

EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #27

- D-T Systems Relationships

Discrete-Time System Relationships

Time Domain

Z / Freq Domain

$$y[n] + a_1 y[n-1] + \dots + a_N y[n-N] = b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

Difference Equation

Impulse Response

Transfer Function

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}}$$

Frequency Response

$$H(\Omega) = \frac{b_0 + b_1 e^{-j\Omega} + \dots + b_M e^{-j\Omega M}}{1 + a_1 e^{-j\Omega} + \dots + a_N e^{-j\Omega N}}$$

Pole/Zero Diagram

Stable if All Poles inside UC

Roots (from Alt. Form)

Inspect

Inspect

ZT

DTFT

Evaluate on Unit Circle

$$H(\Omega) = H(z) \Big|_{z=e^{j\Omega}}$$

Example System Relationships

Time Domain

Difference Equation

Z / Freq Domain

Transfer Function

Frequency Response

ZT (Theory)
Inspect (Practice)

DTFT (Theory)
Inspect (Practice)

$$y(n) - \alpha y(n-1) = \beta x(n)$$

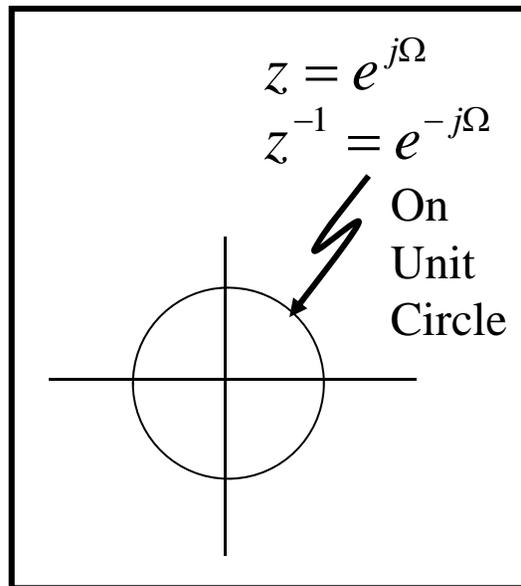
$$H(z) = \frac{\beta}{1 - \alpha z^{-1}}$$

$$y(n) - \alpha y(n-1) = \beta x(n)$$

$$H(\Omega) = \frac{\beta}{1 - \alpha e^{-j\Omega}}$$

Example System Relationships (cont.)

Z / Freq Domain



$$H(z) = \frac{\beta}{1 - \alpha z^{-1}}$$

Transfer
Function

$$z^{-1} = e^{-j\Omega}$$

$$H(\Omega) = \frac{\beta}{1 - \alpha e^{-j\Omega}}$$
$$\Omega \in (-\pi, \pi]$$

Unit Circle

Frequency
Response

Example System Relationships (cont.)

Plotting Transfer Function

$$H(\Omega) = \frac{\beta}{1 - \alpha e^{-j\Omega}} = \frac{\beta}{1 - [\alpha \cos(\Omega) - j\alpha \sin(\Omega)]}$$
$$= \frac{\beta}{[1 - \alpha \cos(\Omega)] + j\alpha \sin(\Omega)}$$

$$|H(\Omega)| = \frac{\beta}{\sqrt{[1 - \alpha \cos(\Omega)]^2 + [\alpha \sin(\Omega)]^2}}$$

$$\angle H(\Omega) = -\tan^{-1} \left[\frac{\alpha \sin(\Omega)}{1 - \alpha \cos(\Omega)} \right]$$

Euler's
Equation

Group into
Real & Imag

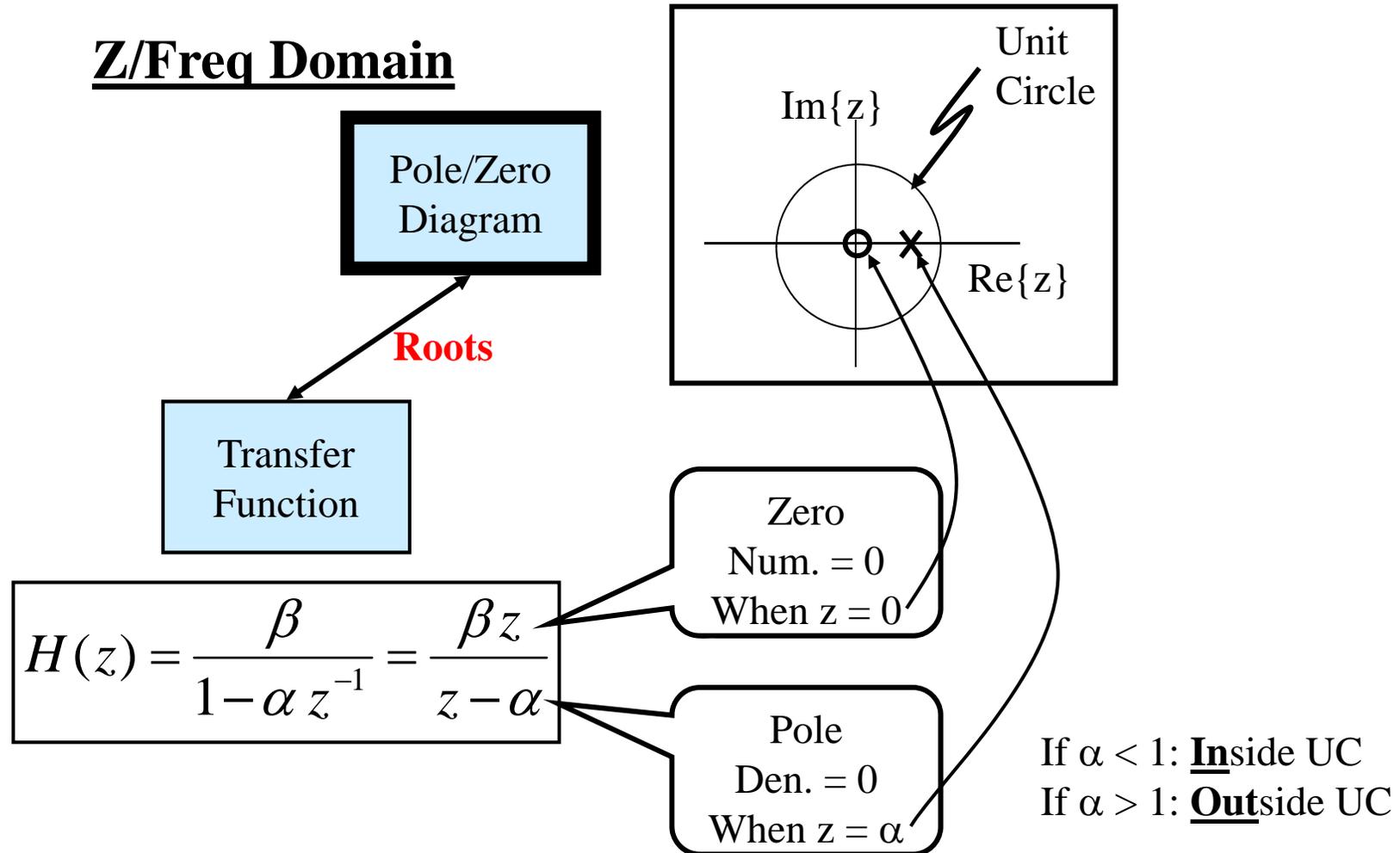
Standard
Eqs for
Mag. & Angle

Or... Use MATLAB:

```
>> omega = -pi:?:pi;  
>> H = freqz(beta,[1 -alpha],omega)  
>> plot(omega/pi,abs(H))  
>> plot(omega/pi,abs(H))
```

Example System Relationships (cont.)

Z/Freq Domain



Or... Using MATLAB:
>> zplane(beta,[1 -alpha])

Example System Relationships (cont.)

