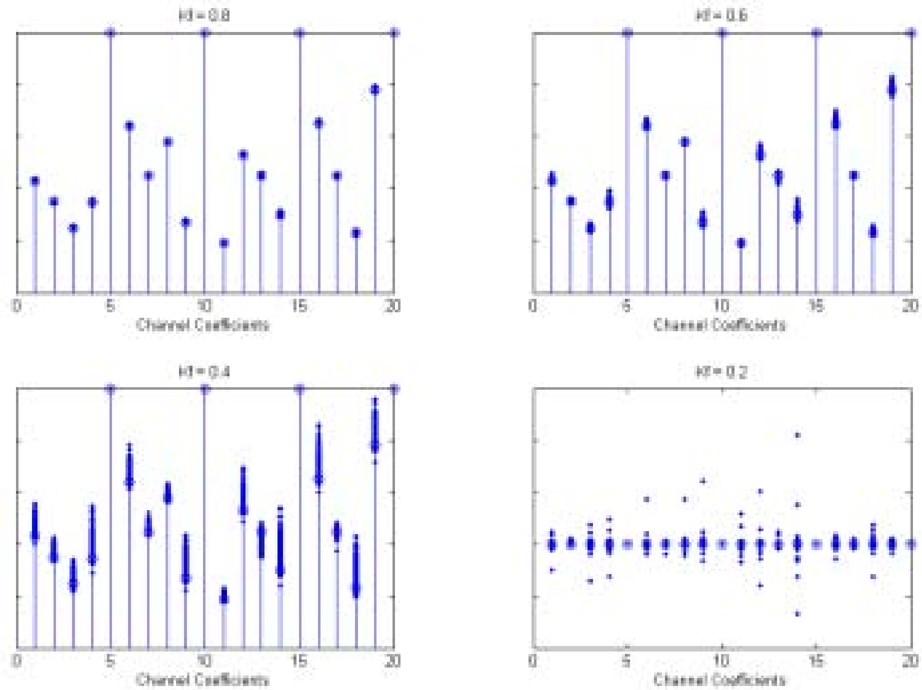
Now, the first identifiability condition of SIMO blind identification is that the bandwidth of the channel input signal must be sufficiently large. All the computer simulations demonstrated in the pervious section are based on a random channel input. Now, by varying the bandwidth of an FM signal, the implication of the identifiability can be tested.

FM (Frequency Modulation) is a form of angle modulation in which the instantaneous frequency is varied linearly with the modulating signal (or the message signal). Thus, in FM the instantaneous frequency f_i is

$$f_i(t) = f_c + k_f m(t), \tag{5.2.1}$$

where k_f is a constant and f_c is the carrier frequency. Equation (5.2.1) shows that the effective bandwidth of a FM signal is proportional to the constant k_f. As the value of k_f increases, the effective bandwidth of a FM signals should also increase.

In this computer simulation, the SNR is fixed at 40 dB and the number of samples used equals to 1000. By varying the value of k_f, the effective bandwidth of the FM signal is also varied. The following diagram was obtained by varying the k_f value (effective bandwidth) of a FM signal.



Again, higher value of k_f corresponds to a higher bandwidth. As the bandwidth of the FM signal decreases, the channel identification performance deteriorates.

5.3 How to estimate the channel length

5.3.1 Theory

The order of each subchannel was assumed to be known in the pervious section. In practice, this assumption may not be true. Therefore, there is a need for an algorithm to estimate the channel length (or the order) L for each subchannel.

By using the singular value decomposition (SVD), the matrix **A** defined in chapter 4 can be rewritten as

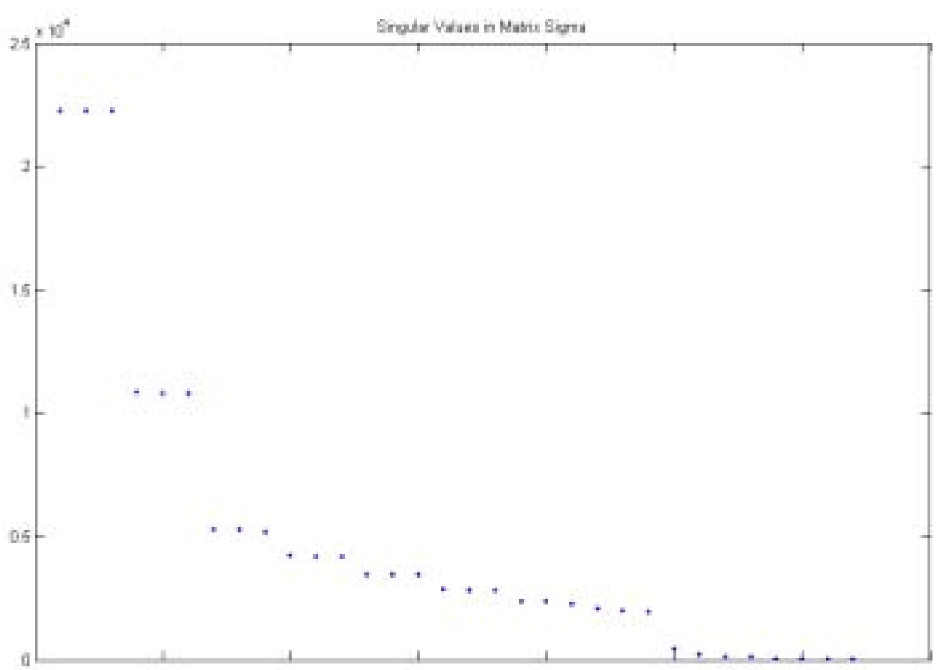
$$\mathbf{A} = \mathbf{U}\,\mathbf{\Sigma}\,\mathbf{V}^{T}, \qquad (6.3.1)$$

where the matrices **U** and **V** are each orthogonal in the sense that their columns are orthonormal. Also, matrix **Σ** is a diagonal matrix with positive or zero singular values (or eigenvalues) as its diagonal elements. Again, the column vector of **U** that corresponds to the smallest eigenvalue in matrix **Σ** is the best estimate of the channel impulse response in the presence of noise. To give some specific dimensions to **Σ**, **U** and **V**, let's consider the following example: Assume that the number of subchannel is four and the order for each subchannel is also four. Also, the number of samples used equal to 1007. Then, from chapter 4, the matrix **X**(L) should be a 6000 by 20 matrix and the matrix **A** should be a 20 by 20 matrix. Therefore, the dimensions for matrices **Σ**, **U** and **V** are all 20 by 20. However, if L = 5, then the dimensions for matrices **Σ**, **U** and **V** become 24 by 24 and the column vector of **U** no longer contain reasonable estimate of the channel impulse response.

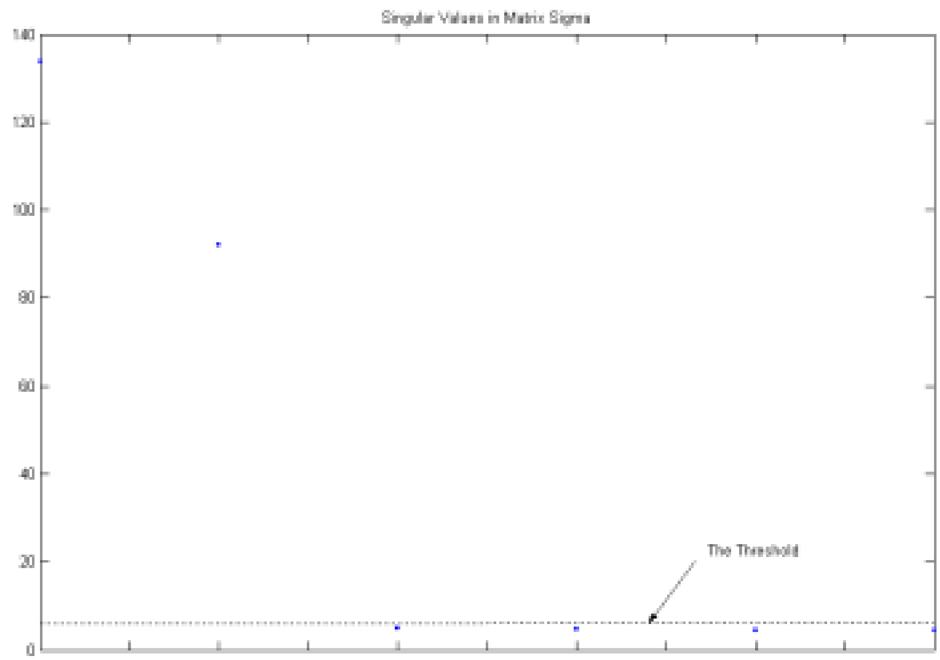
To fix the problem of unknown channel length [14], one can first overestimate the channel length as L_e and form $\mathbf{X}(L_e)$ as in equation (4.10). Then, perform an SVD on $\mathbf{X}(L_e)$ to estimate how many singular values in $\mathbf{\Sigma}$ are correspond to this overestimation. Finally, detect the channel length L and use it to form $\mathbf{X}(L)$ as in equation (4.10).

5.3.2 Computer Simulation

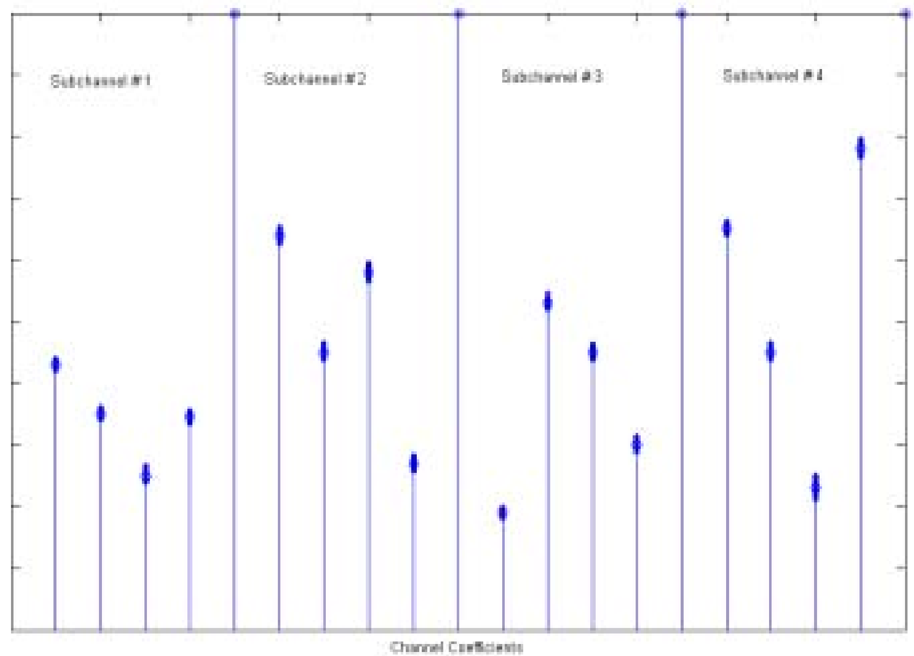
Computer simulation was conducted to test the performance of the algorithm proposed in the pervious section. First, assume that the maximum order of each subchannel is seven. That is, $L_e = 7$. With SNR = 30 dB and the number of samples used equals to 1000, the matrix $\mathbf{X}(L_e)$ is constructed. Then, the matrices $\mathbf{A}$ and $\mathbf{\Sigma}$ are evaluated and the singular values in matrix $\mathbf{\Sigma}$ are plotted in the following diagram.



Since $L_e = 7$, there are 32 singular values in the matrix Σ. In order to isolate the singular values that correspond to the overestimation, a threshold should be set. By trial and error, the threshold is set to be 30% higher than the smallest singular values.



From the diagram above, the threshold has isolated four singular values from the matrix Σ and the overestimation process causes three of the singular values. Therefore, $L = 4$ and the next diagram illustrates how this algorithm performs in 1000 different computer simulations.



The diagram shows that this algorithm has successfully fixed the problem of unknown channel length.

Chapter 6

Conclusion

In this paper, the traditional and blind channel equalization method have been presented. The main focus of this paper is the blind channel identification with unknown deterministic input. Least-Square approach has been used to solve this identification problem and computer simulations have been implemented to demonstrate the potential of this blind algorithm. A series of computer simulation have been used to validate this Least-Square algorithm.

In chapter 5, the computer simulations have shown that as the SNR and the number of samples used increase, the channel identification capability improved. The number of subchannel and the order of each subchannel do not affect the performance of the proposed Least-Square algorithm. The bandwidth of the channel input does concern the performance of the algorithm. As the bandwidth decrease, the estimation error increases. This algorithm can also be used to estimate the channel impulse response even when the channel length of each subchannel is unknown.

Also, the 'Least-Square Approach to Blind Channel Identification' is well suited to the emitter location because it naturally exploits the multiple channels available from the multiple receivers and requires very little prior information about the transmitted signal.

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