Binghamton University
State University of New York

EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #14

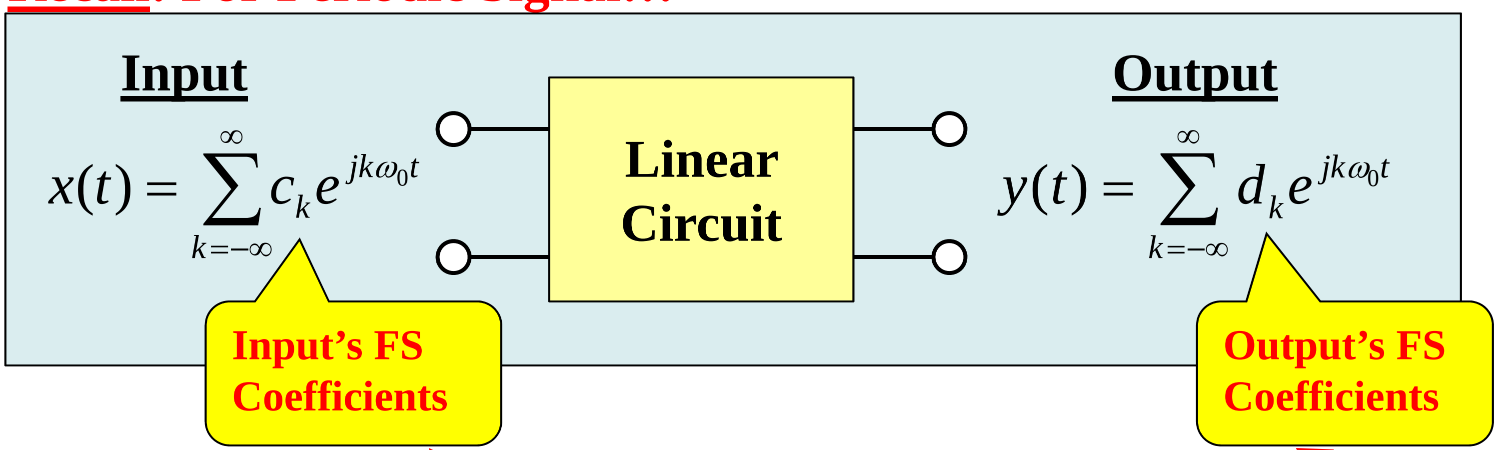
- C-T Signals: Circuits with Non-Periodic Sources

Recall: Convolution Property (The Most Important FT Property!!!)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \quad \leftrightarrow \quad Y(\omega) = X(\omega)H(\omega)$$

Here… we will explore the real-world use of the right side of this result!

Recall: For Periodic Signal…



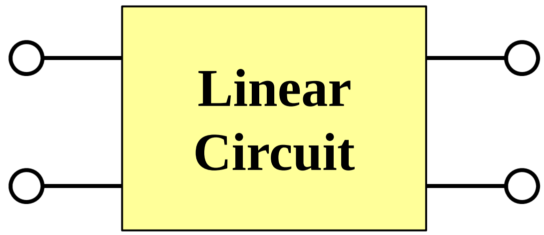
$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \qquad y(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$$

$$d_k = H(k\omega_o)c_k$$

Recall the definition of the frequency response:

Input

$e^{j\omega t}$



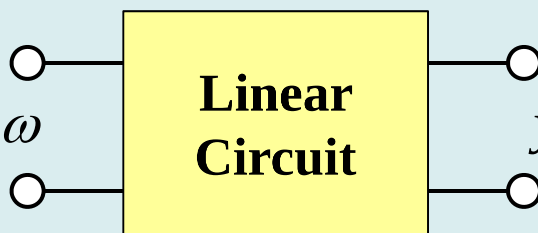
Output

$H(\omega)e^{j\omega t}$

Input $x(t)$ is a linear combo of sinusoids… the output is a linear combo:

Input

$$x(t) = \int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$



Output

$$y(t) = \int_{-\infty}^{\infty} H(\omega)X(\omega)e^{j\omega t}d\omega$$

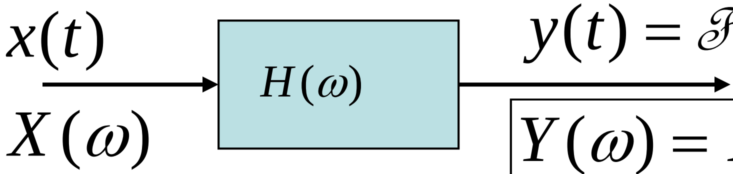
Input's FT

Output's FT

$$Y(\omega) = H(\omega)X(\omega)$$

Unlike for the FS case it is not easy to use these ideas <u>numerically</u> to find the actual y(t)… Rather, we usually use these ideas to help us "visualize" what we need in a circuit design.

So we have as a big picture view:



$$y(t) = \mathscr{F}^{-1}\left\{Y(\omega)\right\}$$

$$Y(\omega) = X(\omega)H(\omega)$$

So…

$$\left|Y(\omega)\right| = \left|X(\omega)\right|\left|H(\omega)\right|$$

$$\angle Y(\omega) = \angle X(\omega) + \angle H(\omega)$$

So…in general we see that the system frequency response re-shapes the input FT's magnitude and phase.

$\Rightarrow$ System can:

-emphasize some frequencies
-de-emphasize other frequencies

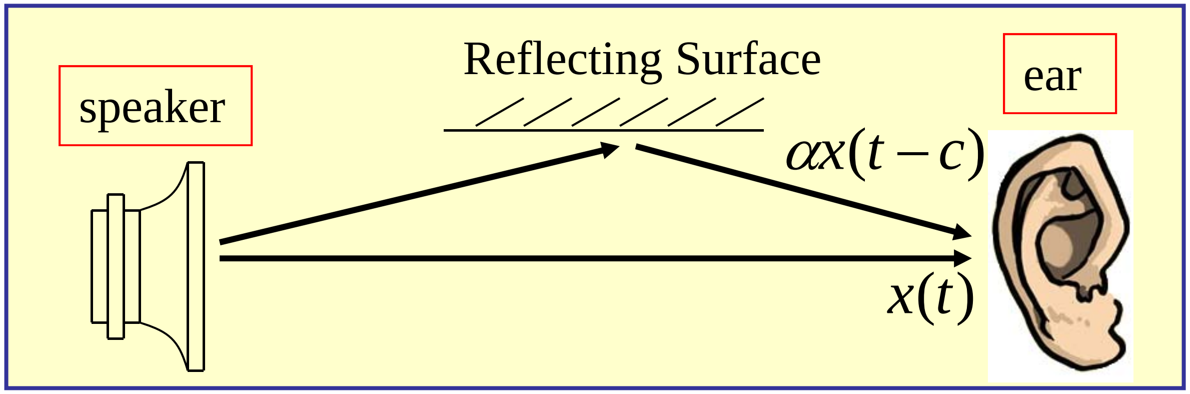
Example Application of Time Shift Property: Room acoustics.

Practical Questions: Why do some rooms sound bad? Why can you fix this by using a "graphic equalizer" to "boost" some frequencies and "cut" others?

Very simple case of a single reflection:

$c > 0$ Delayed signal

$0 < \alpha < 1$ Attenuated signal



So… You hear:

$$y(t) = x(t) + \alpha x(t-c)$$

instead of just $x(t)$

Use linearity and time shift to get the FT at your ear:

$$Y(\omega) = \mathfrak{F}\{x(t) + \alpha x(t-c)\} = \mathfrak{F}\{x(t)\} + \alpha\mathfrak{F}\{x(t-c)\}$$

$$= X(\omega) + \alpha X(\omega)e^{-j\omega c}$$

$$Y(\omega) = X(\omega)\left[1 + \alpha e^{-j\omega c}\right]$$

$H(\omega)$ **of the room!**

This is the FT of what you hear…

It gives an equation that shows how the reflection affects what you hear!!!!

The big picture!

$$|Y(\omega)| = |X(\omega)| \underbrace{|1+\alpha e^{-j\omega c}|}_{\equiv |H(\omega)|}$$

$|H(\omega)|$ changes shape of $|X(\omega)|$

The room changes how much of each frequency you hear…

Let's look closer at |*H*(ω)| to see what it does…

Using Euler's formula gives Rectangular Form

$$|H(\omega)| = |1+\alpha e^{-jc\omega}| = |1+\alpha\cos(c\omega) - j\alpha\sin(c\omega)|$$

$$= \sqrt{(1+\alpha\cos(\omega c))^2 + \alpha^2\sin^2(\omega c)} = \sqrt{1+2\alpha\cos(\omega c) + \underbrace{\alpha^2\cos^2(\omega c)+\alpha^2\sin^2(\omega c)}_{=\alpha^2}}$$

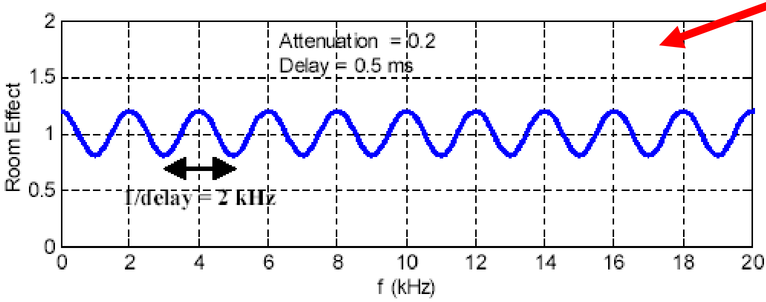
$mag = \sqrt{(\mathrm{Re})^2 + (\mathrm{Im})^2}$

Expand 1st squared term

Use Trig ID

$$|H(\omega)| = \sqrt{(1+\alpha^2) + 2\alpha\cos(\omega c)}$$

$$|Y(\omega)| = |X(\omega)|\sqrt{(1+\alpha^2) + 2\alpha\cos(\omega c)}$$



Effect of the room… what does it look like as a function of frequency?? The cosine term makes it wiggle up and down… and the value of *c* controls how fast it wiggles up and down

Spacing = 1/*c* Hz

"Dip-to-Dip"

"Peak-to-Peak"

c controls spacing between dips/peaks

α controls depth/height of dips/peaks

The next 3 slides explore these effects

What is a typical value for delay *c*???

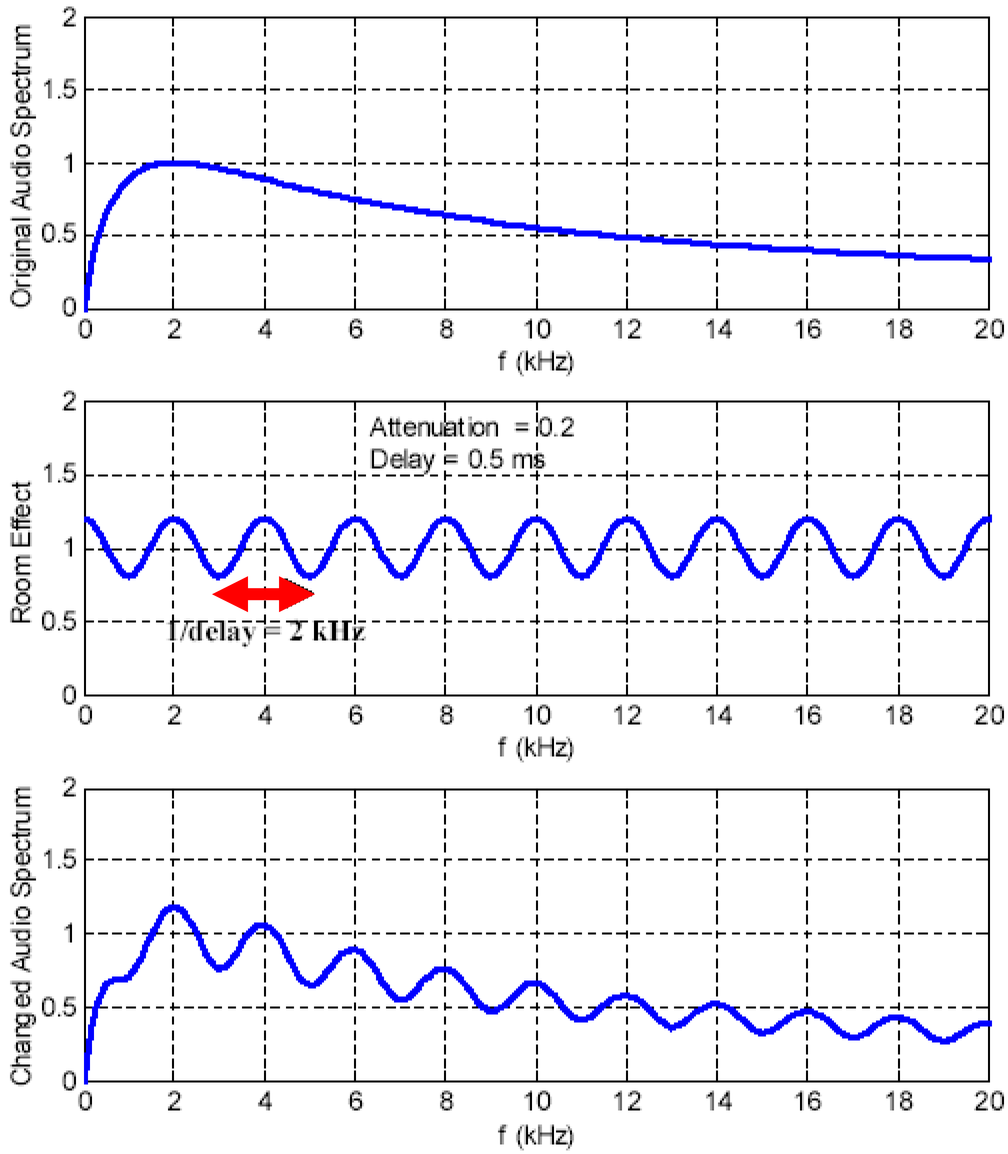
Speed of sound in air ≈ 340 m/s

Typical difference in distance ≈ 0.167m

$$c = \frac{0.167\text{m}}{340\text{m/s}} = 0.5m\text{sec}$$

➔ Spacing = 2 kHz

Attenuation: α = 0.2   Delay: c = 0.5 ms (Spacing = 1/0.5e-3 = 2 kHz)



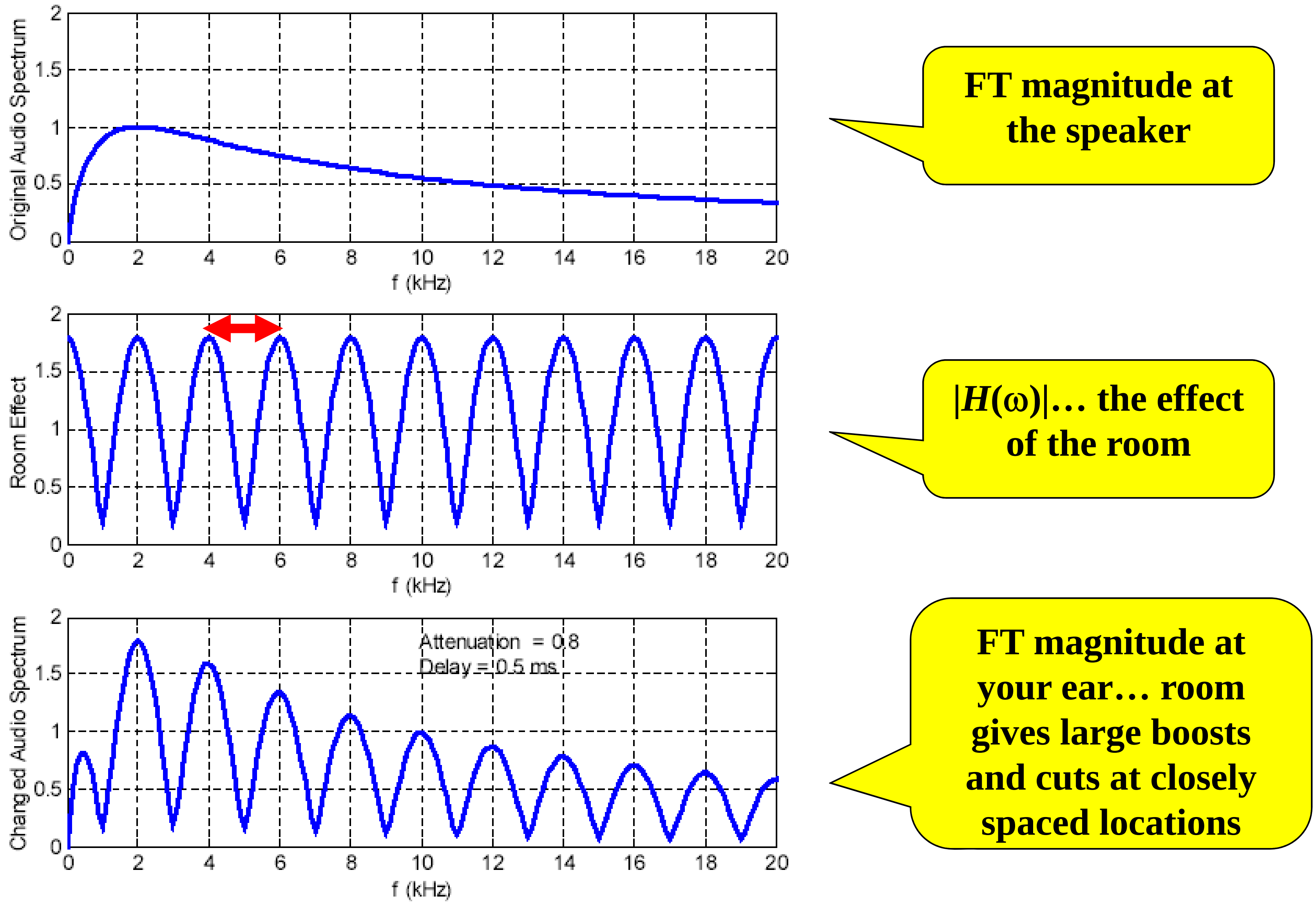
FT magnitude at the speaker (a made-up spectrum… but kind of like audio)

$|H(\omega)|$… the effect of the room

FT magnitude at your ear… room gives slight boosts and cuts at closely spaced locations

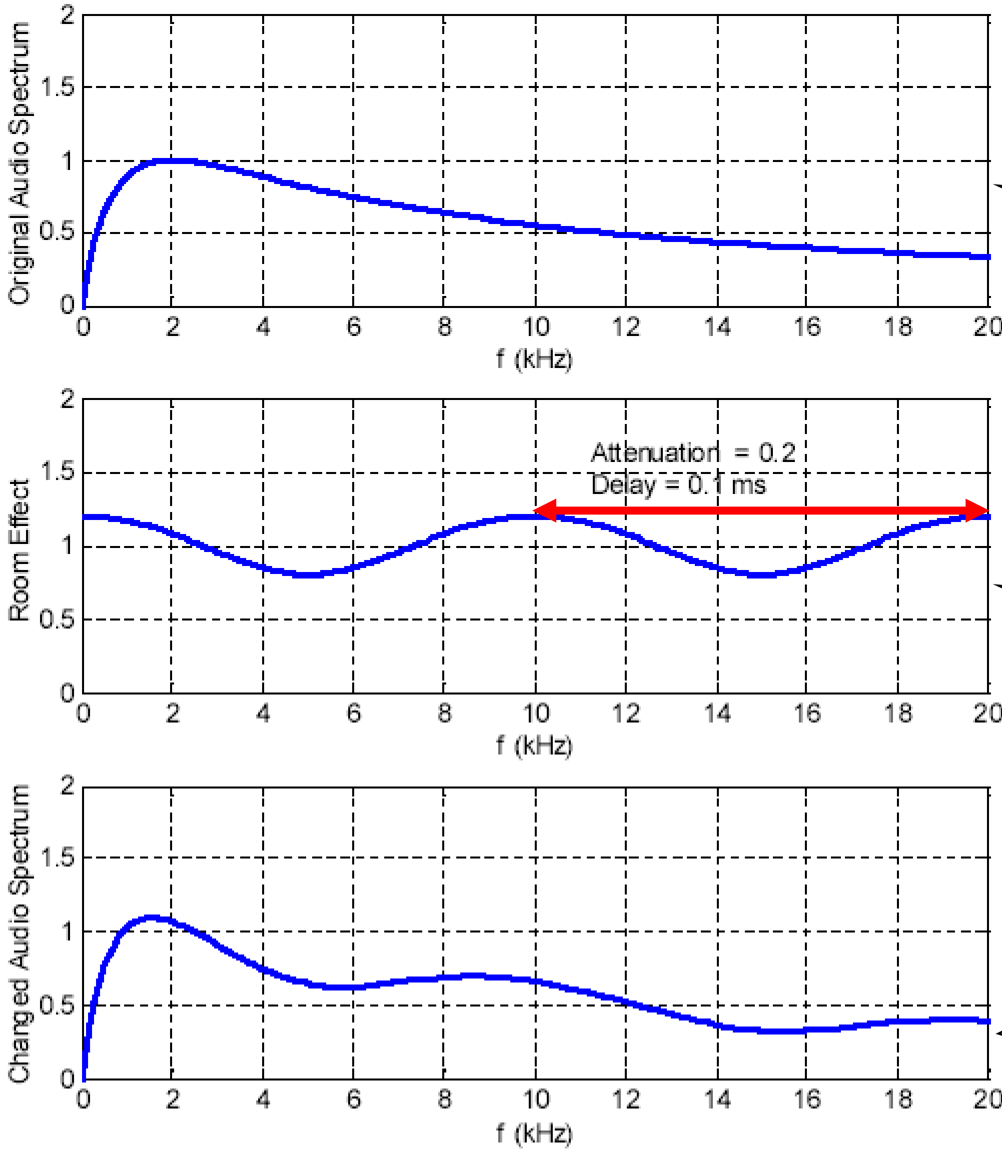
Longer delay causes closer spacing… so more dips/peaks over audio range!

Attenuation: α = 0.8   Delay: c = 0.5 ms (Spacing = 1/0.5e-3 = 2 kHz)



Stronger reflection causes bigger boosts/cuts!!

Attenuation: α = 0.2 Delay: c = 0.1 ms (Spacing = 1/0.1e-3 = 10 kHz)



FT magnitude at the speaker

$|H(\omega)|$… the effect of the room

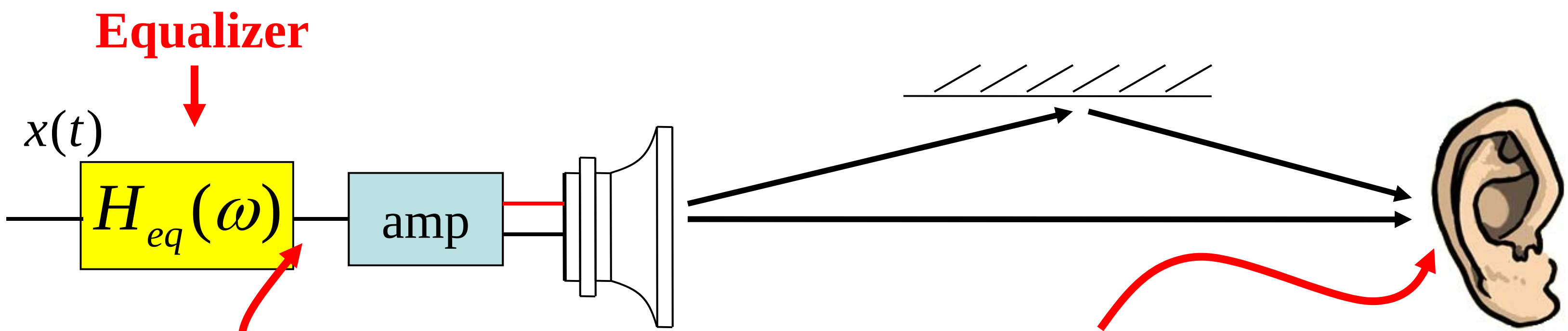
FT magnitude at your ear… room gives small boosts and cuts at widely spaced locations

Shorter delay causes wider spacing… so fewer dips/peaks over audio range!

Matlab Code to create the previous plots

```
function room_delay(atten,delay)

f=0:100:20000;  %     Freq range: 0 Hz to 20 kHz
w=2*pi*f;       %     convert to rad/sec

H=abs(1 + atten*exp(-j*w*delay));  % Compute Room Effect

% Make up a fictitious audio spectrum
X=50000*w./((2*pi*2000+w)).^2;

% Now do plots
subplot(3,1,1)   % splits figure into 3 subplots, pick 1st one
plot(f/1000,X)   % note f converted into k Hz
xlabel('f (kHz)')
ylabel('Original Audio Spectrum')
axis([0 20 0 2])  % set axis ranges as desired
grid   % put grid lines on

subplot(3,1,2)   % splits figure into 3 subplots, pick 2nd one
plot(f/1000,H)
xlabel('f (kHz)')
ylabel('Room Effect')
axis([0 20 0 2])
grid

subplot(3,1,3)   % splits figure into 3 subplots, pick 3rd one
plot(f/1000,H.*X)
xlabel('f (kHz)')
ylabel('Changed Audio Spectrum')
axis([0 20 0 2])
grid
```

Room boosts and cuts various frequencies… So, fix it using an "equalizer"



$$X_2(\omega) = X(\omega)H_{eq}(\omega)$$

$$Y(\omega) = H_{room}(\omega)X_2(\omega)$$
$$= H_{room}(\omega)H_{eq}(\omega)X(\omega)$$

Then: $$|Y(\omega)| = |X(\omega)||H_{eq}(\omega)||1+\alpha e^{-j\omega c}|$$

Recall: Peaks and dips

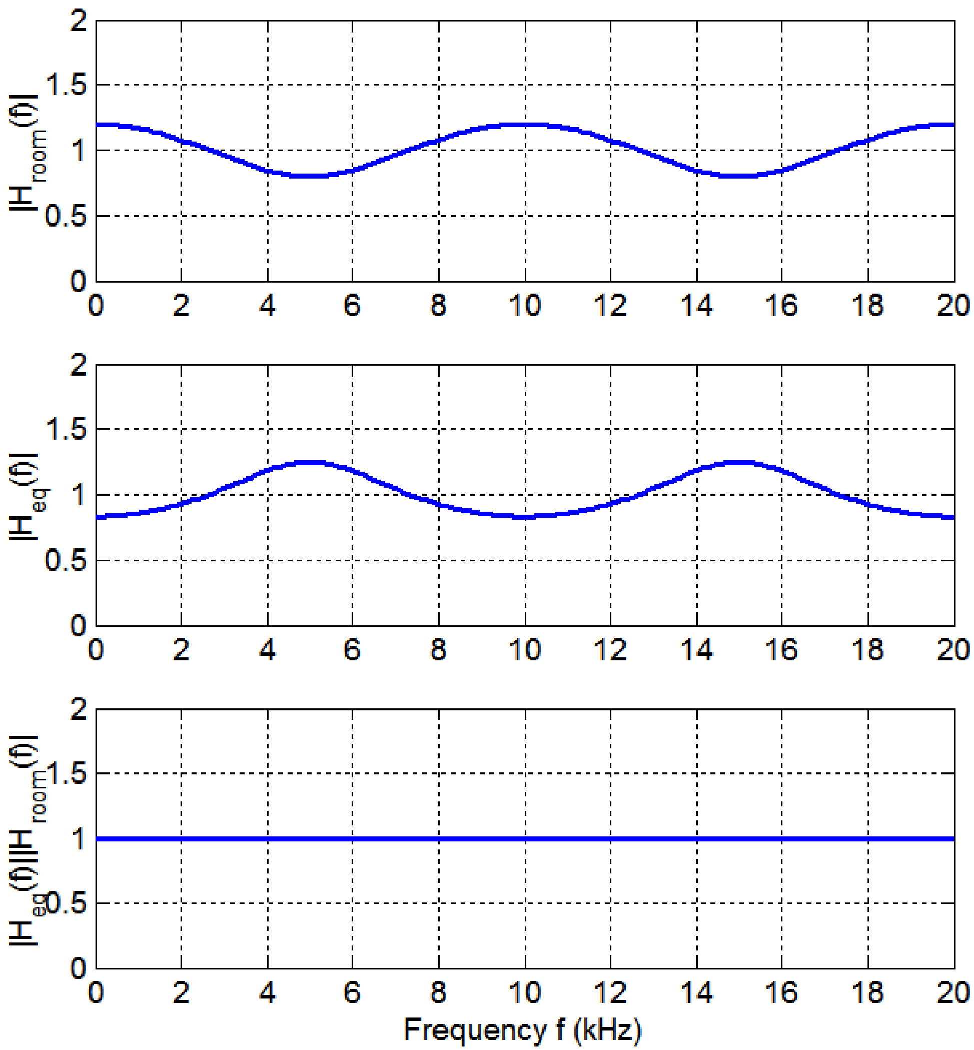
Want this whole thing to be = 1 so $|Y(\omega)| = |X(\omega)|$

Equalizer's $|H_{eq}(\omega)|$ should peak at frequencies where the room's $|H_{room}(\omega)|$ dips and vice versa

Room

Equalizer

Room & Equalizer



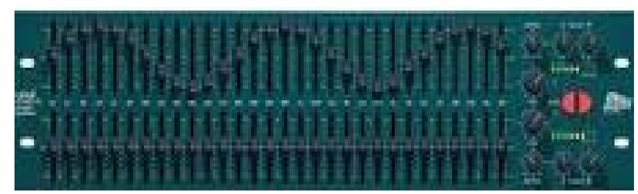


Image from musiciansfriend.com