Binghamton University
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EECE 301
Signals & Systems
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Note Set #36

- C-T Systems: Bode Plots

What are Bode Plots?

"Bode Plot" = Freq. Resp. plotted with $|H(\omega)|$ in dB on a log frequency axis.

They do also include phase plots but we'll focus here on the magnitude part

Its easy to use computers to make Bode plots… we already saw that!

- **Use MATLAB's freqs command**

But good engineers need *insight* to:

- understand the results of an analysis

- make decisions for design

Learning how to make "by hand" a Bode magnitude plot will help give you the needed insight!

Key Idea of the Method

Consider a Freq Resp Magntiude factored into terms of its poles and zeros…

$$|H(\omega)| = \frac{|K|\,|j\omega - z_1|\,|j\omega - z_2|\cdots\left|(j\omega)^2 + (2\zeta_1\omega_{n,1})j\omega + \omega_{n,1}^2\right|}{|j\omega - p_1|\,|j\omega - p_2|\cdots\left|(j\omega)^2 + (2\zeta_2\omega_{n,2})j\omega + \omega_{n,2}^2\right|}$$

Complex conjugate roots are kept together

Convert to dB form and use log property:

(dB converts multiplications to additions!)

log(ab) = log(a) + log(b)
log(a/b) = log(a) – log(b)

$$20\log_{10}|H(\omega)| =$$

$$20\log_{10}|K| + 20\log_{10}|j\omega - z_1| + \cdots + 20\log_{10}\left|(j\omega)^2 + (2\zeta_1\omega_{n,1})j\omega + \omega_{n,1}^2\right|$$

$$-20\log_{10}|j\omega - p_1| - \cdots - 20\log_{10}\left|(j\omega)^2 + (2\zeta_2\omega_{n,2})j\omega + \omega_{n,2}^2\right|$$

So… <u>Key Idea</u> is that we <u>have a sum of terms</u> and only need to figure out two things:

1. **What does each of these terms look like?**
2. **How do we add them together?**

One Little Trick First

Complex conjugate roots are kept together

After factoring: $$H(s)=\frac{Ks(s+2)(s+10)(s^2+60s+40,000)}{(s+5)(s+20)(s^2+6s+6,400)}$$

Note zero at origin (might not have one... Or could be pole at origin)

Our trick is to pull out "constants" for each term (except a pole or zero @ origin)… like this:

$$H(s)=\left[\frac{K\times 2\times 10\times 40,000}{5\times 20\times 6,400}\right]\frac{s(s/2+1)(s/10+1)\left[(s/200)^2+(60/40,000)s+1\right]}{(s/5+1)(s/20+1)\left[(s/80)^2+(6/6,400)s+1\right]}$$

$$H(\omega)=\left[\frac{K\times 2\times 10\times 40,000}{5\times 20\times 6,400}\right]\frac{j\omega(j\omega/2+1)(j\omega/10+1)\left[(j\omega/200)^2+(60/40,000)j\omega+1\right]}{(j\omega/5+1)(j\omega/20+1)\left[(j\omega/80)^2+(6/6,400)j\omega+1\right]}$$

The whole point of doing this is so that each term (once in dB form) is at 0 dB at low frequecies… this will make it easy to add the dB terms together!

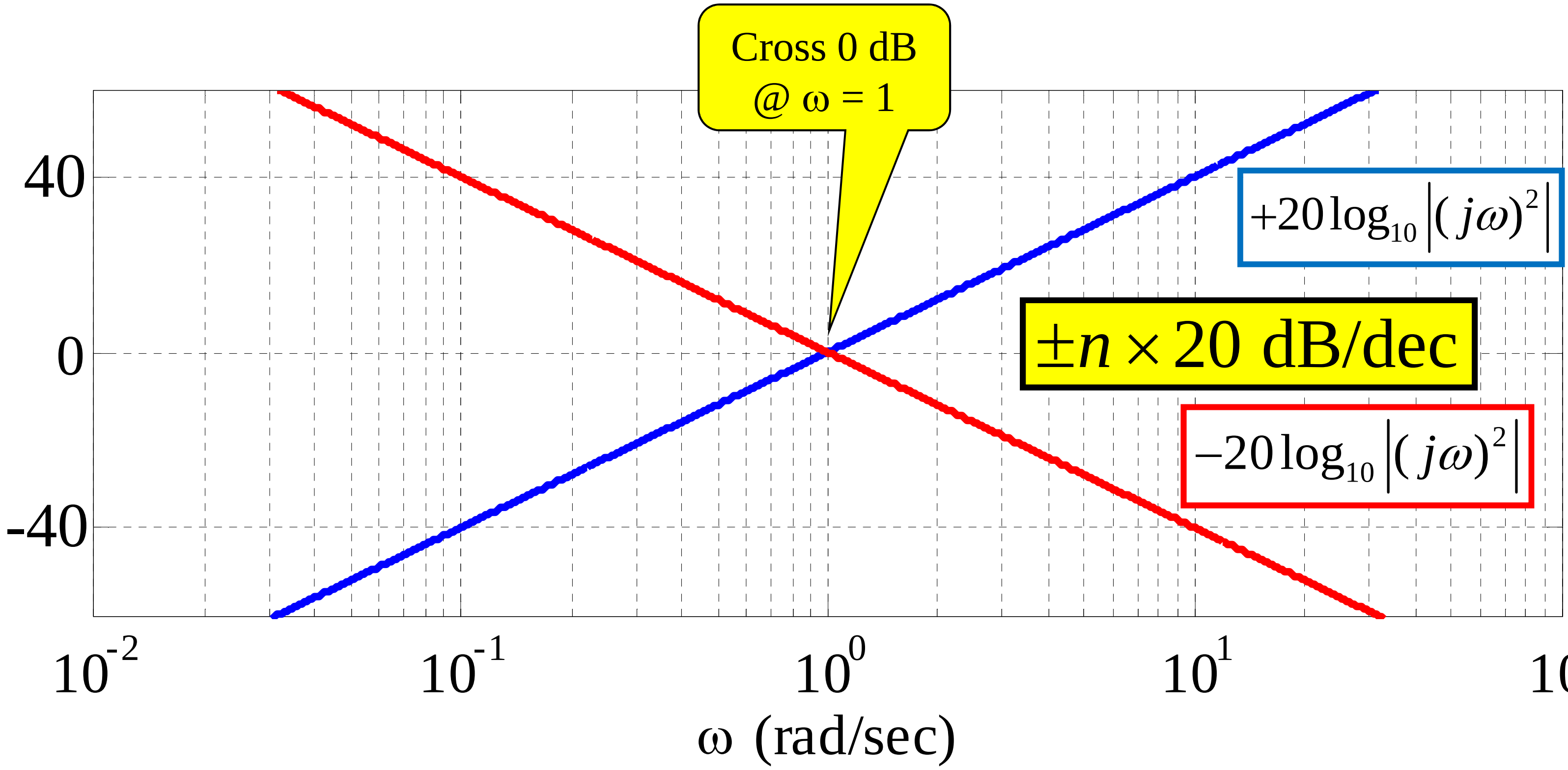
What the Terms Look Like Plotted

$$20\log_{10}|K|$$

Just a constant… Flat Plot

$$\pm 20\log_{10}|(j\omega)^n|$$

An n^{th} order Pole/Zero @ Origin (Plot shown for n = 2)



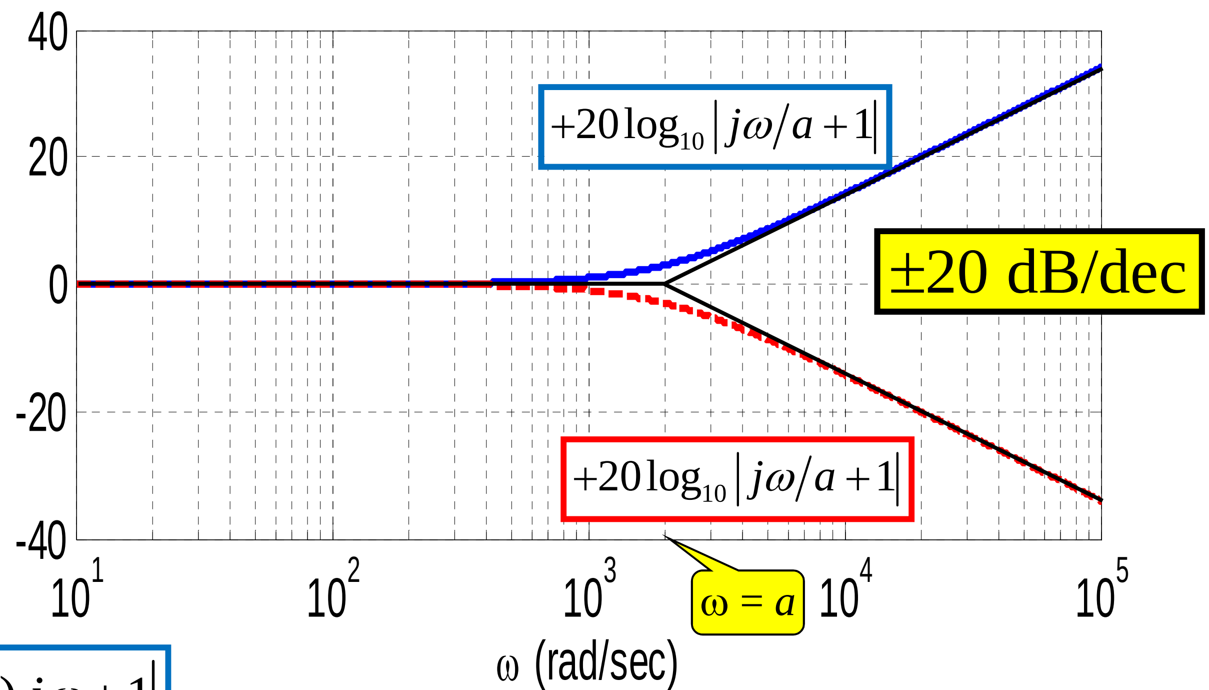
$\pm 20 \log_{10} \left| j\omega / a + 1 \right|$

1^{st} order Pole/Zero @ *a*

Straight-Line Approx

Flat at 0 dB until $\omega = a$

Then "break" to a slope of ± 20 dB/dec



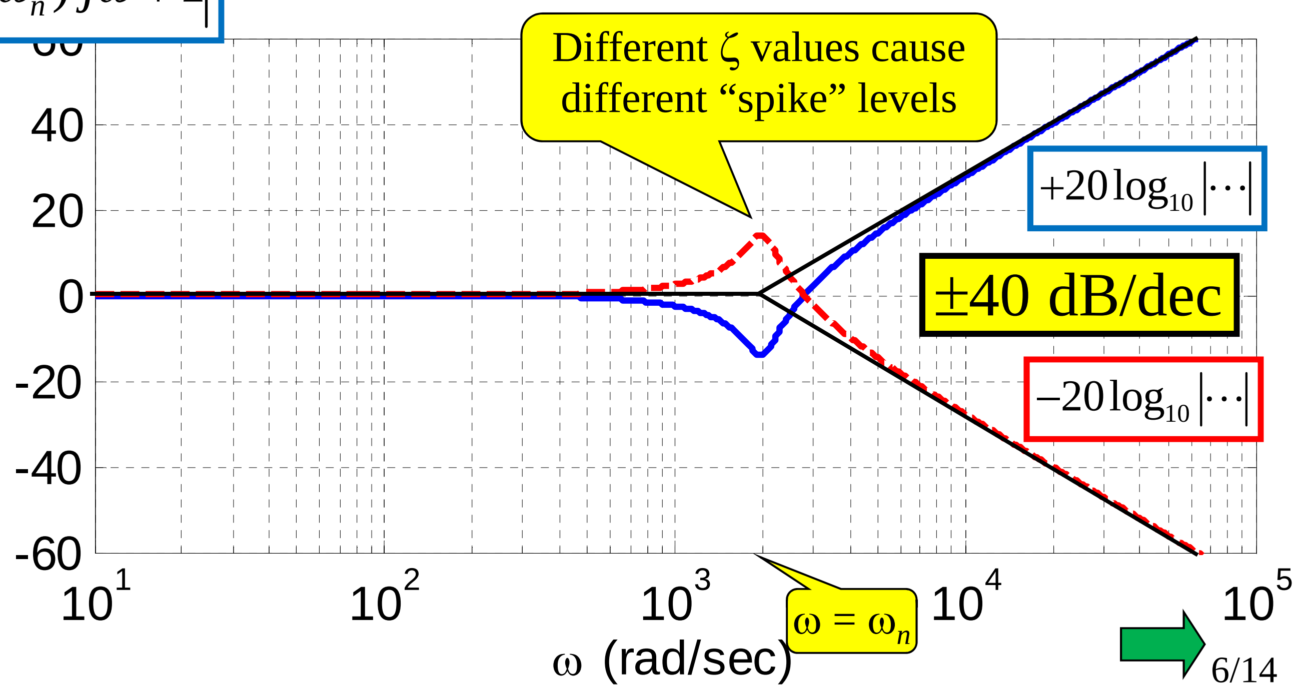
$\pm 20 \log_{10} \left| (j\omega/\omega_n)^2 + (2\zeta_1/\omega_n) j\omega + 1 \right|$

2^{nd} order Pole/Zero @ ω_n

Straight-Line Approx

Flat at 0 dB until $\omega = \omega_n$

Then "break" to a slope of ± 40 dB/dec



What This Does For Us…

Notice that the 1st and 2nd Order Non-Origin terms contribute nothing (i.e., they add 0 dB) at frequencies below their "break points"

So… if we start below all break points it is as if these terms don't exist…

So… if we start plotting at a low enough frequency all we have to do is plot the effect of the constant term and any origin-located poles/zeros (if any exist).

Then… as we go up in frequency… at each break point we change the slope by the amount of slope the new term provides!

Sum of two Lines has Slope equal to Sum of Slopes!

General Steps to "Sequentially" Build Bode Plots

1. Factor $H(s)$… leave complex-root terms as quadratics
2. Convert to $j\omega$ form
3. Pull out "constants" into a "gain" term
4. Combine constant term with "$j\omega$" terms (if any)
5. Identify "break points" and put in ascending order
6. Plot constant term with "$j\omega$" terms at ω values below the lowest "break point"
7. At "break point", change slope by ±20dB/decade or ±40dB/decade for 1st order or 2nd order terms, repectively.
   - Repeat this step through ordered list of "breakpoints".
8. Make "resonant corrections" for "under damped" 2nd order terms (i.e. when $\zeta < 0.5$).

Example

$$H(s)=\frac{0.1s^3+25s^2+1000s}{s^3+4s^2+104s+200}$$

1. Factor:

$$H(s)=\frac{0.1s^3+25s^2+1000s}{s^3+4s^2+104s+200}=\frac{0.1s(s^2+250+10000)}{s^3+4s^2+104s+200}=\frac{0.1s(s+50)(s+200)}{(s+2)(s^2+2s+100)}$$

```
>> roots([1 250 10000])
ans =
  -200
   -50
```

```
>> roots([1 4 104 200])
ans =
 -1.0000 + 9.9499i
 -1.0000 - 9.9499i
 -2.0000
```

Leave as 2nd order term

$$s^2+2\zeta\omega_n s+\omega_n^2$$

$$\omega_n=10\Rightarrow 2\zeta\omega_n=2$$

$$\Rightarrow \zeta=0.1$$

2. Convert to *jω*:

$$H(\omega)=\frac{0.1j\omega(j\omega+50)(j\omega+200)}{(j\omega+2)((j\omega)^2+2j\omega+100)}$$

3. Pull Out "Constants":

$$H(\omega)=\frac{0.1j\omega(j\omega+50)(j\omega+200)}{(j\omega+2)((j\omega)^2+2j\omega+100)}$$

$$H(\omega)=\underbrace{\frac{0.1\times 50\times 200}{2\times 100}}_{=5}\left[\frac{j\omega(1+j\omega/50)(1+j\omega/200)}{(1+j\omega/2)\left(1+2j\omega/100+(j\omega/10)^2\right)}\right]$$

Gain Term

4. Combine gain term with $j\omega$ term :

$$H(\omega) = \left[\frac{(5j\omega)(1 + j\omega/50)(1 + j\omega/200)}{(1 + j\omega/2)\left(1 + 2j\omega/100 + (j\omega/10)^2\right)} \right]$$

5. Identify Breakpoints and List in Ascending Order:

$$H(\omega) = \left[\frac{(5j\omega)(1 + j\omega/50)(1 + j\omega/200)}{(1 + j\omega/2)\left(1 + 2j\omega/100 + (j\omega/10)^2\right)} \right]$$

List breakpoints in ascending order:

Break Points	*Change* in slope
2	-20dB/decade – 1st order term in denominator
10	-40dB/decade – 2nd order term in denominator
50	+20dB/decade – 1st order term in numerator
200	+20dB/decade – 1st order term in numerator

6. Plot constant term with "$j\omega$" terms at ω values below the lowest "break point" :

$$H(\omega) = \left[\frac{(5j\omega)(1 + j\omega/50)(1 + j\omega/200)}{(1 + j\omega/2)\left(1 + 2j\omega/100 + (j\omega/10)^2\right)} \right]$$

-Evaluate $|5j\omega|$ in dB at ω value that is (at least) 1 decade below the lowest BP (Here used $\omega = 0.1$ which is more than a decade below 2):

$$20\log_{10}(5 \times 0.1) = 20\log_{10}(0.5) = -6dB$$

-Plot a point at -6 dB at $\omega = 0.1$

-Draw a line of slope +20dB/decade from this point up to the first BP

It is + slope because here the $j\omega$ term is in numerator
It would be – slope if it were in the denominator

It is 20 dB slope because here there is a first order $j\omega$ term
It would be px20 slope if it were $(j\omega)^p$

7. At "break point", change slope by ±20dB/decade or ±40dB/decade for 1st order or 2nd order terms, repectively. :

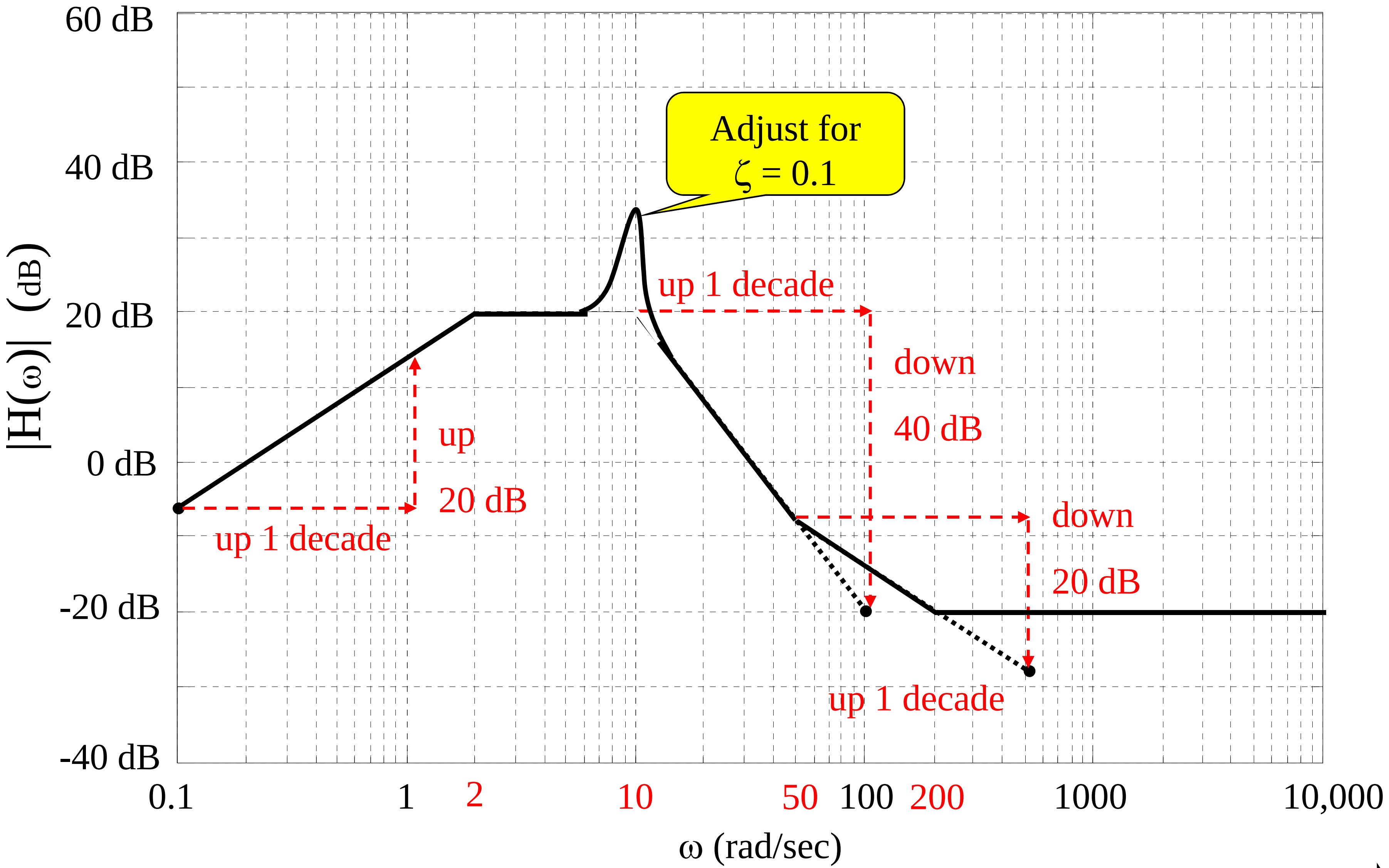
Break Points	*Change* in slope
2	-20dB/decade – 1st order term in denominator
10	-40dB/decade – 2nd order term in denominator
50	+20dB/decade – 1st order term in numerator
200	+20dB/decade – 1st order term in numerator

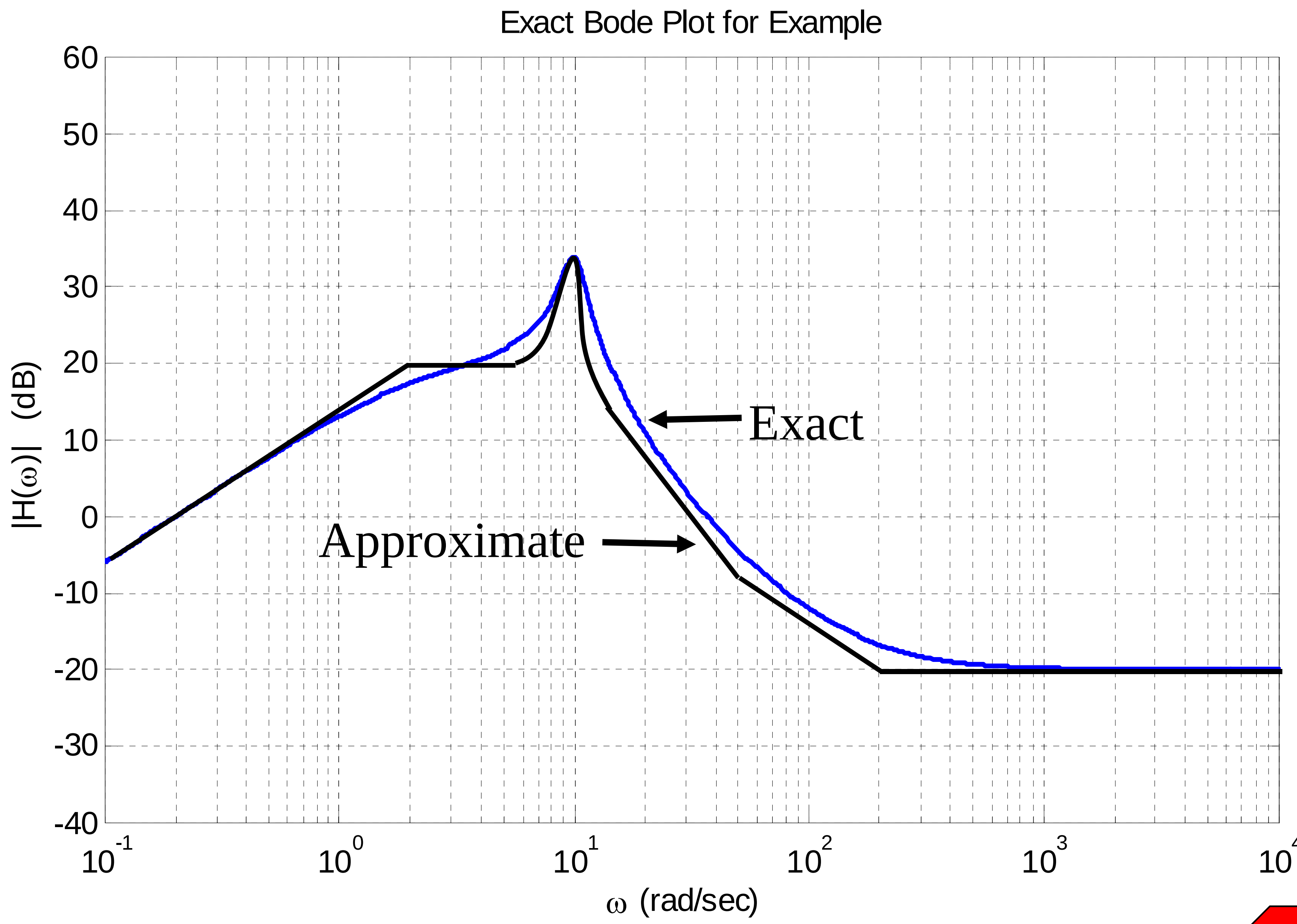
8. Make "resonant corrections" for "under damped" 2nd order terms (i.e. when $\zeta < 0.5$). :

Finally: Make adjustment for the ζ value from the plot of the 2nd order term: $\zeta = 0.1$ gives peak ≈ 14dB up

ζ value	Adjustment
0.1	14 dB
0.2	8 dB
0.3	5 dB
0.4	3 dB
0.5	1 dB

Approximate Bode Plot for Example in Notes





Exact Bode Plot for Example
60
50
40
30
20
10
0
-10
-20
-30
-40
|H(ω)| (dB)
10^-1
10^0
10^1
10^2
10^3
10^4
ω (rad/sec)
Exact
Approximate