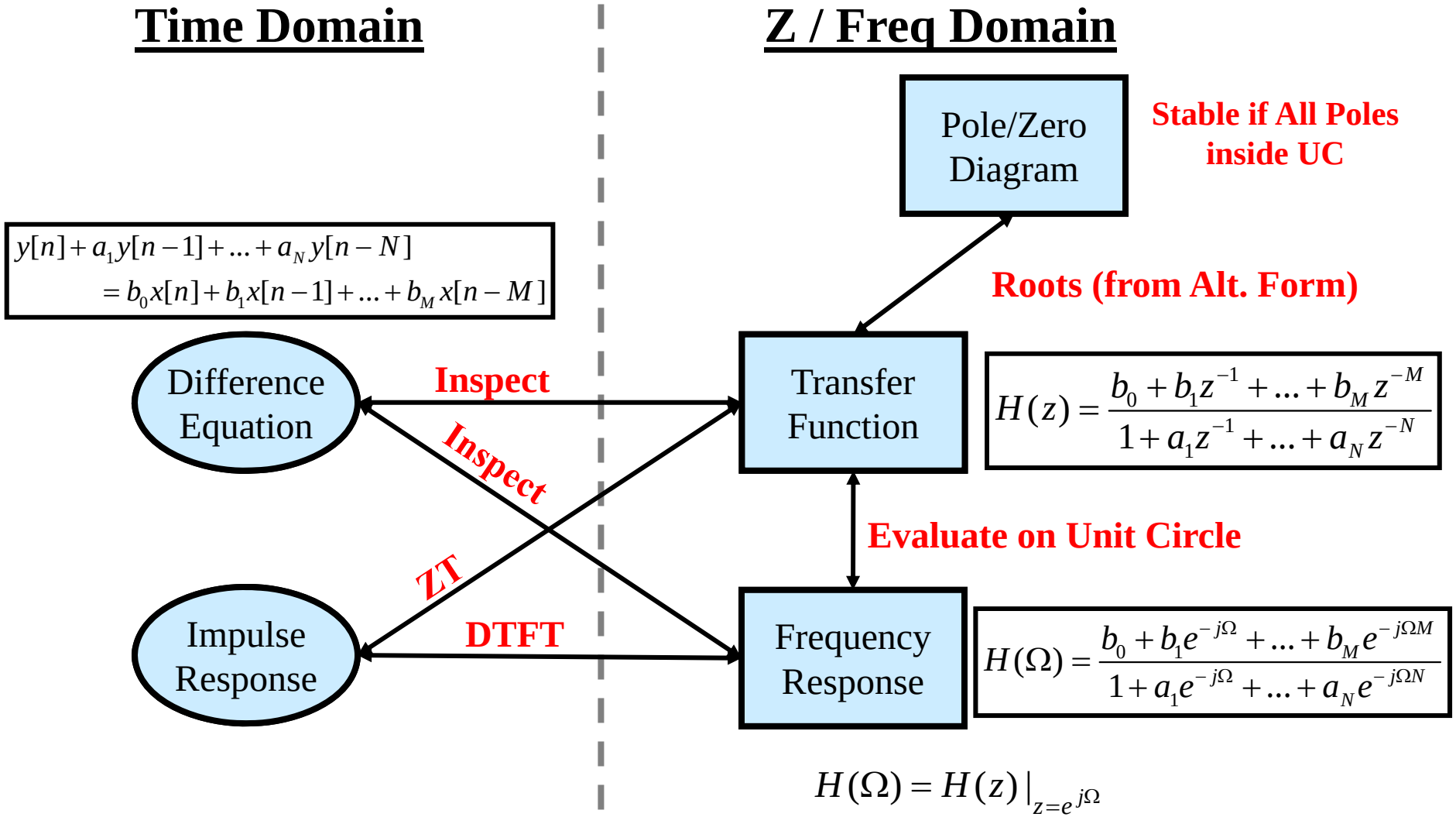
BINGHAMTON UNIVERSITY
State University of New York

EECE 301
Signals & Systems
Prof. Mark Fowler

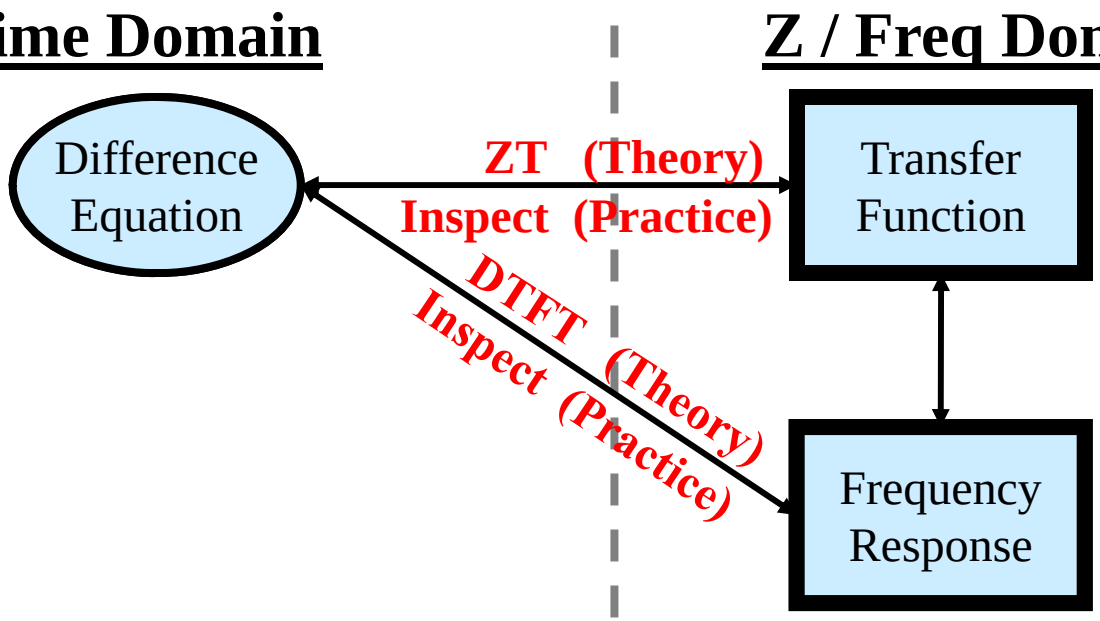
Note Set #27

- D-T Systems Relationships

Discrete-Time System Relationships



Example System Relationships

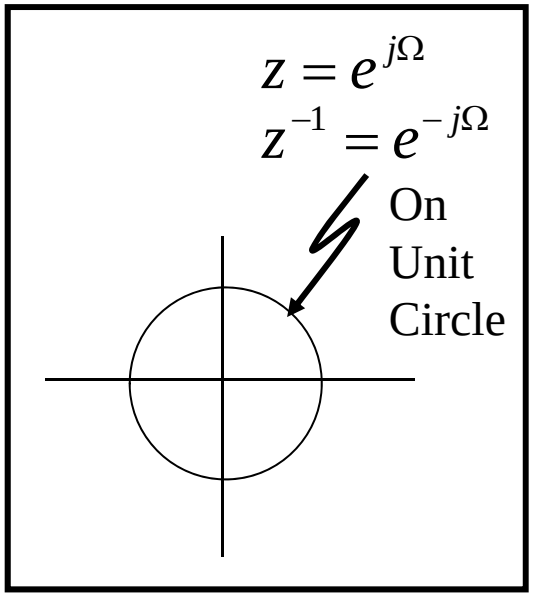


$$y(n) - \alpha y(n-1) = \beta x(n) \quad \Rightarrow \quad H(z) = \frac{\beta}{1 - \alpha z^{-1}}$$

$$y(n) - \alpha y(n-1) = \beta x(n) \quad \Rightarrow \quad H(\Omega) = \frac{\beta}{1 - \alpha e^{-j\Omega}}$$

Example System Relationships (cont.)

Z / Freq Domain



$$H(z) = \frac{\beta}{1 - \alpha z^{-1}}$$

Transfer Function

$z^{-1} = e^{-j\Omega}$

Unit Circle

$$H(\Omega) = \frac{\beta}{1 - \alpha e^{-j\Omega}}$$

$$\Omega \in (-\pi, \pi]$$

Frequency Response

Example System Relationships (cont.)

Plotting Transfer Function

$$H(\Omega) = \frac{\beta}{1-\alpha e^{-j\Omega}} = \frac{\beta}{1-[\alpha\cos(\Omega) - j\alpha\sin(\Omega)]}$$

$$= \frac{\beta}{[1-\alpha\cos(\Omega)] + j\alpha\sin(\Omega)}$$

$$|H(\Omega)| = \frac{\beta}{\sqrt{[1-\alpha\cos(\Omega)]^2 + [\alpha\sin(\Omega)]^2}}$$

$$\angle H(\Omega) = -\tan^{-1}\left[\frac{\alpha\sin(\Omega)}{1-\alpha\cos(\Omega)}\right]$$

Euler's Equation

Group into Real & Imag

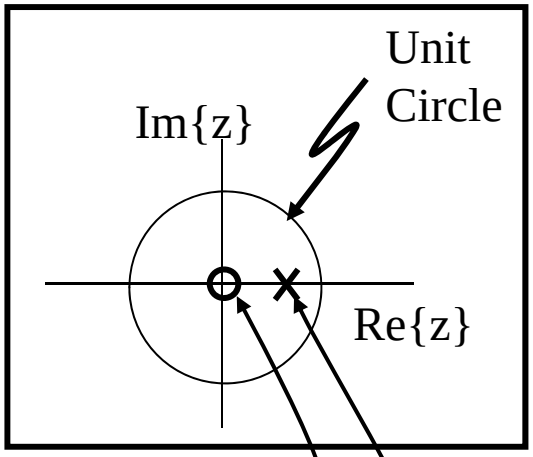
Standard Eqs for Mag. & Angle

Or… Use MATLAB:

```
>> omega = -pi:??:pi;
>> H = freqz(beta,[1 –alpha],omega)
>> plot(omega/pi,abs(H))
>> plot(omega/pi,abs(H))
```

Example System Relationships (cont.)

Z/Freq Domain



Pole/Zero Diagram

Roots

Transfer Function

$$H(z) = \frac{\beta}{1-\alpha z^{-1}} = \frac{\beta z}{z-\alpha}$$

Zero
Num. = 0
When z = 0

Pole
Den. = 0
When z = α

If α < 1: **In**side UC
If α > 1: **Out**side UC

Or… Using MATLAB:
>> zplane(beta,[1 -alpha])

Example System Relationships (cont.)