BINGHAMTON
UNIVERSITY

State University of New York

EECE 301
Signals & Systems
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Note Set #30

- D-T Systems: IIR Filters

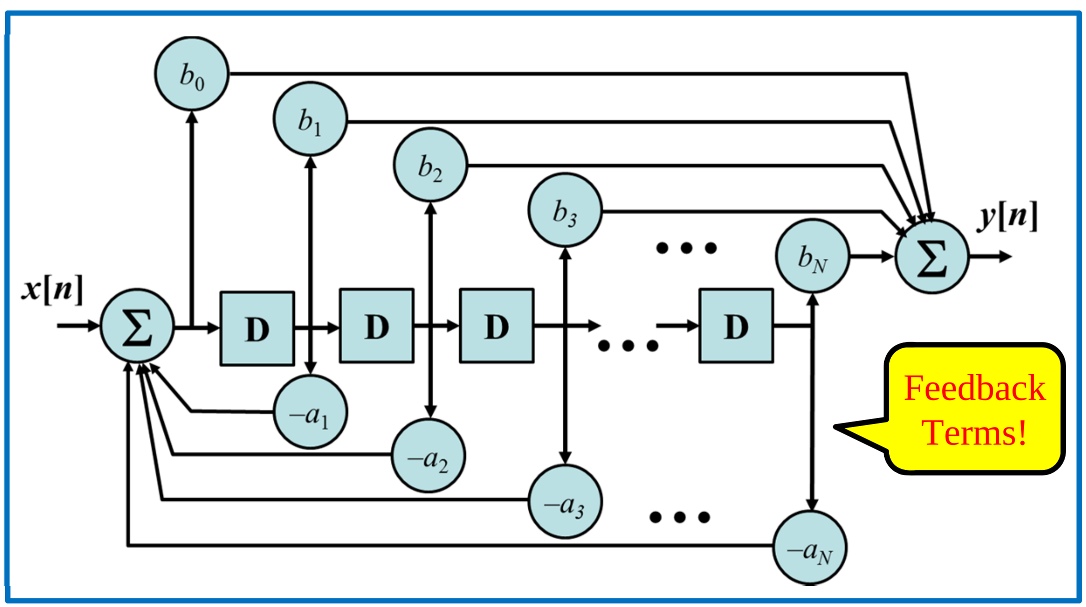
IIR Filters (Recursive Filters)

IIR (Recursive) filters have two issues that constrain their use…

1. They do not have linear phase (*can* get approx. linear phase w/ special designs)
   - linear phase is more crucial in certain areas…like digital comm & radar (which involve pulses) or filtering images (which involves edges).
2. May not be inherently stable (feedback gives poles other than at origin)
   - This can be a serious issue when implementing IIR filters

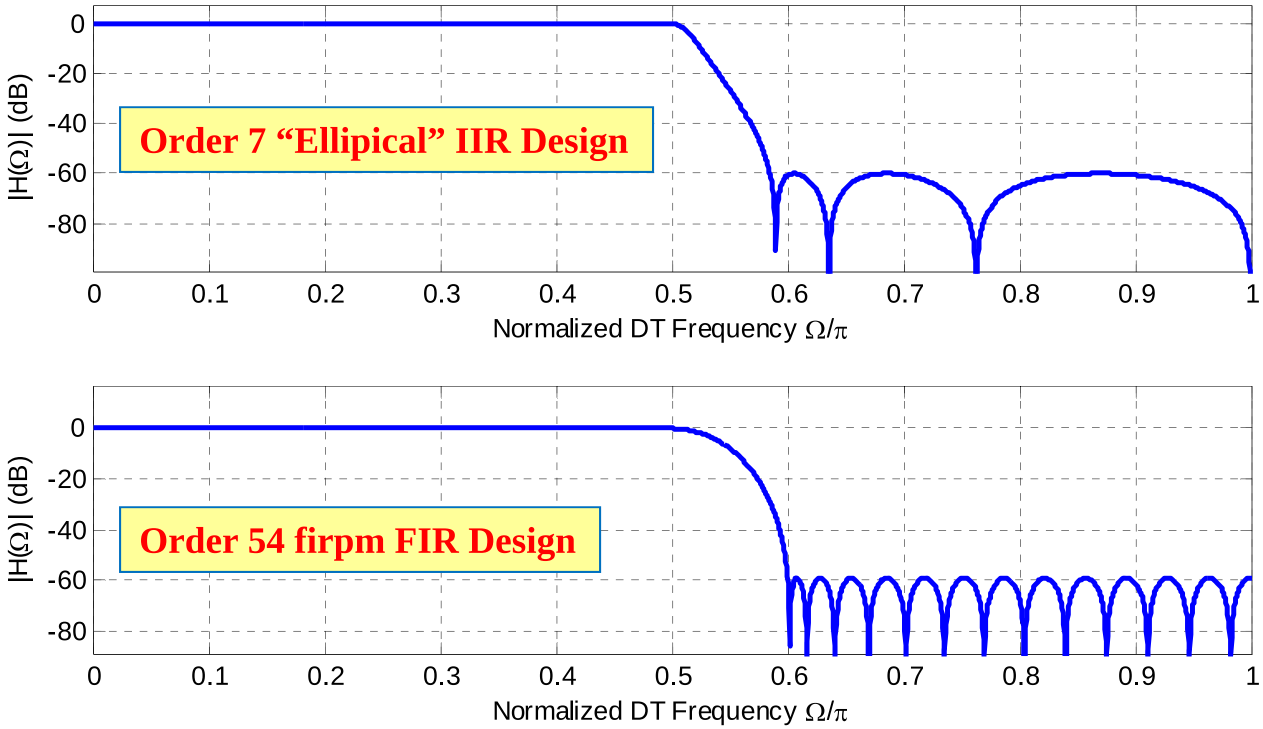
IIR Filters have one main advantage over FIR filters:

- can get good magn. resp. w/o high computational complexity



You can usually get quite good filters even with fairly low orders (like 10 or so).

Complexity Comparison: IIR vs FIR



This IIR requires:

- 15 multiplies
- 13 additions

This FIR requires:

- 55 multiplies
- 54 additions

Almost 4x as much computation for the FIR filter!

MATLAB-Based IIR Design

MATLAB has several easy commands for IIR design, including:

- butter, cheby1, cheby2, ellip

Chebyshev IIR

Sets Order

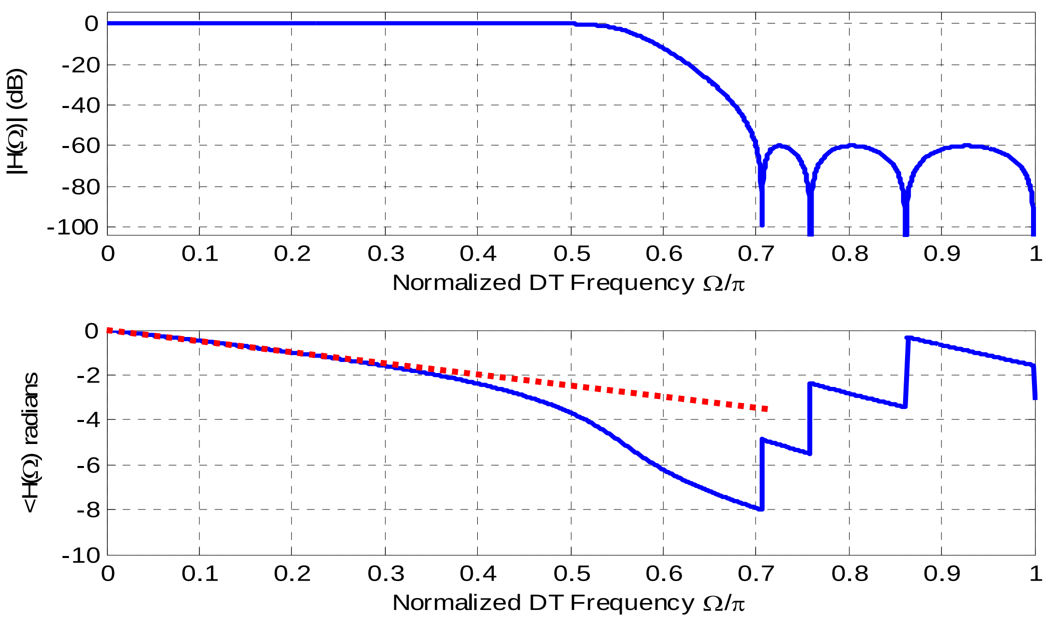
Sets Stopband Height

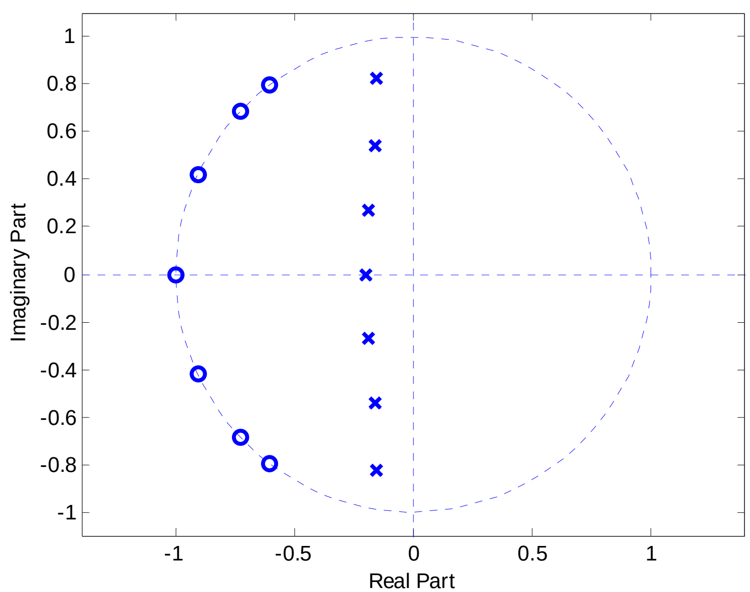
Sets Stopband Edge

```
[b,a]=cheby2(7,60,0.7);
```

b = [0.0692   0.3789   0.9728   1.5028   1.5028   0.9728   0.3789   0.0692]

a = [1.0000   1.2028   1.6599   1.0991   0.6240   0.2098   0.0473   0.0048]





Elliptical IIR

[b,a] = ellip(7,0.1,60,0.5);

Sets Order

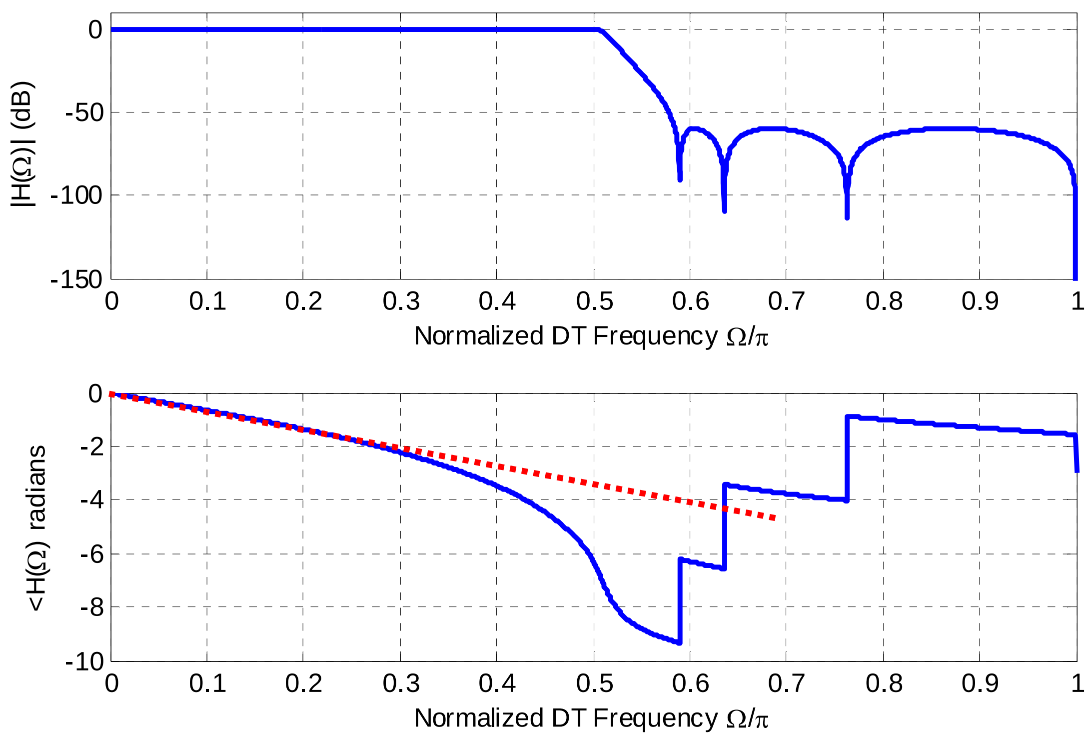
Sets Passband Ripple

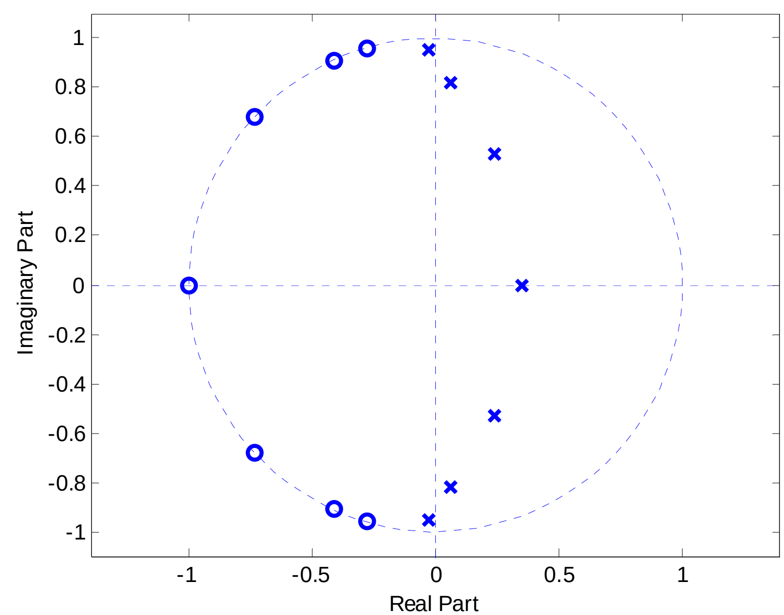
Sets Stopband Height

Sets Passband Edge

b = [0.0338 0.1302 0.2821 0.4013 0.4013 0.2821 0.1302 0.0338]

a = [1.0000 -0.8994 2.1386 -1.5364 1.4793 -0.7327 0.3178 -0.0725]





Butterworth IIR

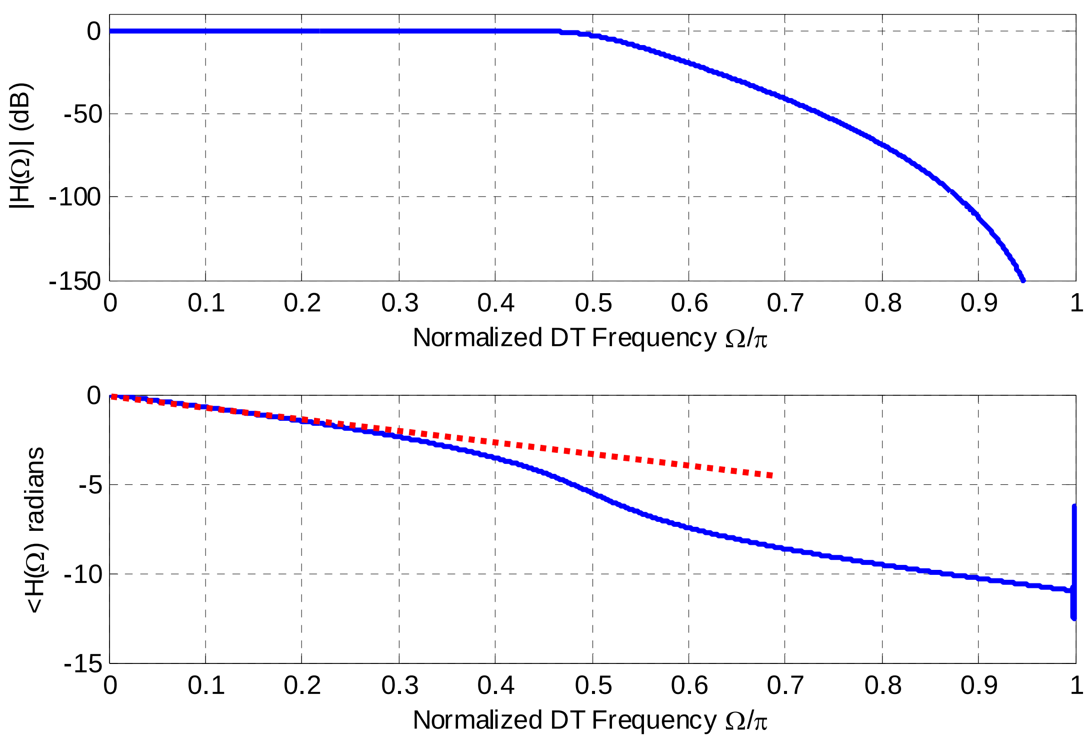
Sets Order

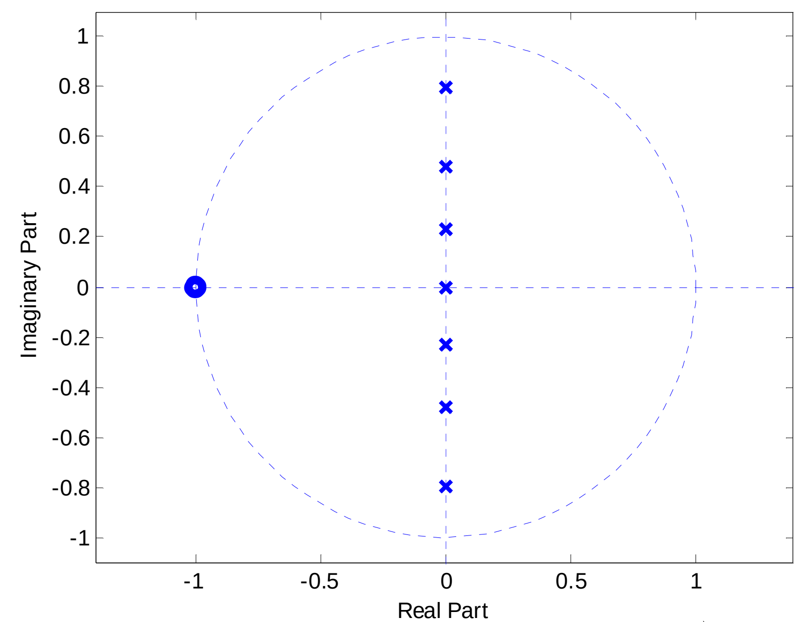
Sets Passband Edge

```
[b,a] = butter(7,0.5);
```

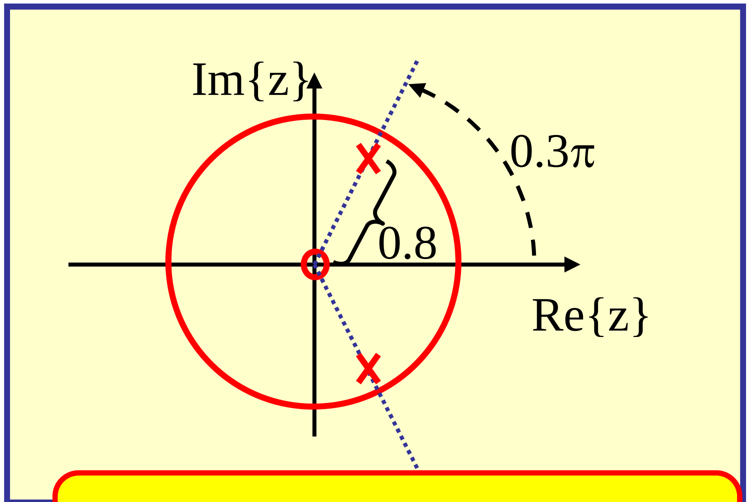
b = [0.0166 0.1160 0.3479 0.5798 0.5798 0.3479 0.1160 0.0166]

a = [1.0000 -0.0000 0.9200 -0.0000 0.1927 -0.0000 0.0077 -0.0000]

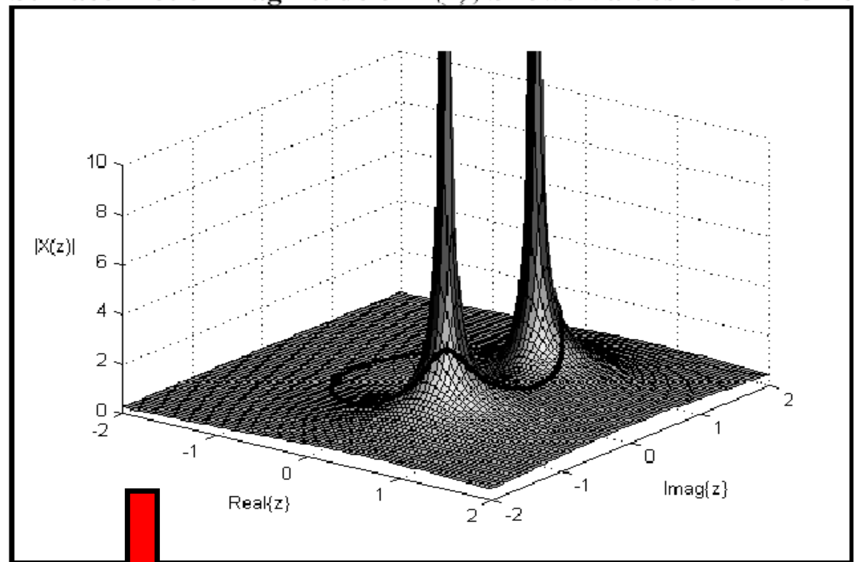




Visualizing IIR Freq Resp.

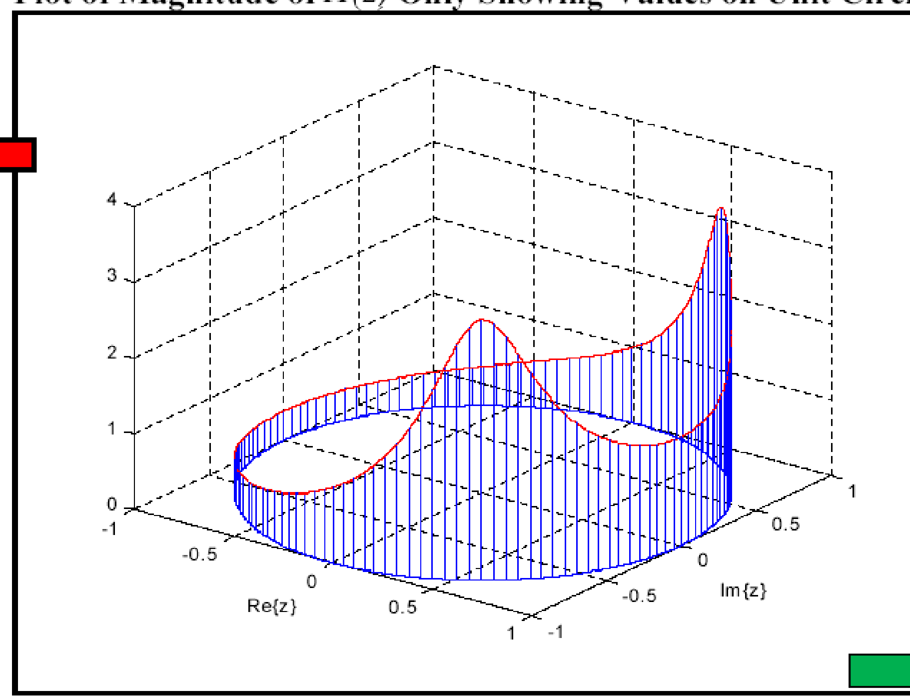


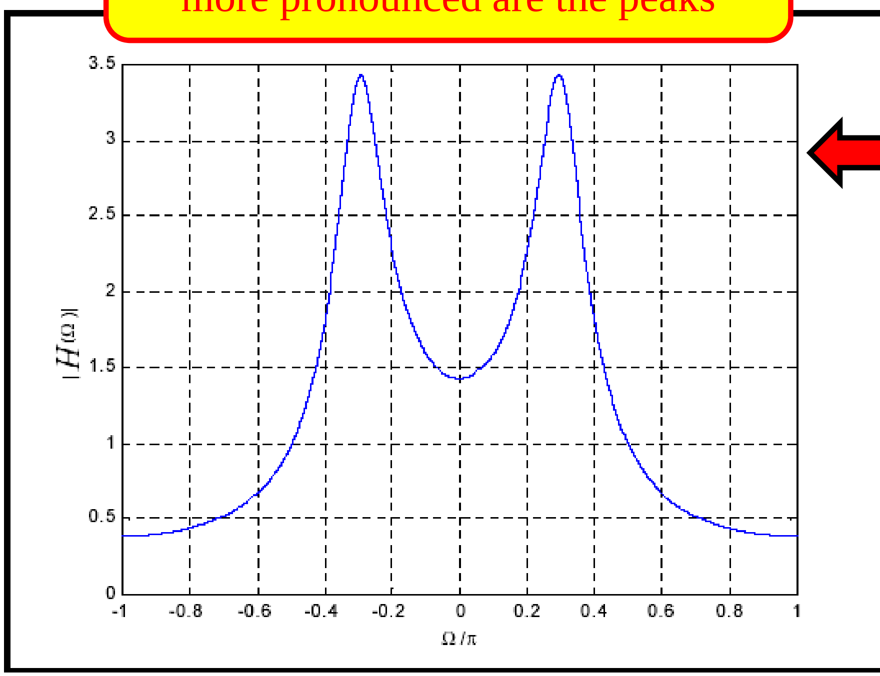
Surface Plot of Magnitude of $H(z)$; Shows Values on Unit Circle



Closer the poles are to the UC the more pronounced are the peaks

Plot of Magnitude of $H(z)$ Only Showing Values on Unit Circle





Effect of Poles & Zeros on Frequency Response of DT filters

Note: Including a pole or zero at the origin …

…doesn't change the magnitude but does change the phase

Placing a zero on UC…

…makes |H| = 0 at angle where zero is placed

Placing more zeros/poles…

… gives sharper transitions.

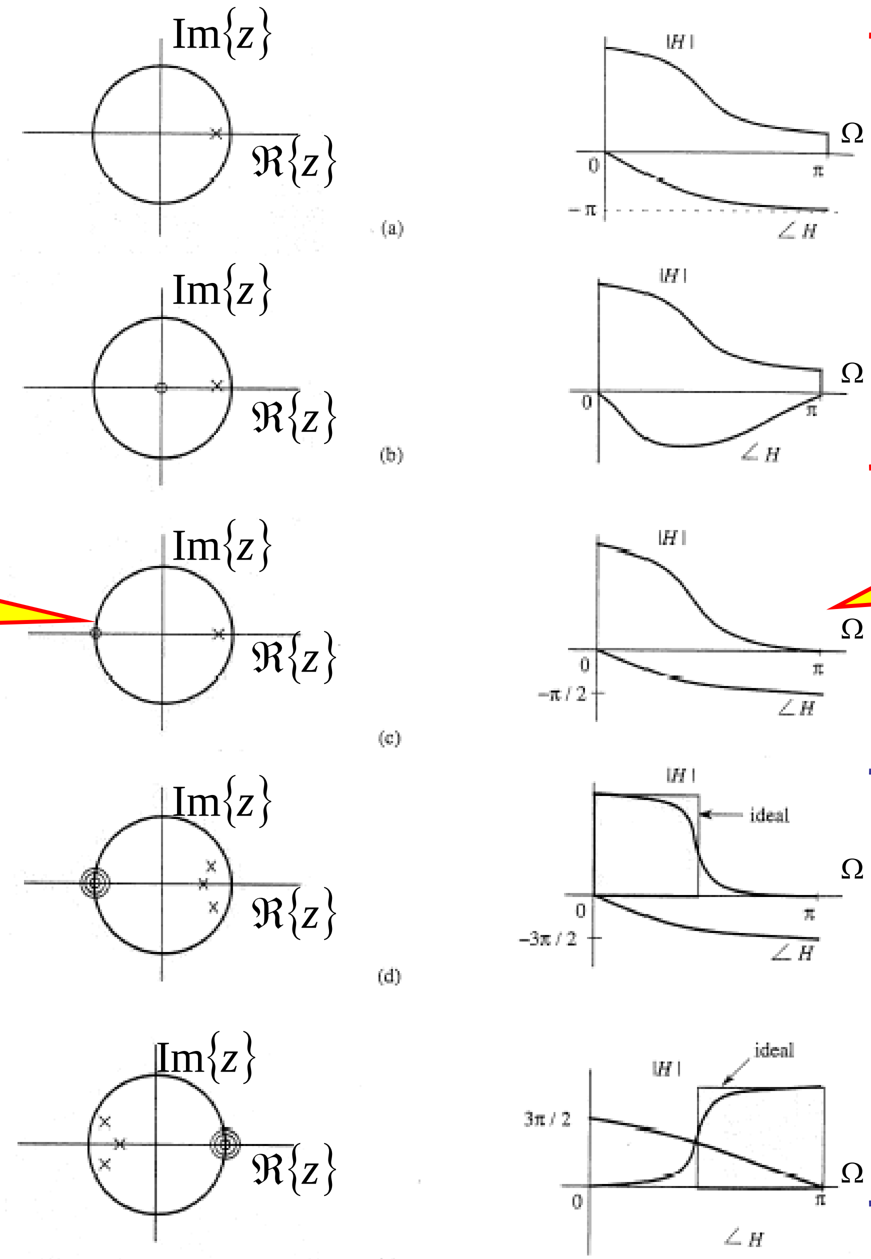


Figure from B.P. Lathi, Signal Processing and Linear Systems

Effect of Zero at Origin

$H_1(z)$ has an extra zero at the origin:

$$H_1(z) = zH(z)$$

$$H_1(\Omega) = e^{j\Omega} H(\Omega)$$

$$\begin{aligned} \left|H_1(\Omega)\right| &= \left|e^{j\Omega} H(\Omega)\right| \\ &= \left|e^{j\Omega}\right| \left|H(\Omega)\right| \\ &= \left|H(\Omega)\right| \end{aligned}$$

$$\begin{aligned} \angle H_1(\Omega) &= \angle e^{j\Omega} H(\Omega) \\ &= \angle H(\Omega) + \Omega \end{aligned}$$

Effect of Pole at Origin

$H_1(z)$ has an extra pole at the origin:

$$H_1(z) = z^{-1} H(z)$$

$$H_1(\Omega) = e^{-j\Omega} H(\Omega)$$

$$\begin{aligned} \left|H_1(\Omega)\right| &= \left|e^{-j\Omega} H(\Omega)\right| \\ &= \left|e^{-j\Omega}\right| \left|H(\Omega)\right| \\ &= \left|H(\Omega)\right| \end{aligned}$$

$$\begin{aligned} \angle H_1(\Omega) &= \angle e^{-j\Omega} H(\Omega) \\ &= \angle H(\Omega) - \Omega \end{aligned}$$

No Effect on Magnitude

Effect on Phase

Effect of Feedback

We know two important things:

- Feedback is what can give us poles other than at the origin
- Any pole outside the UC causes the system to be unstable

Thus…. It is feedback that *can* cause a system to be stable.

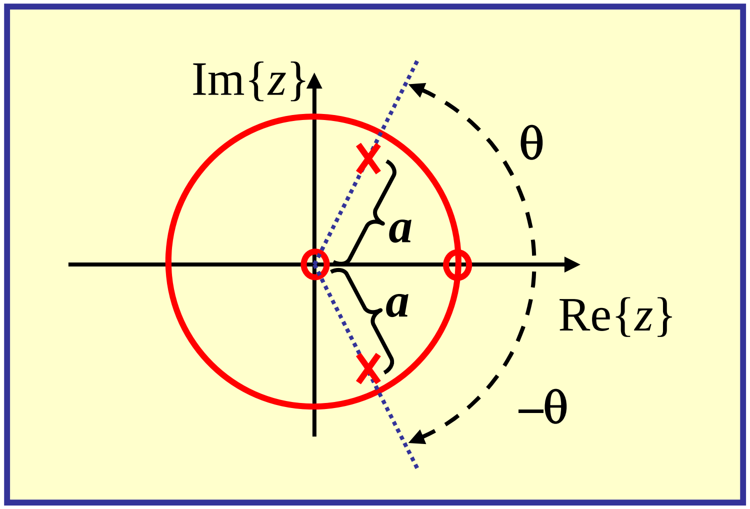
$$y[n]-2a\cos(\theta)y[n-1]+a^2y[n-2]=x[n]-x[n-1]$$

For *a*>0 and 0 ≤ θ ≤ π

$$H(z)=\frac{1-z^{-1}}{1-2a\cos(\theta)z^{-1}+a^2z^{-2}}=\frac{z(z-1)}{z^2-2a\cos(\theta)z+a^2}$$

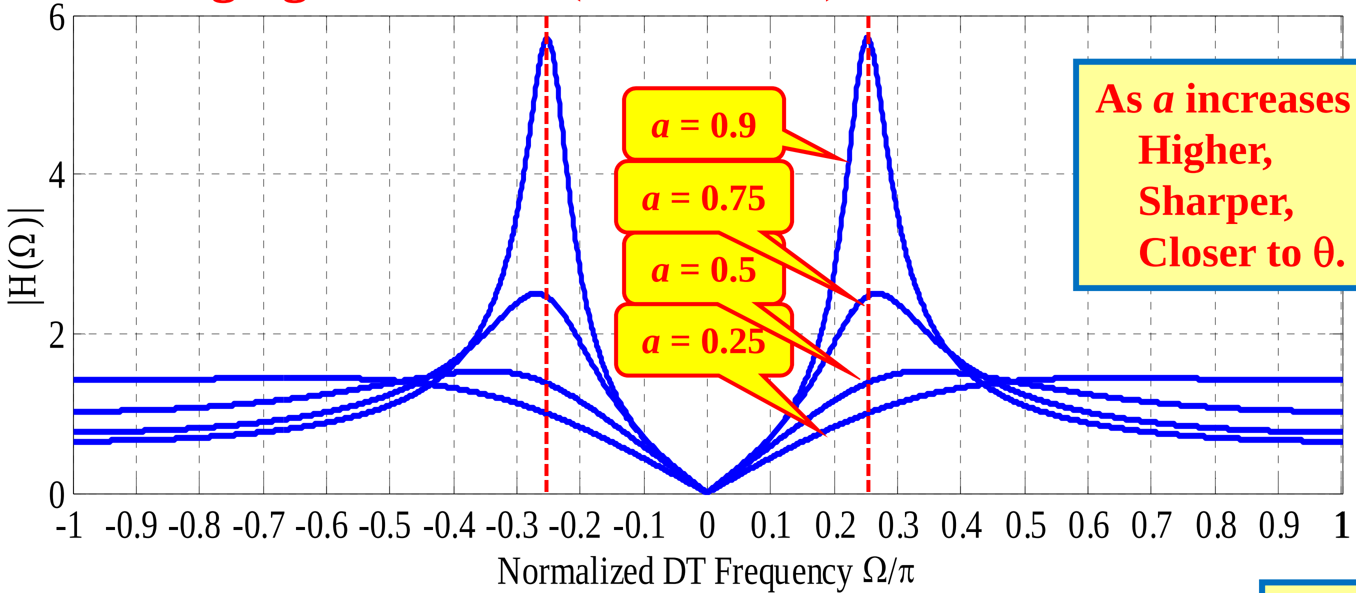
Has roots (poles!) at:

$$z=ae^{\pm j\theta}$$



So… if *a* is increased to be 1 or larger then the system is unstable!

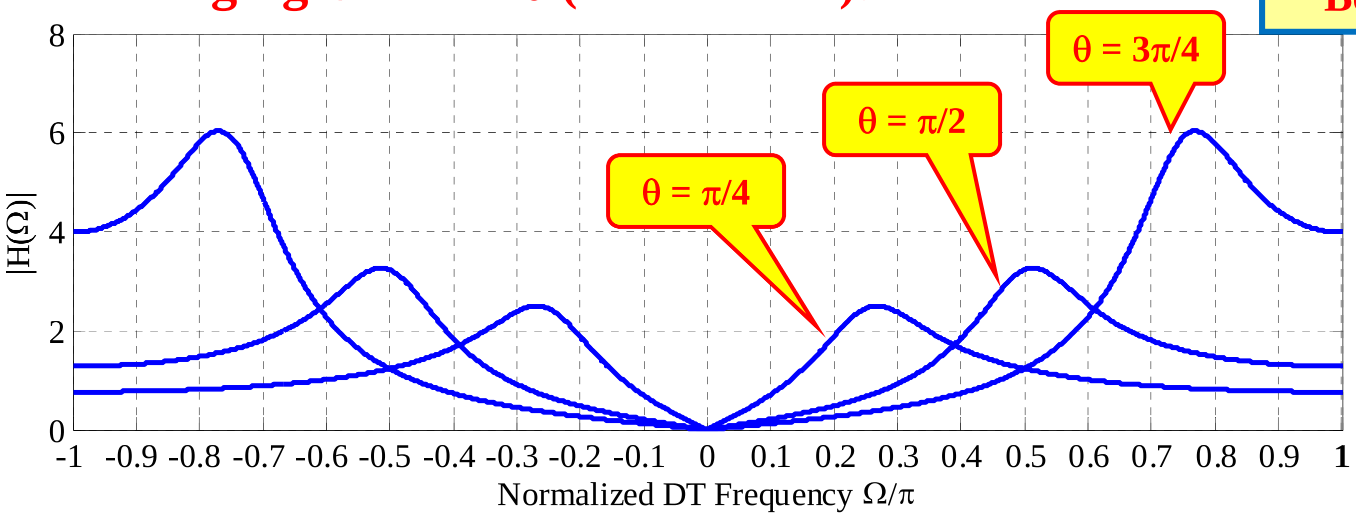
Effect of Changing Value of *a* (with θ = π/4):



As *a* increases spike becomes:
- **Higher,**
- **Sharper,**
- **Closer to θ.**

As θ increases spike:
- **Moves up in freq.,**
- **Becomes Higher.**

Effect of Changing Value of θ (with *a* = 0.75):



WHY???