EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #35

- C-T Systems: CT Filters - Passive

Introductory Comments

"CT Filters" are also called "Analog Filters"

Recall that we already talked about ideal CT filters:

- $|H(\omega)|$ is Constant in Pass band
- $|H(\omega)|$ is zero in Stop band   (Transition Band has zero width)
- $\angle H(\omega)$ is linear in Pass band

We also saw that such ideal filters can not really exist because they would need to be non-causal!!

Here we'll take a brief look at some of the kinds of CT filters that ***can*** be made…

- Note… all CT filter behavior exploits the fact that capacitors and inductors have an impedance that varies with frequency!

And we'll illustrate how to describe such filters using:

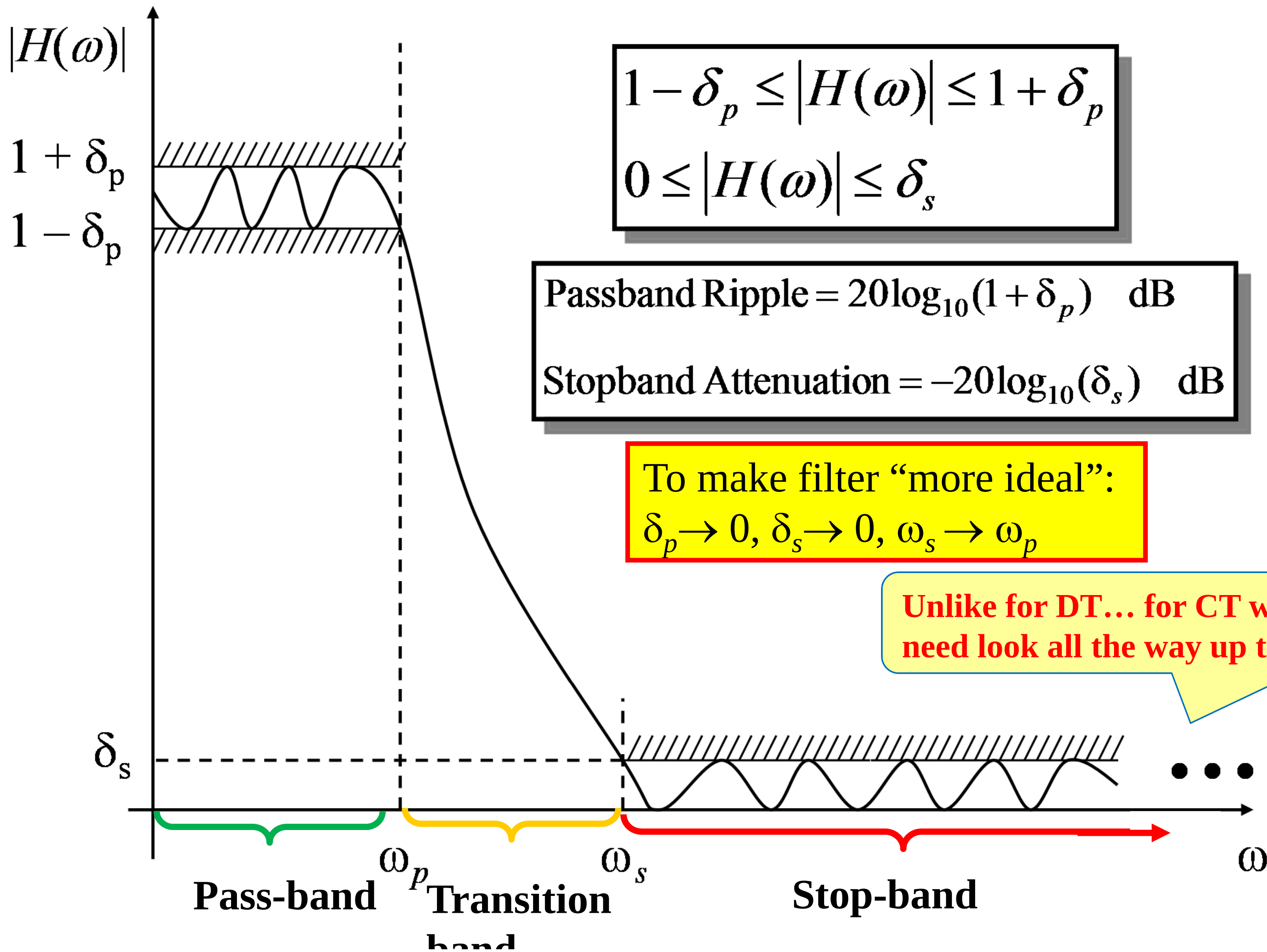
- Transfer Function
- Frequency Response
- Pole-Zero Diagrams

Also… keep in mind that although DT filters only need to be examined over $-\pi$ to π rad/*sample* (their Freq Resp repeats outside of that)… CT filters need to be examined for how they behave over $-\infty$ to ∞ rad/*second*. Thus, we will mostly plot them on a log frequency axis… with dB for the magnitude.

Practical Filter Specification

LPF Spec – Version 1

Specs for HPF, BPF, & BSF are similar…



$$1-\delta_p \le |H(\omega)| \le 1+\delta_p$$

$$0 \le |H(\omega)| \le \delta_s$$

$$\text{Passband Ripple} = 20\log_{10}(1+\delta_p) \quad \text{dB}$$

$$\text{Stopband Attenuation} = -20\log_{10}(\delta_s) \quad \text{dB}$$

To make filter "more ideal":
$\delta_p \rightarrow 0,\ \delta_s \rightarrow 0,\ \omega_s \rightarrow \omega_p$

Unlike for DT… for CT we need look all the way up to ∞

LPF Spec – Version 2

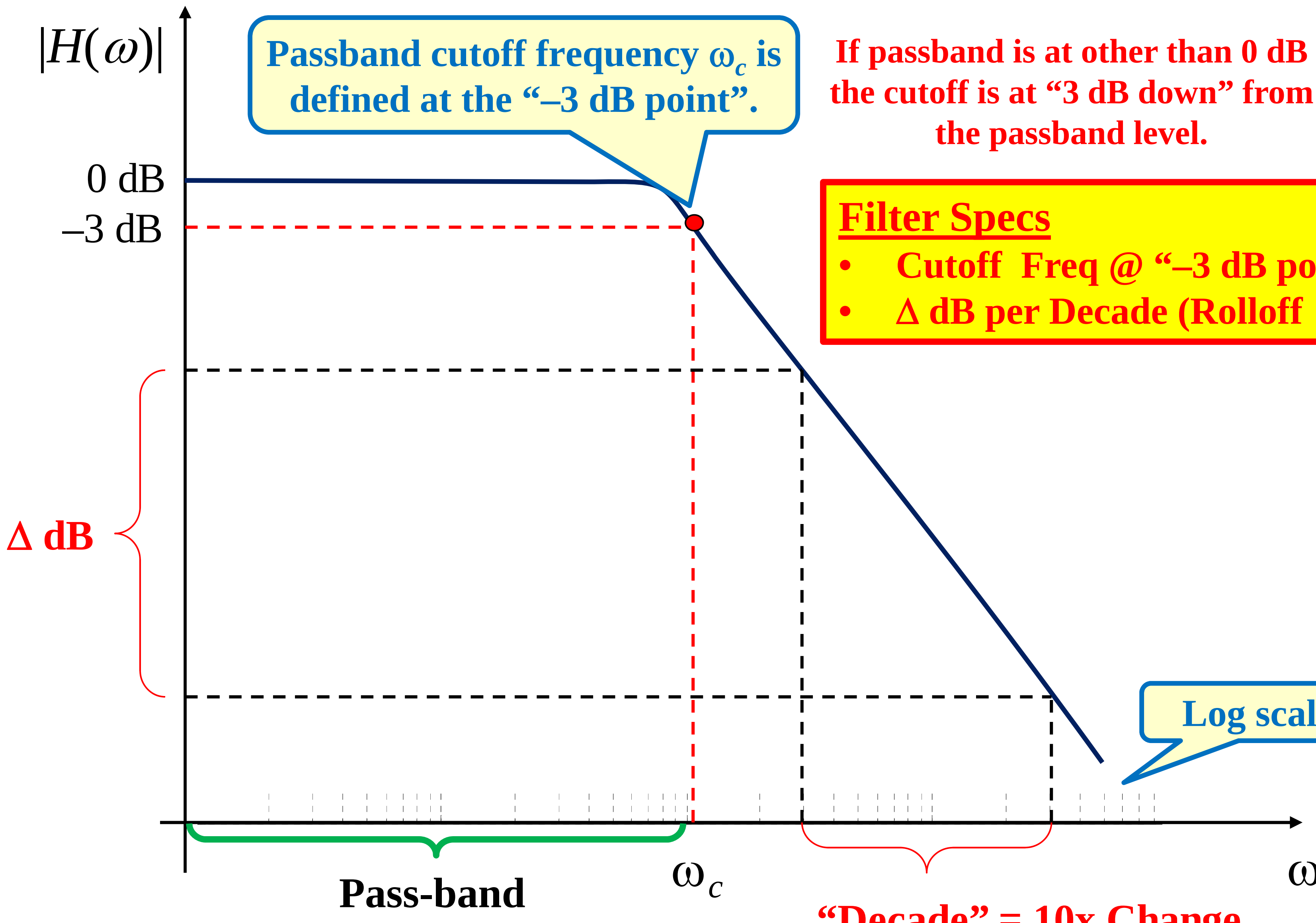
Specs for HPF, BPF, & BSF are similar…

Passband cutoff frequency ω_c is defined at the "–3 dB point".

If passband is at other than 0 dB the cutoff is at "3 dB down" from the passband level.

Filter Specs

- Cutoff Freq @ "–3 dB point"
- Δ dB per Decade (Rolloff Rate)



CT Filter Types

Recall that DT filters were categorized as recursive (IIR) vs. non-recursive (FIR).

CT filters don't have a corresponding categorization… they all have infinite duration impulse responses!!!

Instead the main way to categorize CT filters is: **Passive** vs. **Active**

Passive: These filters use only "passive components" (resistors, capacitors, and inductors) and do not contain any op amps or transistors.

- One main advantage of such filters is that they can be used in places where access to a power supply is not available (e.g., inside a stereo speaker to separate the audio into bass and treble before sending it to the woofer & tweeter).

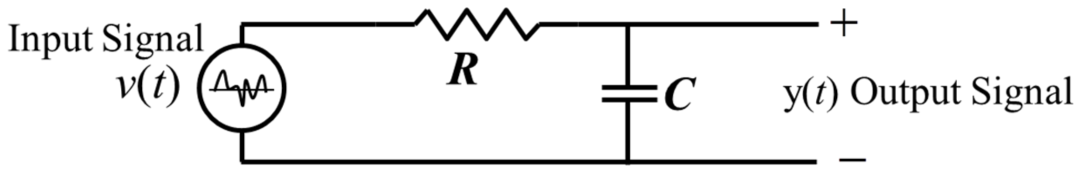
Active: These filters use op amps (and/or transistors) together with resistors, capacitors, and inductors.

- Allows filters to be designed without inductors
- Op amp characteristics enable design by cascading several "stages"

Heavy, Bulky, Expensive

- **Large Input Impedance**
- **Small Output Impedance**

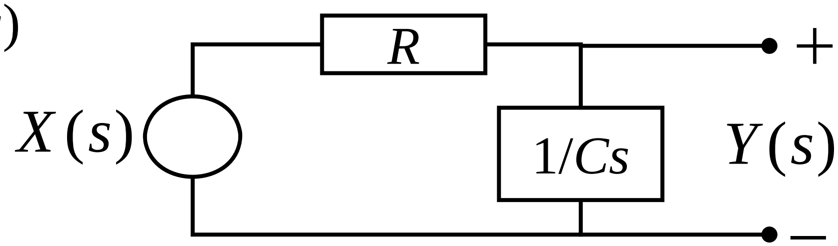
"First-Order" Lowpass Filter: RC Circuit



We already analyzed this filter using phasor ideas… but we'll take another look here.

To analyze this filter in the s-domain:

- Replace input and output by their LT symbols
- Replace components by their **s-domain impedances**
- Solve for output $Y(s)$ in terms of input $X(s)$… the thing that multiplies $X(s)$ is the TF $H(s)$



$$Y(s) = \left[\frac{1/Cs}{1/Cs + R}\right] X(s)$$

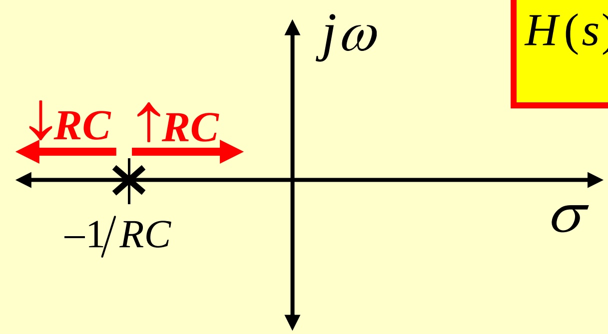
By voltage divider (the best approach here) we get this

$$H(s) = \frac{1/Cs}{1/Cs + R}$$

$$H(s) = \frac{1}{1 + RCs} = \frac{1/RC}{s + 1/RC}$$

1 Pole @ s = -1/*RC*

1st Order



$$H(s) = \frac{1/RC}{s + 1/RC}$$

$$H(s)=\frac{1}{1+RCs}=\frac{1/RC}{s+1/RC}$$

$$H(\omega)=\frac{1}{1+jRC\omega}$$

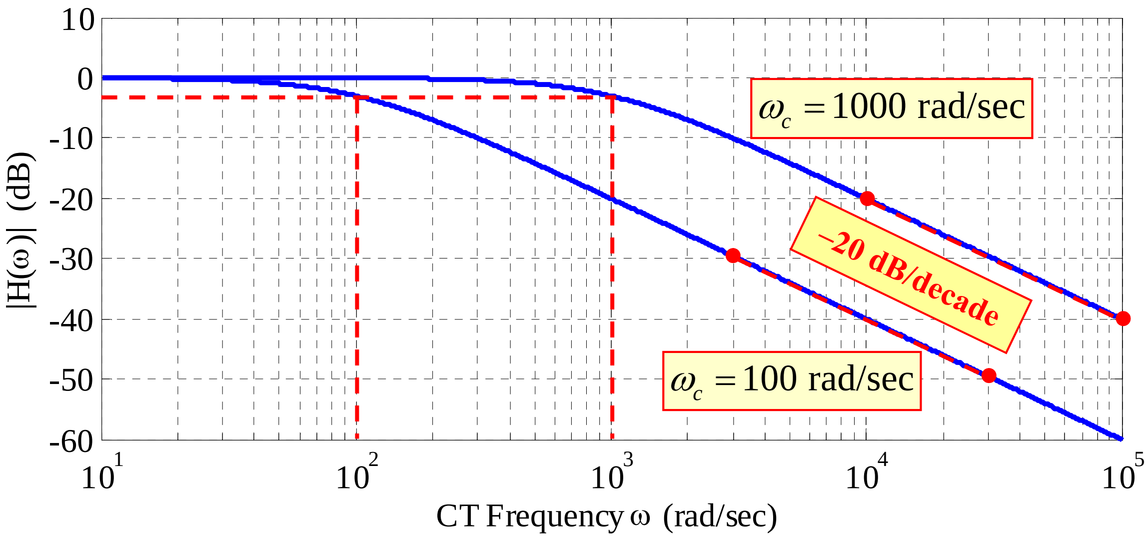
When $\omega = 1/RC$ then

Magn is $1/\sqrt{2}$ (-3dB)

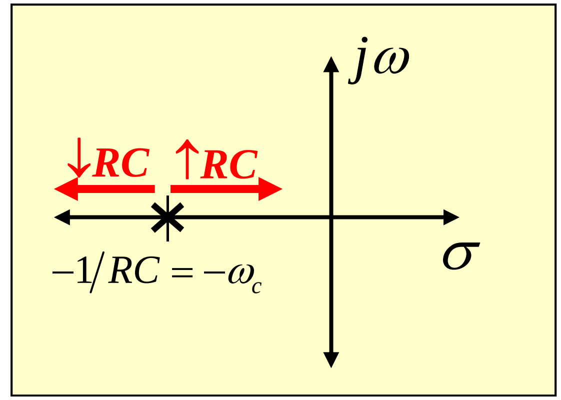
$$|H(\omega)|=\frac{1}{\sqrt{1+(RC)^2\,\omega^2}}$$

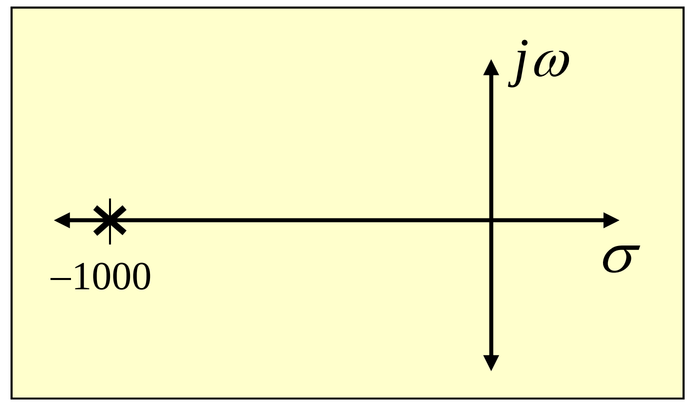
So… $\omega_c = 1/RC$

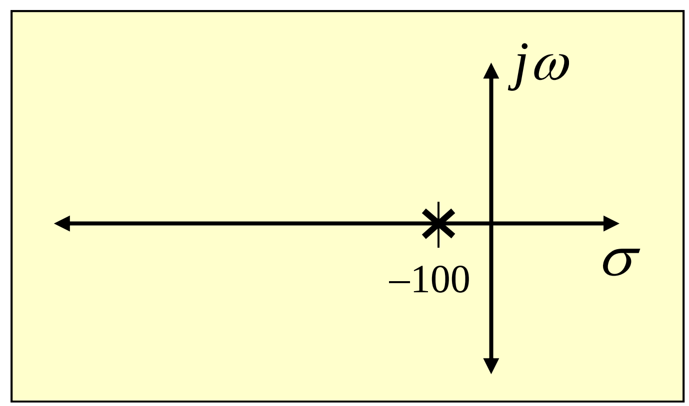
$$H(s)=\frac{\omega_c}{s+\omega_c}$$



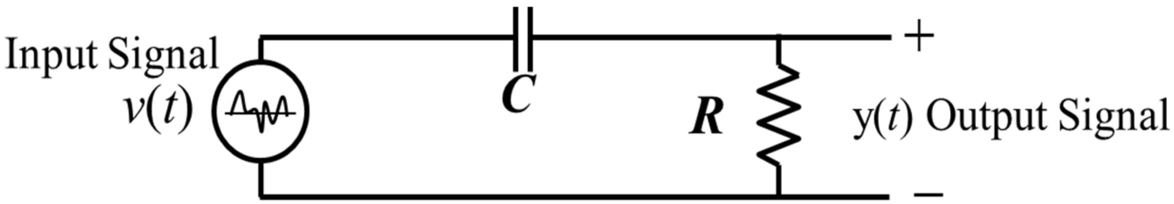

```
>> w=logspace(1,5,1000);
>> wc=100;H=freqs(wc,[1 wc],w);
>> semilogx(w,20*log10(abs(H)))
```

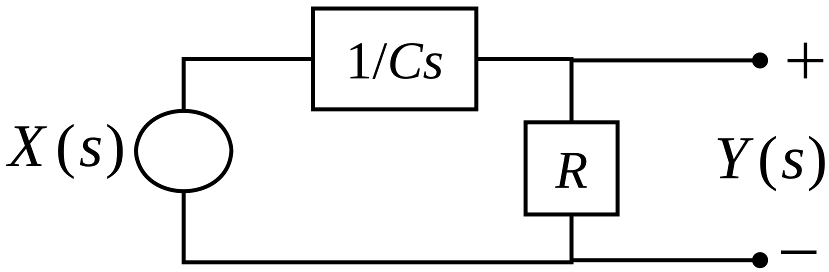






"First-Order" Highpass Filter: RC Circuit





$$Y(s) = \left[\frac{R}{1/Cs + R}\right] X(s)$$

By voltage divider (the best approach here) we get this

$$H(s) = \frac{R}{1/Cs + R}$$

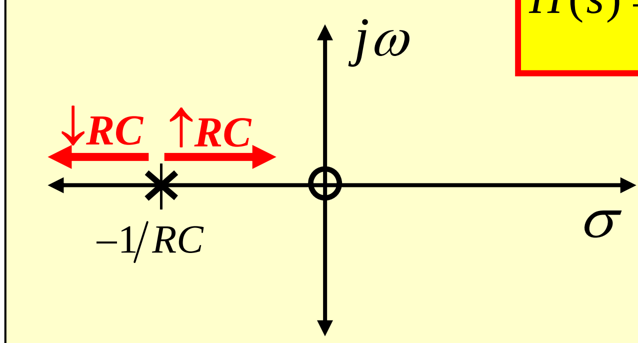
$$H(s) = \frac{RCs}{1 + RCs} = \frac{s}{s + 1/RC}$$

1 Zero @ s = 0

1 Pole @ s = -1/*RC*

1st Order

$$H(s) = \frac{s}{s + 1/RC}$$



$$H(s) = \frac{RCs}{1+RCs} = \frac{s}{s+1/RC}$$

$$H(\omega) = \frac{jRC\omega}{1+jRC\omega}$$

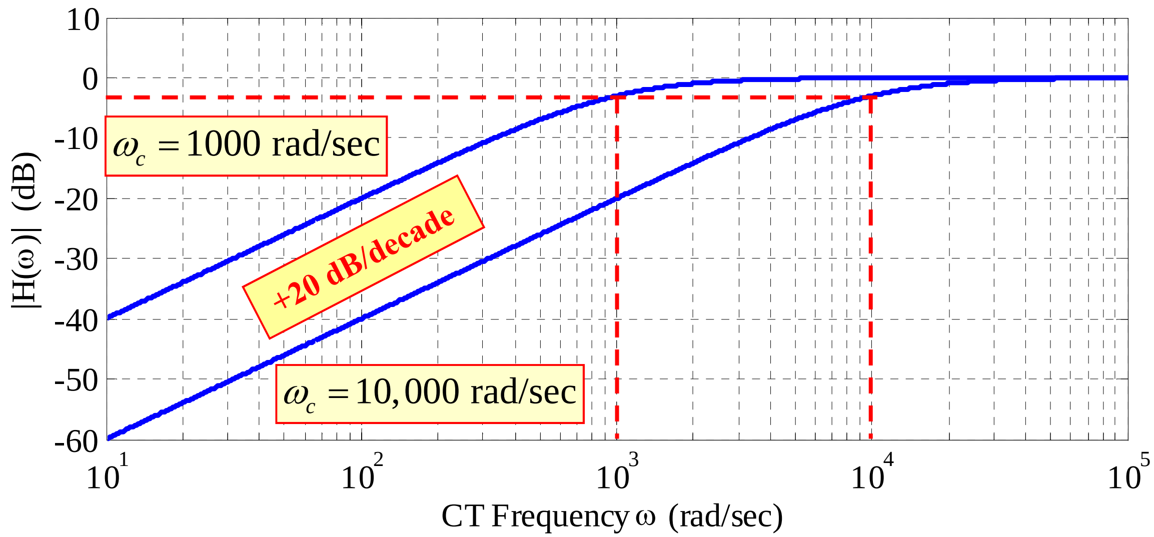
When $\omega = 1/RC$ then

Magn is $1/\sqrt{2}$ (-3dB)

$$|H(\omega)| = \frac{RC|\omega|}{\sqrt{1+(RC)^2\omega^2}}$$

So… $\omega_c = 1/RC$

$$H(s) = \frac{s}{s+\omega_c}$$




```
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>> wc=1000;H=freqs([1 0],[1 wc],w);
>> semilogx(w,20*log10(abs(H)))
```
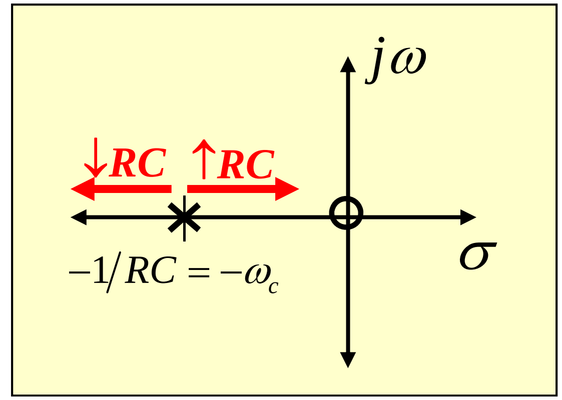
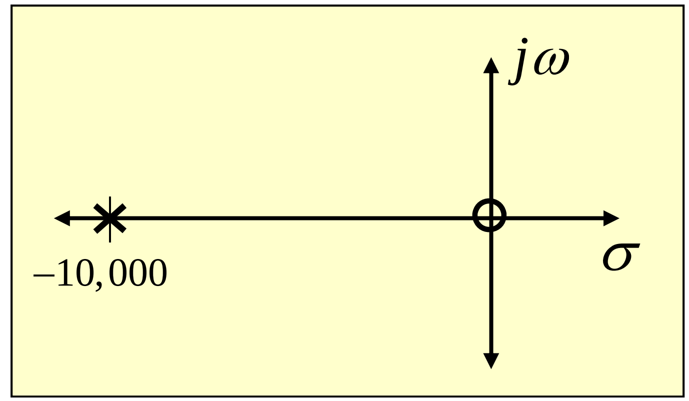
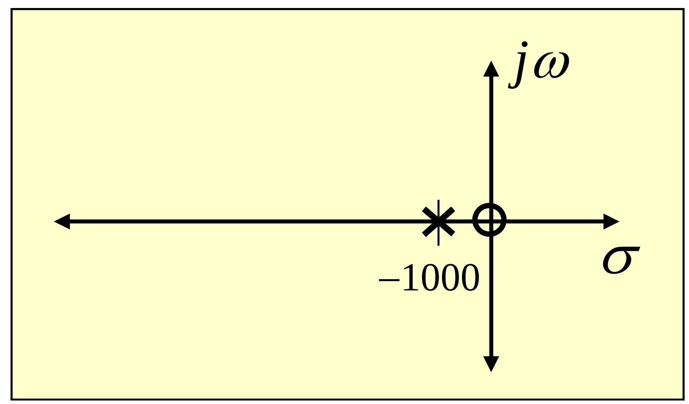
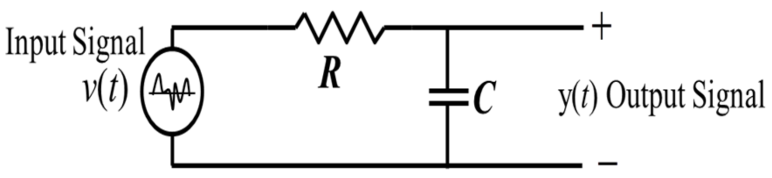







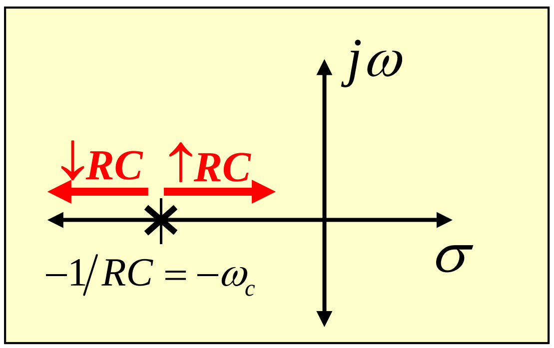
Lowpass Filter

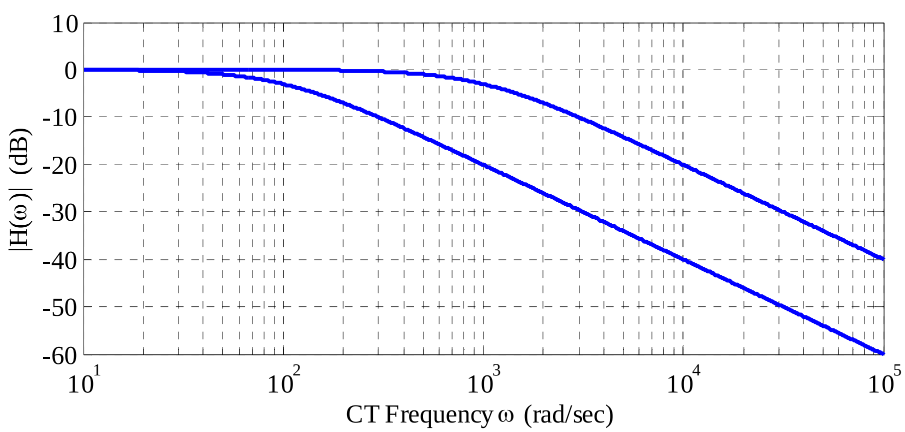


At high freqs… C is like a short…
Stops high frequencies!!

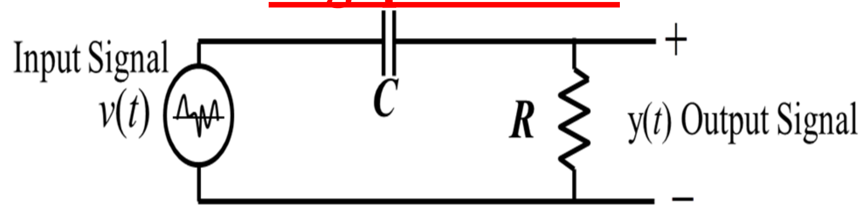
$$H(s) = \frac{\omega_c}{s + \omega_c}$$

$$H(\omega) = \frac{1}{1 + jRC\omega}$$





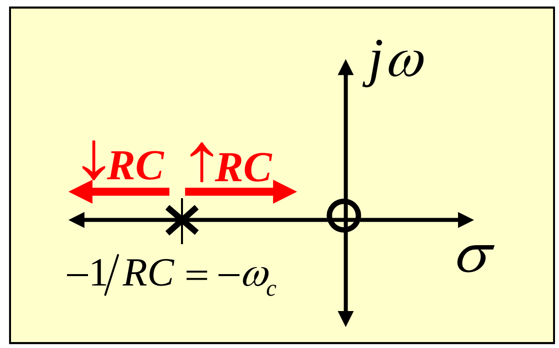
Highpass Filter

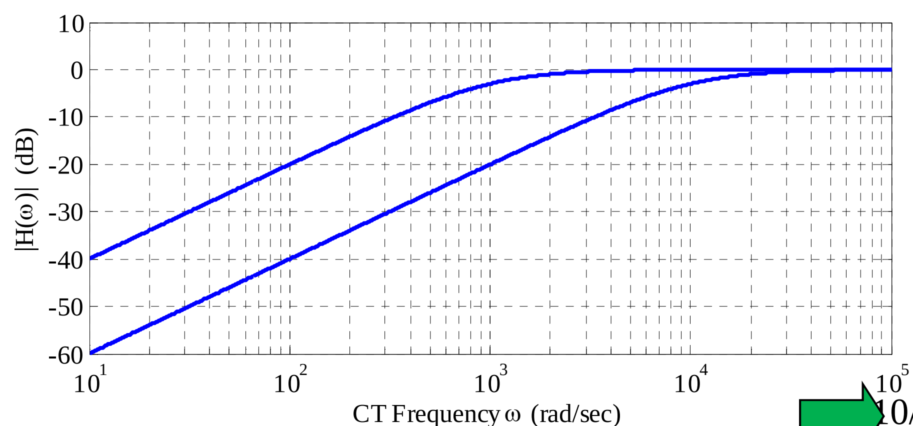


At low freqs… C is like an open…
Stops low frequencies!!

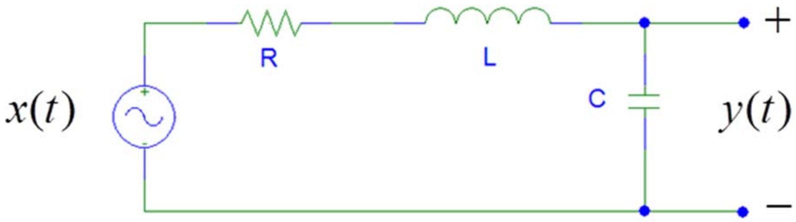
$$H(s) = \frac{s}{s + \omega_c}$$

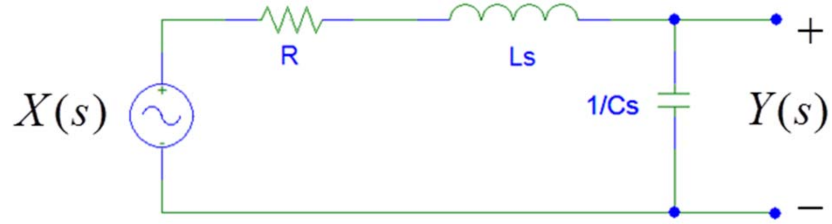
$$H(\omega) = \frac{jRC\omega}{1 + jRC\omega}$$





A "Second-Order" Lowpass Filter: RLC Circuit





By voltage divider (the best approach here) we get this:
$$Y(s) = \left[\frac{1/Cs}{1/Cs + Ls + R}\right] X(s)$$

$$H(s) = \frac{1/Cs}{1/Cs + Ls + R} \Rightarrow H(s) = \frac{1/LC}{s^2 + (R/L)s + 1/LC} \Rightarrow H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

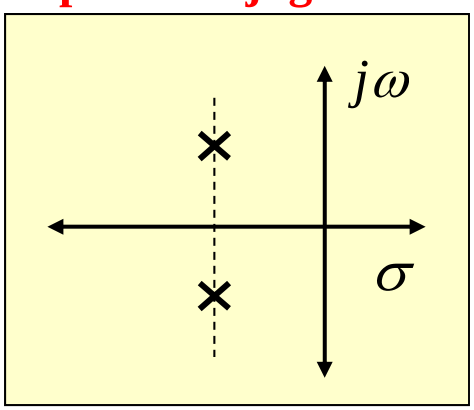
2nd Order

2 Poles (3 Possible Ways)

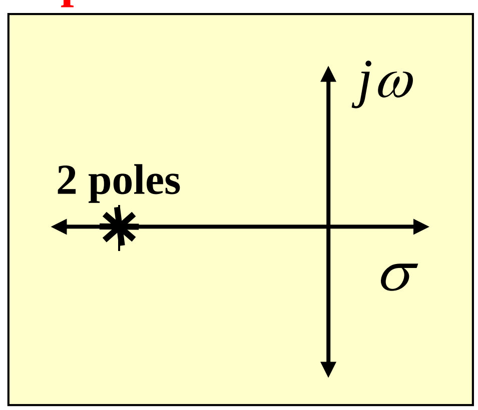
Damping Ratio
$$\zeta \triangleq \frac{R}{2L}$$

Natural Freq.
$$\omega_n \triangleq \frac{1}{\sqrt{LC}}$$

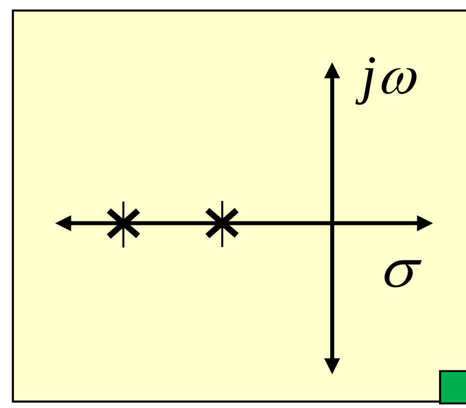
Complex-Conjugate Poles



Repeated "Real" Poles



Distinct "Real" Poles



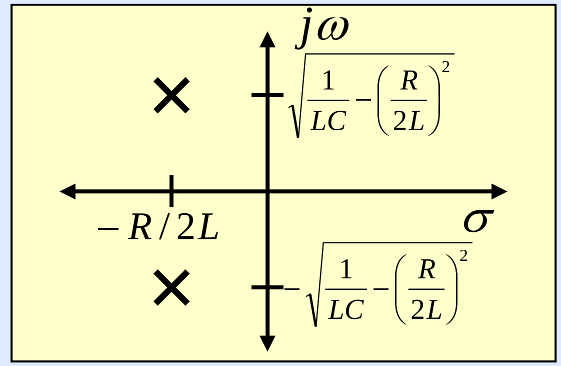
The poles are the roots of $s^2 + (R/L)s + 1/LC$: $p_{1,2} = -R/2L \pm \sqrt{(R/2L)^2 - 1/LC}$

Complex Roots

$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ ⇒ $0 \le R < \frac{2L}{\sqrt{LC}}$ ⇒ $0 \le \zeta < 1$

⇒ $p_{1,2} = \frac{-R}{2L} \pm j\omega_0$ $\quad \omega_0 \triangleq \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

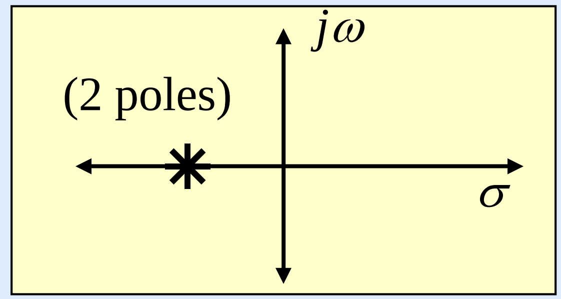
When $R = 0$, poles are <u>on</u> $j\omega$ axis



Repeated Real Roots

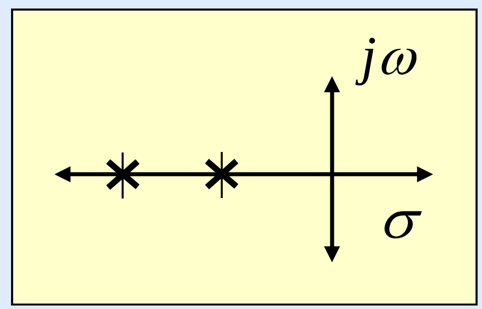
$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ ⇒ $R = \frac{2L}{\sqrt{LC}}$ ⇒ $\zeta = 1$

⇒ $p_{1,2} = \frac{-R}{2L}$

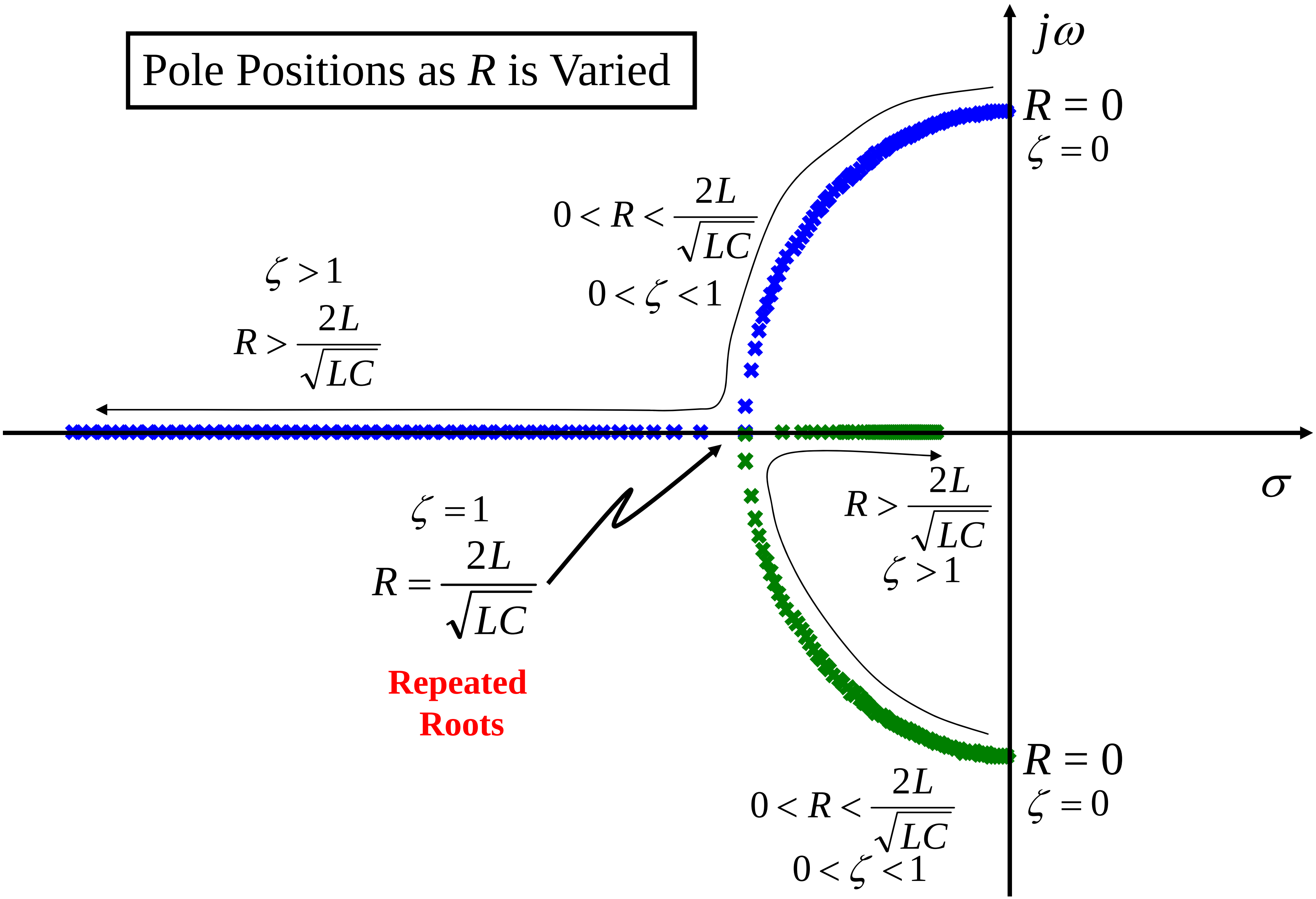


Distinct Real Roots

$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ ⇒ $R > \frac{2L}{\sqrt{LC}}$ ⇒ $\zeta > 1$

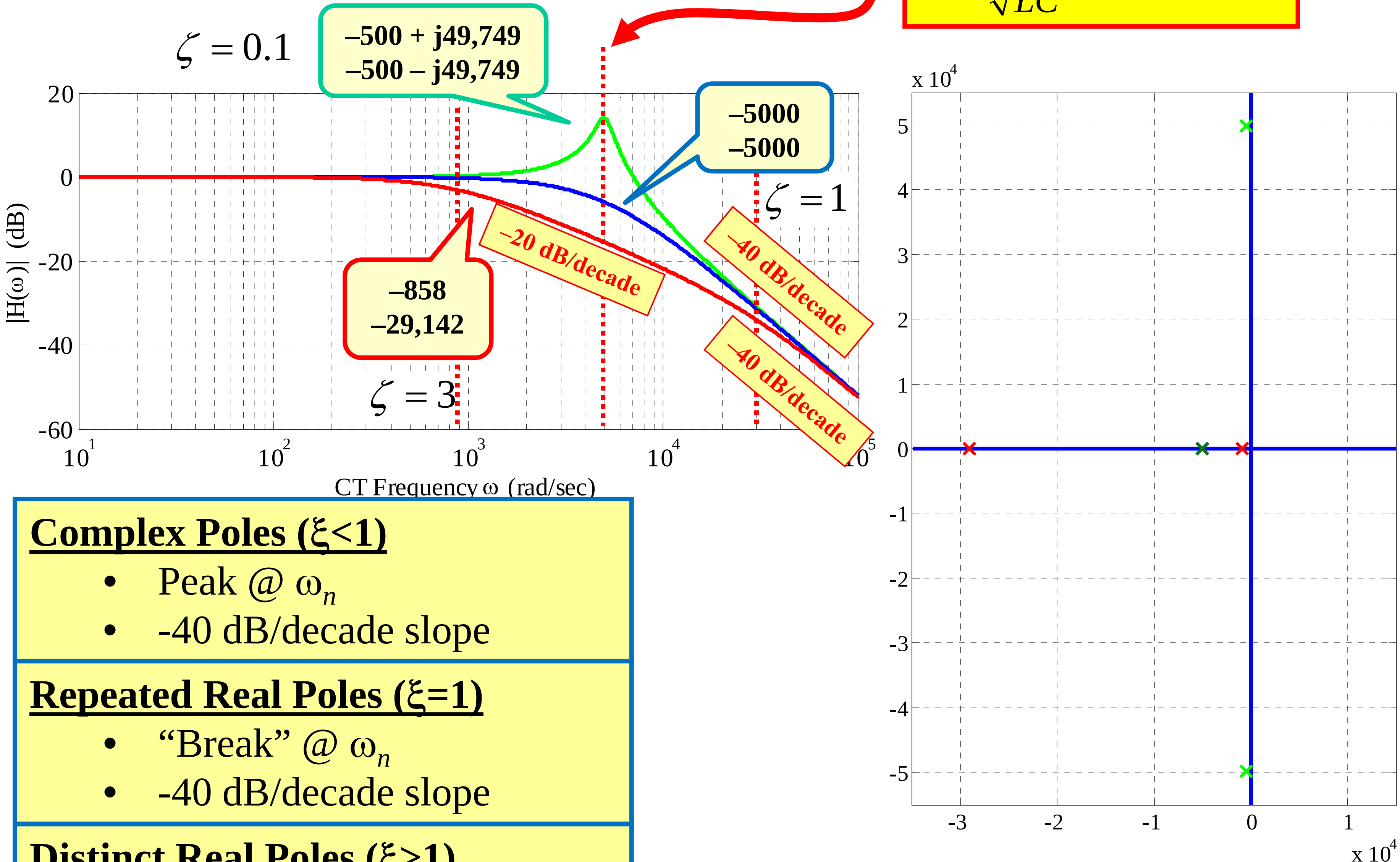


Pole Positions as R is Varied



Freq Resp Magnitude for Three Cases

$$\omega_n = \frac{1}{\sqrt{LC}} = 5000 \text{ rad/sec}$$



Complex Poles (ξ<1)

- Peak @ ω_n
- -40 dB/decade slope

Repeated Real Poles (ξ=1)

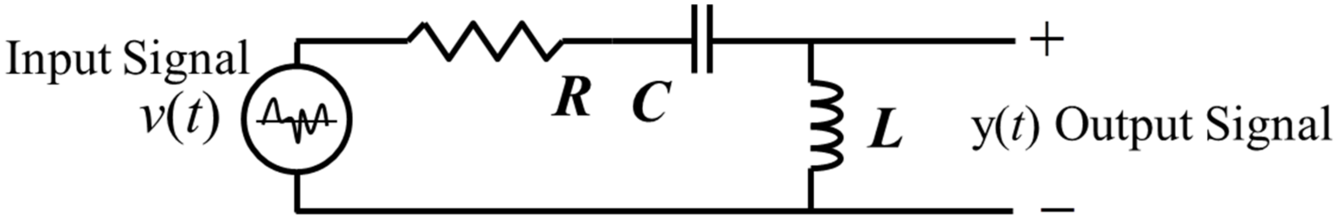
- "Break" @ ω_n
- -40 dB/decade slope

Distinct Real Poles (ξ>1)

- Two Breaks @ "Poles"
- -20 dB/dec then -40 dB/dec

2nd Order has faster rolloff vs 1st Order (-40 dB/dec vs. -20 dB/dec)

A "Second-Order" Highpass Filter: RLC Circuit



By voltage divider (the best approach here) we get this:

$$Y(s) = \left[\frac{Ls}{1/Cs + Ls + R}\right] X(s)$$

$$H(s) = \frac{Ls}{1/Cs + Ls + R} \Rightarrow H(s) = \frac{s^2}{s^2 + (R/L)s + 1/LC} \Rightarrow H(s) = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2 Zeros @ Origin

Same Denominator!!

2^{nd} Order

2 Poles (3 Possible Ways)

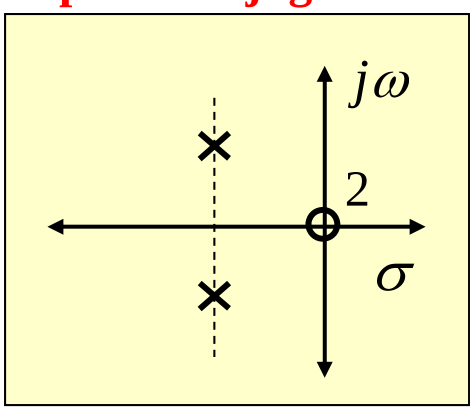
Damping Ratio

$$\zeta \triangleq \frac{R}{2L}$$

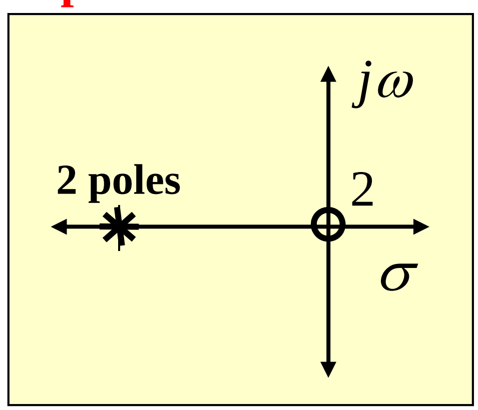
Natural Freq.

$$\omega_n \triangleq \frac{1}{\sqrt{LC}}$$

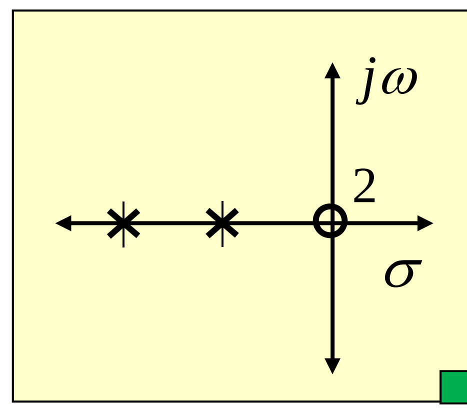
Complex-Conjugate Poles

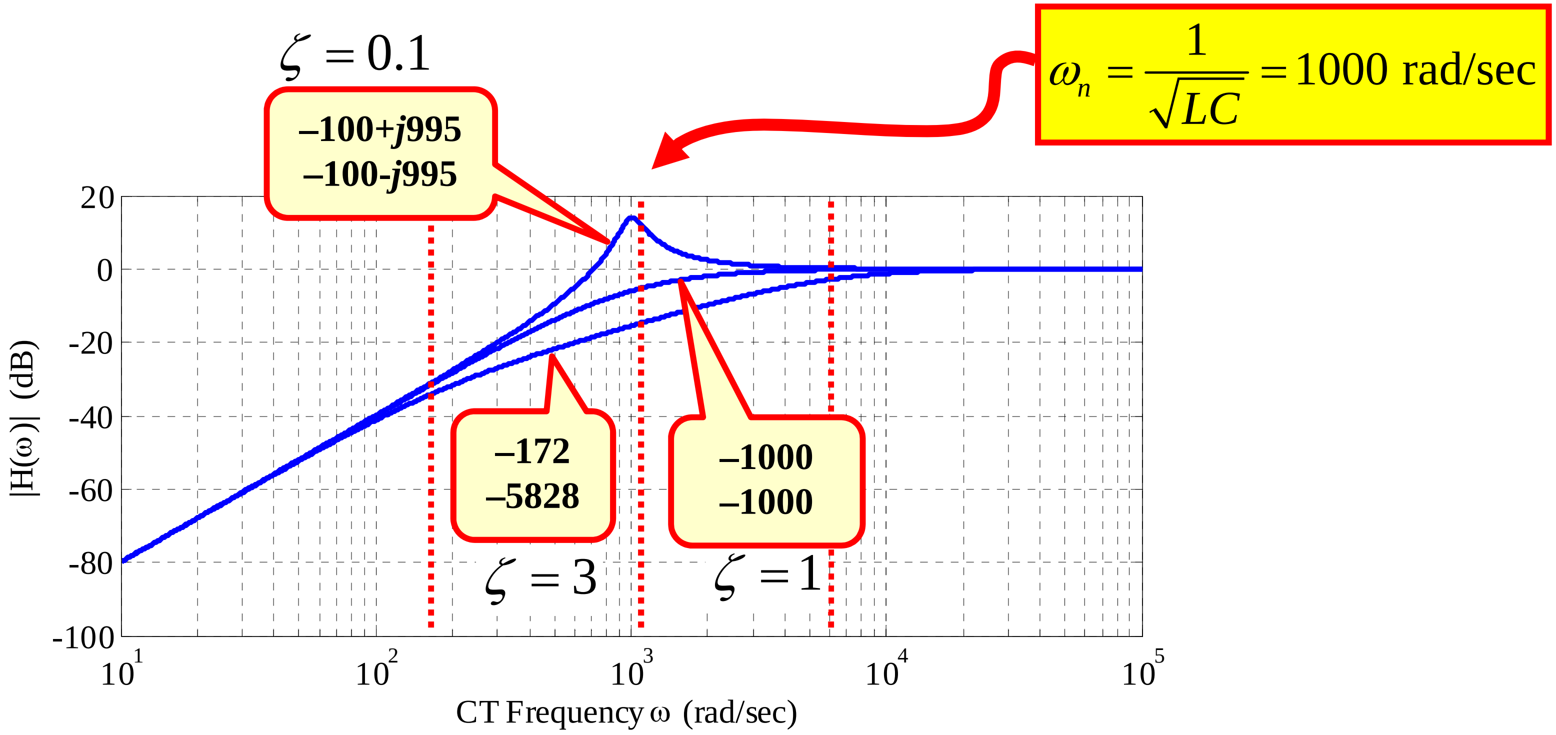


Repeated "Real" Poles



Distinct "Real" Poles





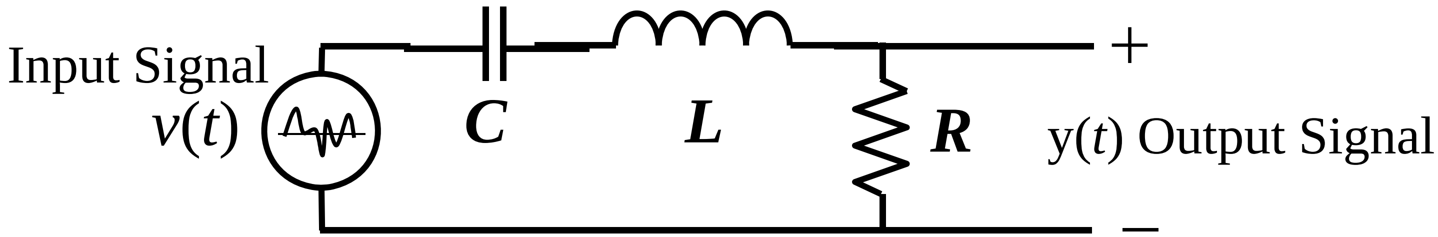
2nd Order has faster rolloff vs 1st Order (-40 dB/dec vs. -20 dB/dec)

What we are seeing is that we get 20 dB of slope for each order!!! (For LPF and HPF… But see next for BPF…)

So a 3rd Order LPF would (eventually) rolloff at -60 dB/decade!!!

So… the main advantage of higher order filters is that your stop band is better due to the faster rolloff!!

A "Second-Order" Bandpass Filter: RLC Circuit



By voltage divider (the best approach here) we get this:

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$$H(s) = \frac{R}{1/Cs + Ls + R} \Rightarrow H(s) = \frac{(R/L)s}{s^2 + (R/L)s + 1/LC} \Rightarrow H(s) = \frac{2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

1 Zero @ Origin

Same Denominator!!

2nd Order

2 Poles (3 Possible Ways)

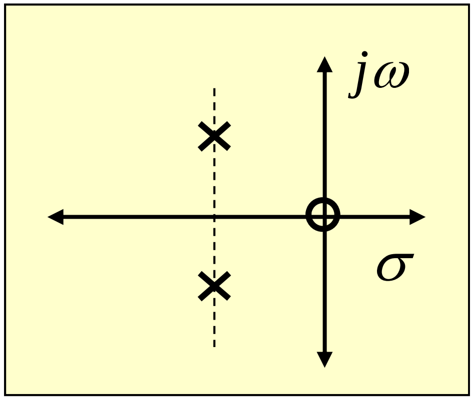
Damping Ratio

$$\zeta \triangleq \frac{R}{2L}$$

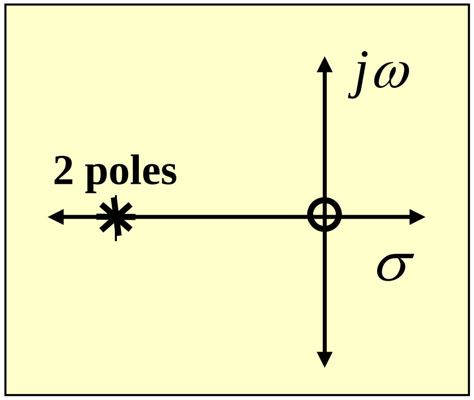
Natural Freq.

$$\omega_n \triangleq \frac{1}{\sqrt{LC}}$$

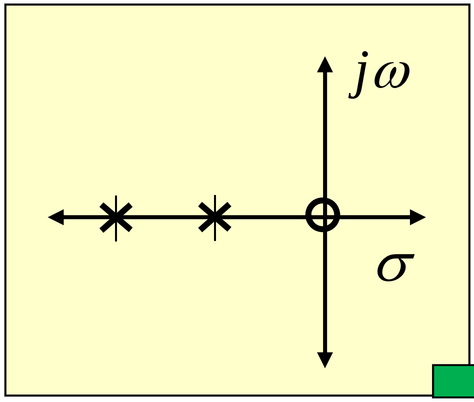
Complex-Conjugate Poles



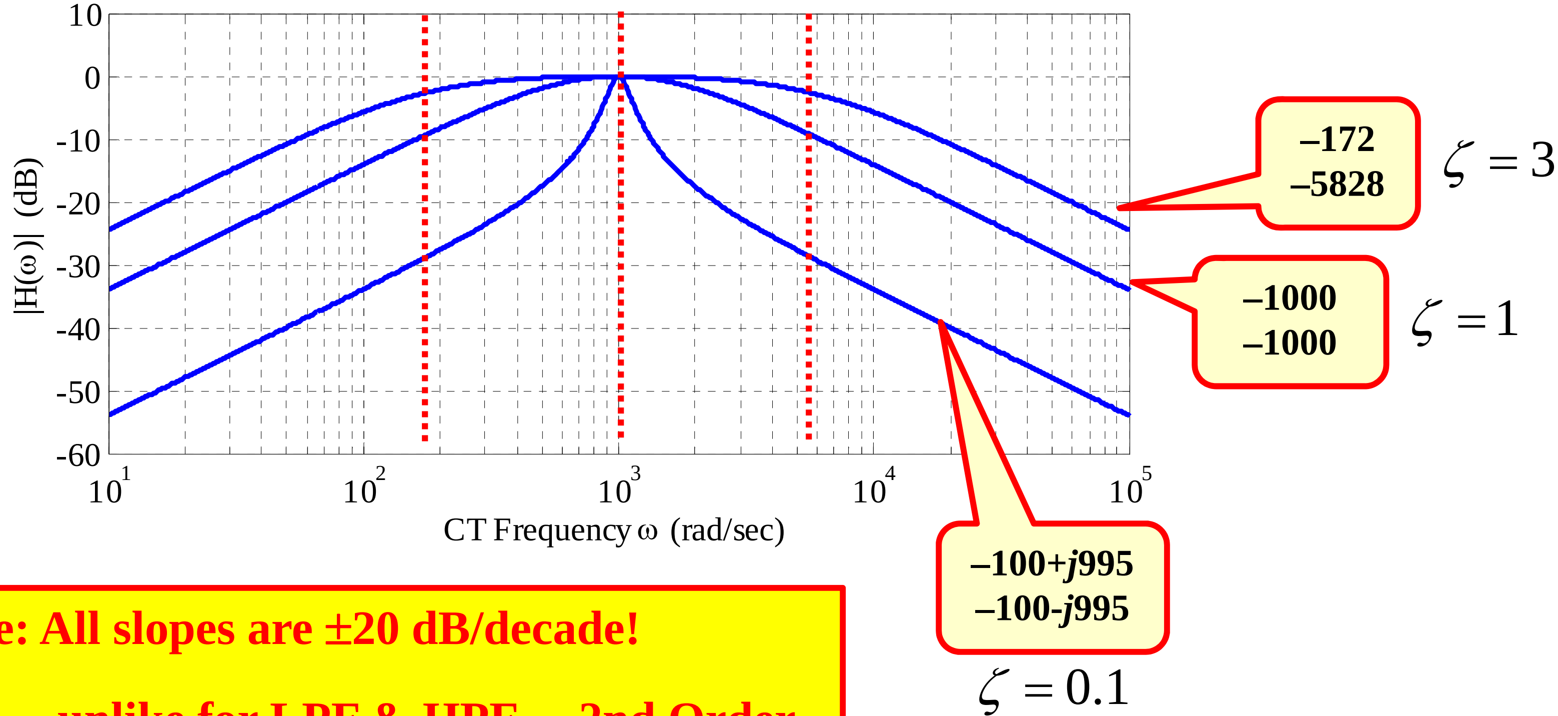
Repeated "Real" Poles



Distinct "Real" Poles



$$\omega_n = \frac{1}{\sqrt{LC}} = 1000 \text{ rad/sec}$$



Note: All slopes are ±20 dB/decade!

So… unlike for LPF & HPF… 2nd Order BPF does NOT have the faster rolloff… But, 1st Order can't even GIVE a BPF!!!

What is happening is that the second order gives you two 20 dB/dec slopes "available"…

But for a BPF you need one going up and one going down… so each only gets one of the two 20 dB slopes!