(A) Find the FT of ~~y(t)~~

$$y(t) = \text{sinc}(3000t)\sin(10{,}000\pi t)$$

Recognize 2 things

1. $\text{Sinc}(mt)$ is on table

2. Mult. by sinusoid $\Rightarrow$ Modulation Property

From knowledge of Mod. Prop.... first find

$$\mathcal{F}\{\text{Sinc}(3000t)\}$$

From table... $\tau \, \text{Sinc}\left[\frac{\tau t}{2\pi}\right] \longleftrightarrow 2\pi \, p_\tau(\omega)$

$$\frac{\tau}{2\pi} = 3000 \Rightarrow \tau = 2\pi \times 3000$$

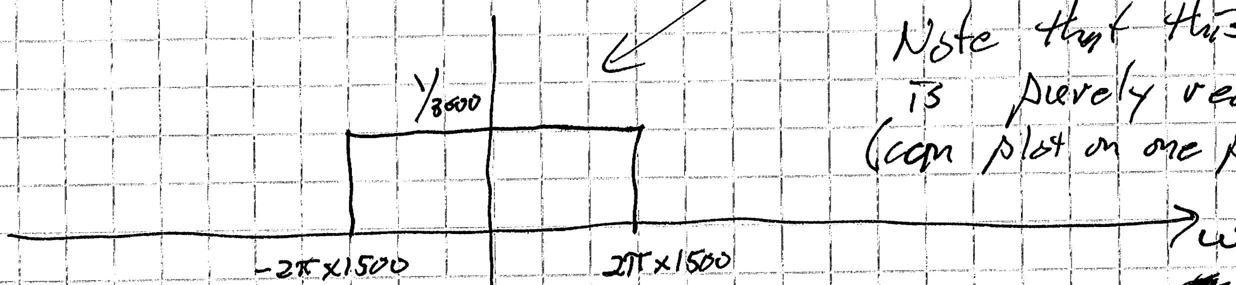
By linearity

$$\frac{1}{2\pi \times 3000} \mathcal{F}\left\{2\pi \times 3000 \, \text{Sinc}(3000t)\right\}$$

$$= \frac{1}{\cancel{2\pi} \times 3000} \cdot \cancel{2\pi} \, p_\tau(\omega)$$

So... $\mathcal{F}\{\operatorname{sinc}(3000t)\} = \underbrace{\frac{1}{3000} p_{2\pi\times3000}(\omega)}$

Note that this is purely real (can plot on one plot)

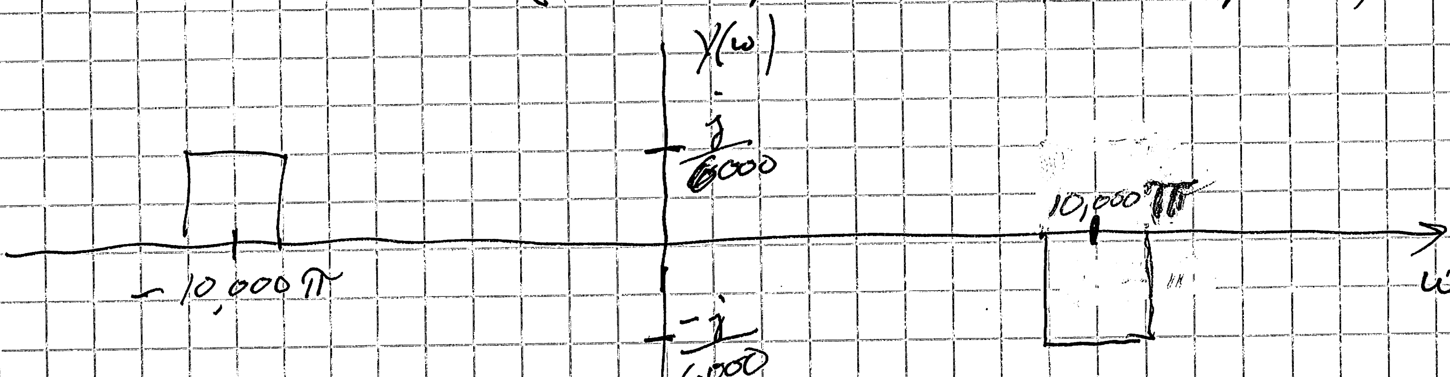


Now by mod prop. w/ sine we get

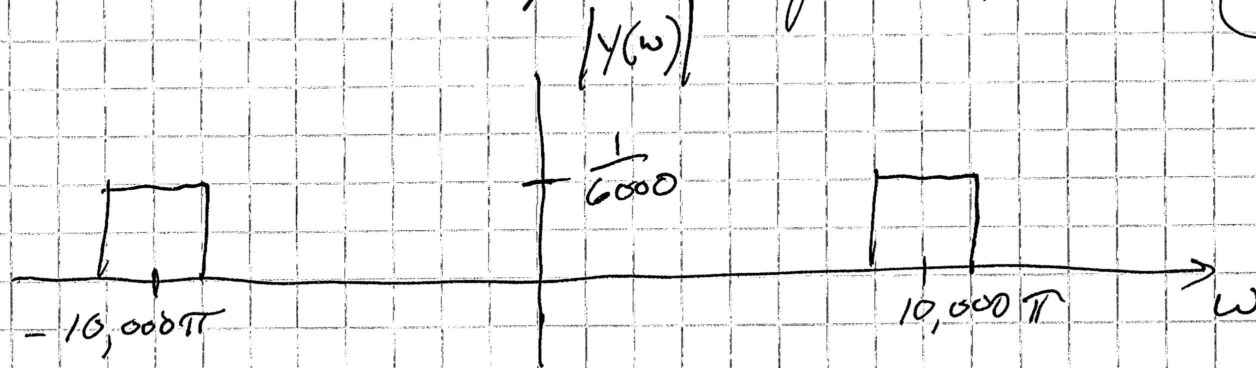
$$Y(\omega) = \frac{j}{2}\left[\frac{1}{3000} p_{2\pi\times3000}(\omega + 10{,}000\pi) - \frac{1}{3000} p_{2\pi\times3000}(\omega - 10{,}000\pi)\right]$$

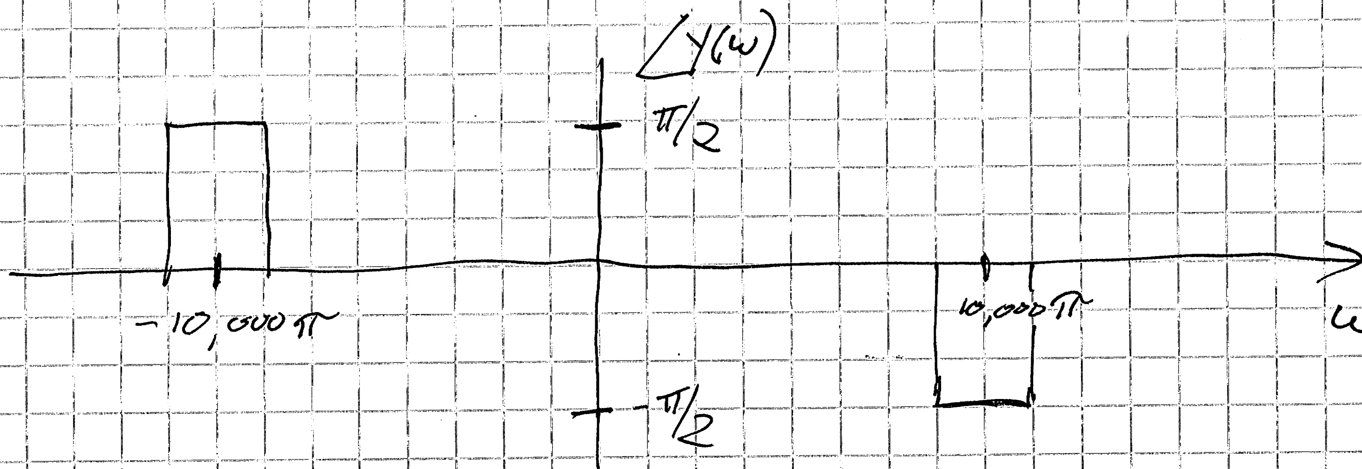
$$= \frac{j}{6000}\left[p_{2\pi\times3000}(\omega + 10{,}000\pi) - p_{2\pi\times3000}(\omega - 10{,}000\pi)\right]$$

Note that this is purely imag. (can plot w/ only one plot)

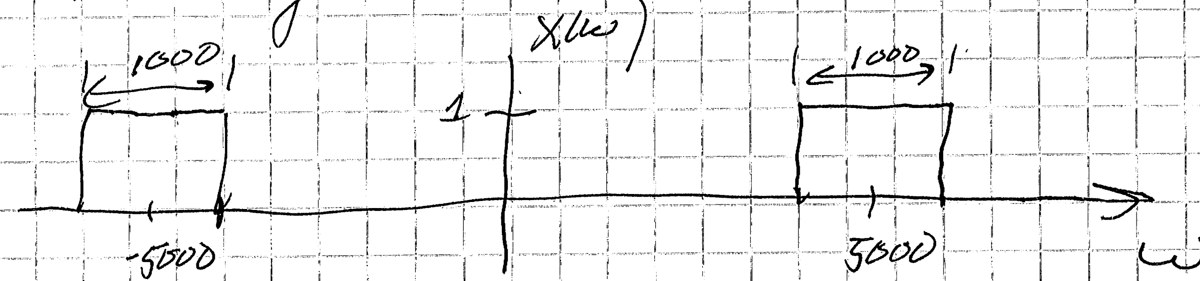


If we need to plot magn. & phase: (A-3)





(B) Find the Inverse FT of the following



Recognize 2 things:

1. Two identical things ... one shifted up & one shifted down

   $\Rightarrow$ Mod property

2. Rectangle in freq. domain is a sinc in time domain

$$X(\omega) = p_{1000}(\omega + 5000) + p_{1000}(\omega - 5000)$$

So ~~need to~~ from cosine mod property

we know

$$x(t) = \underbrace{\mathcal{F}^{-1}\left\{ p_{1000}(\omega) \right\}}_{\text{this is some sinc in } t} \cos(5000t)$$

(8-2)

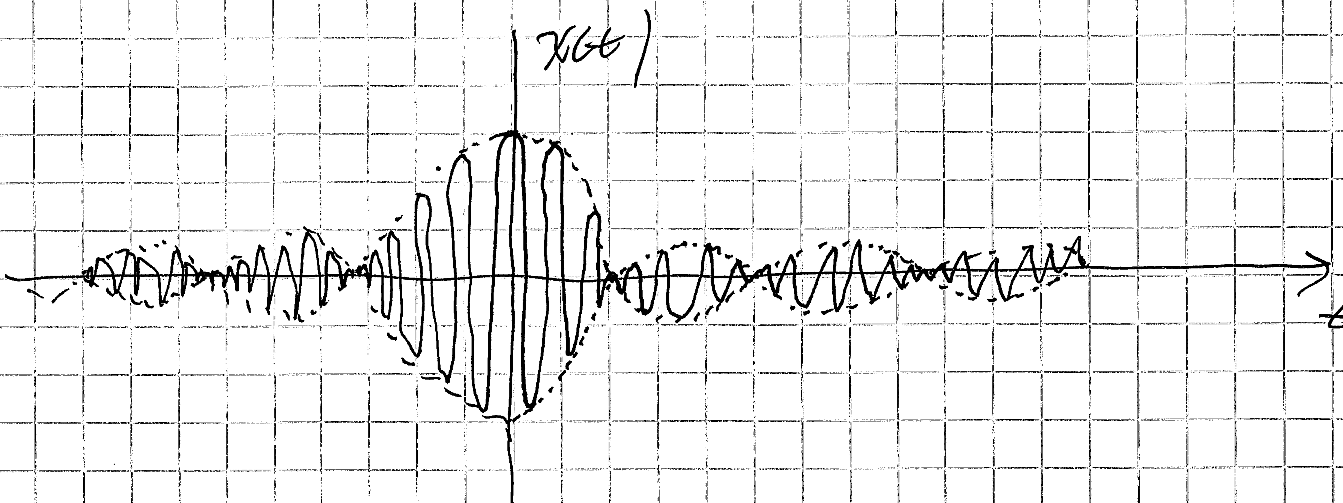
From alternate form of FT of sinc

$$\mathcal{F}^{-1}\left\{ P_{1000}(\omega) \right\} = \frac{500}{\pi}\,\mathrm{sinc}\left(\frac{500}{\pi}t\right)$$

$2\pi a = 1000$

$a = \frac{500}{\pi}$

So... $$x(t) = \frac{500}{\pi}\,\mathrm{sinc}\left(\frac{500}{\pi}t\right)\cos(5000t)$$



(c) Find FT of signal

$$x(t) = t\,e^{-20t}\,u(t)\cos(100t)$$

Recognize 3 things:

1. Mult. by cosine smacks of the modulation property

2. Mult. by t smacks of the "Multiply by t^n" property

3. That leaves $e^{-20t}u(t)$ ... which is on the FT table!

So start @ #3:

$$\mathcal{F}\left\{e^{-20t}u(t)\right\} = \frac{1}{j\omega + 20}$$

Now we can do #1 & #2 in either order to get $x(t)$ so it does not matter which we do first in the frequency domain!

C-2

So $\mathcal{F}\left\{t e^{-20t} u(t)\right\}$

$$= j \frac{d}{d\omega}\left[\frac{1}{j\omega+20}\right]$$

Recall: $\frac{d}{dx}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dx}$

$$= j\left[\frac{-1}{(j\omega+20)^2}\right] j = \frac{1}{(j\omega+20)^2}$$

Now do modulation property

$$\boxed{X(\omega) = \frac{1}{2}\left[\frac{1}{[j(\omega+100)+20]^2} + \frac{1}{[j(\omega-100)+20]^2}\right]}$$

Find FT of $x(t) = e^{j100t} e^{-5|t-3|}$

1. See e^{j100t} & think MODULATION

2. See the delay $t-3$

3. Left w/ $e^{-5|t|}$

From Table in Schaum's: $\mathcal{F}\{e^{-5|t|}\} = \frac{2\times 5}{\omega^2 + 5^2}$

Must do delay First, then Mod !!

(See next page)

$$e^{-5|t-3|} \longleftrightarrow \frac{10}{\omega^2+5^2} e^{-j\omega 3}$$

$$\boxed{e^{+j100t} e^{-5|t-3|} \longleftrightarrow \frac{10}{(\omega-100)^2+5^2} e^{-j(\omega-100)3}}$$

Next page shows that delay then mod gives a different signal than mod then delay ...

Delay then Mod

$x(t)$

$\downarrow$ Delay

$x(t-c)$

$\downarrow$ Mod.

$\underbrace{e^{j\omega_0 t} x(t-c)}_{\text{We had this one!}}$

Slightly Different

$\longleftrightarrow$

Mod then Delay

$x(t)$

$\downarrow$ Mod.

$e^{j\omega_0 t} x(t)$

$\downarrow$ Delay

$e^{j\omega_0 (t-c)} x(t-c)$

$= \boxed{e^{-j\omega_0 c}} e^{j\omega_0 t} x(t-c)$