Binghamton University
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EECE 301
Signals & Systems

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Note Set #5

- Basic Properties of Systems

1.5 Basic System Properties

There are some fundamental properties that many (but not all!) systems share regardless if they are C-T or D-T and regardless if they are electrical, mechanical, etc.

An understanding of these fundamental properties allows an engineer to develop tools that can be widely applied… rather than attacking each seemingly different problem anew!!

The three main fundamental properties we will study are:

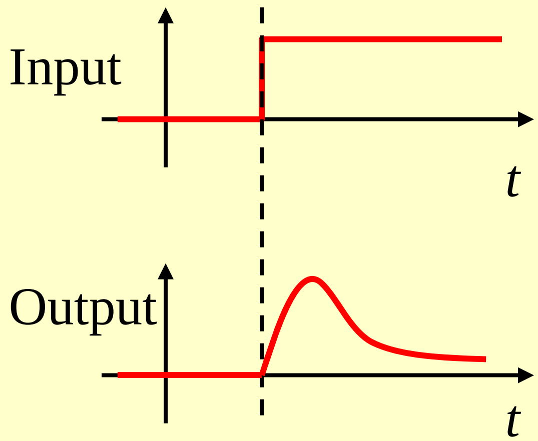
1. Causality
2. Linearity
3. Time-Invariance

<u>Causality</u>: A causal (or non-anticipatory) system's output at a time t_1 does not depend on values of the input $x(t)$ for $t > t_1$

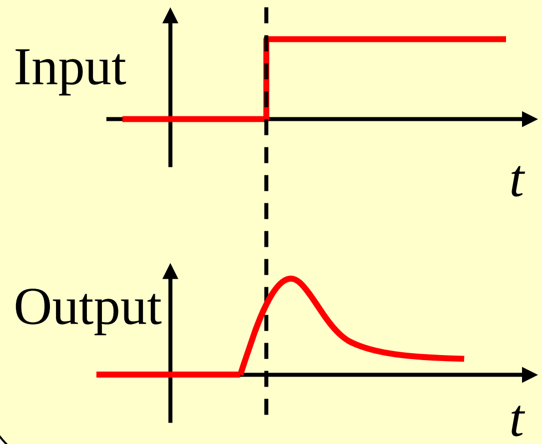
The "future input" cannot impact the "now output"

$\Rightarrow$ A Causal system (with zero initial conditions) cannot have a non-zero output until a non-zero input is applied.

Causal System



Non-Causal System



Most systems in nature are causal

But… we need to understand non-causal systems because theory shows that the "best" systems are non-causal! So we need to find causal systems that are as close to the best non-causal systems!!!

Linearity: A system is linear if superposition holds:

$$x_1(t) \longrightarrow \boxed{\textbf{Linear System}} \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow \boxed{\textbf{Linear System}} \longrightarrow y_2(t)$$

$$x(t) = a_1\, x_1(t) + a_2\, x_2(t) \longrightarrow \boxed{\textbf{Linear System}} \longrightarrow y(t) = a_1\, y_1(t) + a_2\, y_2(t)$$

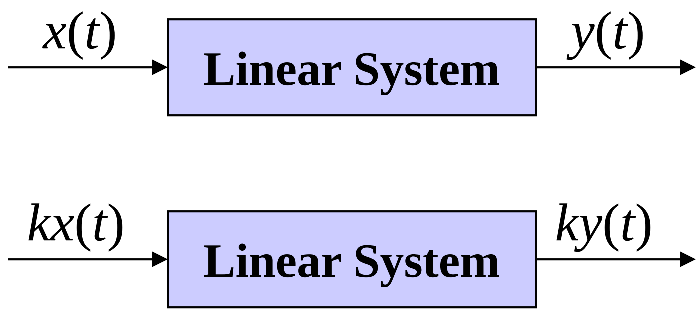
$$x_1(t) \longrightarrow \boxed{\textbf{Non-Linear}} \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow \boxed{\textbf{Non-Linear}} \longrightarrow y_2(t)$$

$$x(t) = a_1\, x_1(t) + a_2\, x_2(t) \longrightarrow \boxed{\textbf{Non-Linear}} \longrightarrow y(t) \neq a_1\, y_1(t) + a_2\, y_2(t)$$

If a system is linear: "a scaled input gives a scaled output"

Same scaling factor



When superposition holds, it makes our life easier!
We then can <u>decompose</u> <u>complicated signals</u> into a <u>sum</u> of <u>simpler signals</u>… and then find out how each of these simple signals goes through the system!!

This is exactly what the so-called Frequency Domain Methods of later chapters do!!!

<u>Systems with only *R*, *L*, and *C* are linear systems</u>!
Systems with electronics (diodes, transistors, op-amps, etc.) <u>may</u> be non-linear, but they <u>could</u> be linear… at least for inputs that do not exceed a certain range of inputs.

Time-Invariance

Physical View: The system itself does not change with time

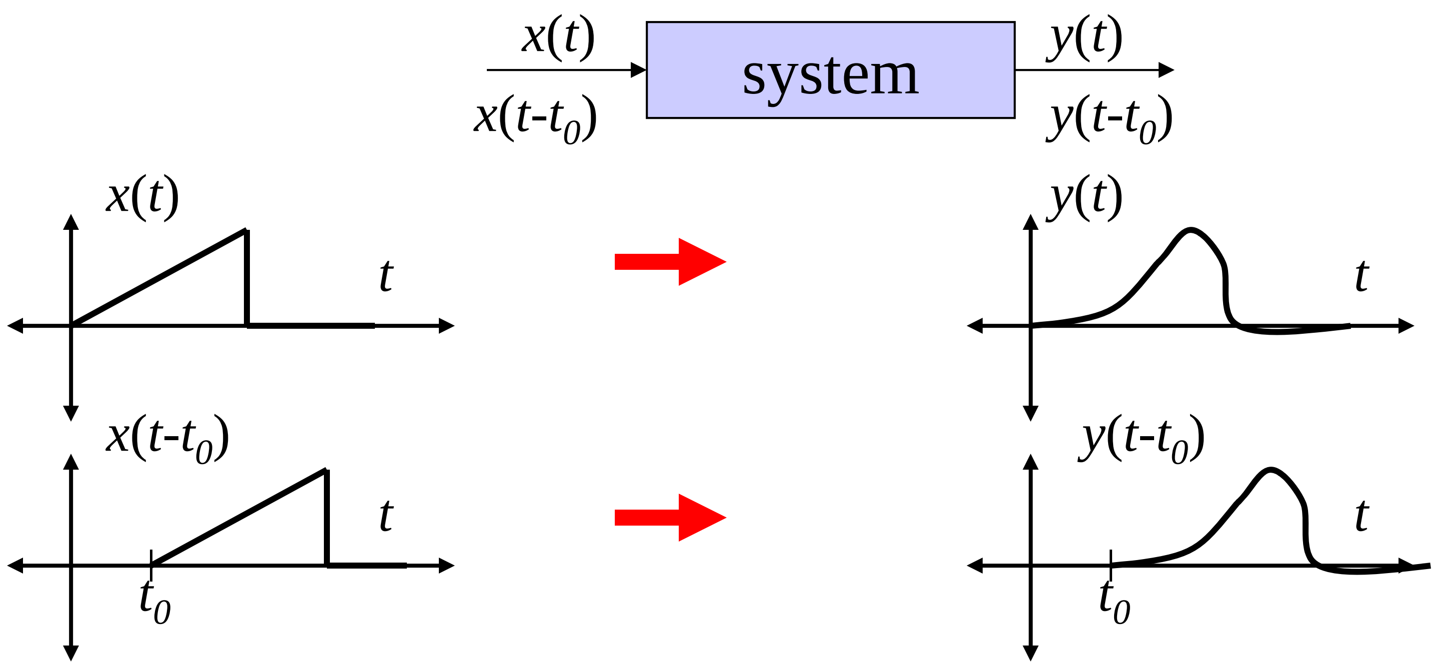
Ex. A circuit with fixed *R*,*L*,*C* is time invariant.

Actually, *R*,*L*,*C* values change slightly over time due to temperature & aging effects.

A circuit with, say, a variable *R* is time variant

(assuming that someone or something is changing the *R* value)

Technical View: A system is time invariant (TI) if:



Systems described by Linear, Constant-Coefficient Differential Equations are Continuous-Time, Linear Time-Invariant (LTI) Systems

Differential equations like this are LTI

$$a_N \frac{d^N y(t)}{dt^N} + a_{N-1} \frac{d^{N-1} y(t)}{dt^{N-1}} + \ldots + a_0 y(t) = b_M \frac{d^M x(t)}{dt^M} + \ldots + b_1 \frac{dx(t)}{dt} + b_0 x(t)$$

- coefficients (a's & b's) are constants $\Rightarrow$ TI

- No nonlinear terms $\Rightarrow$ Linear

Examples Of Nonlinear terms

$$x^n(t), \quad \left[\frac{d^k x(t)}{dt^k}\right]\left[\frac{d^p x(t)}{dtp}\right], \quad y^n(t), \quad \left[\frac{d^k y(t)}{dt^k}\right]\left[\frac{d^p y(t)}{dt^p}\right], \quad etc.$$

Systems described by Linear, Constant-Coefficient Difference Equations are Discrete-Time, Linear Time-Invariant (LTI) Systems

Difference equations like this are LTI

$$a_0 y[n] + a_1 y[n-1] + \ldots + a_N y[n-N] = b_0 x[n] + b_1 x[n-1] + \ldots + b_M x[n-M]$$

- coefficients (a's & b's) are constants $\Rightarrow$ TI

- No nonlinear terms $\Rightarrow$ Linear

Examples Of Nonlinear terms

$$x^n[n], \quad x[n-p]x[n-m] \quad , \quad y^n[n], \quad y[n-p]y[n-m] \quad , \quad etc.$$

Summary

Our focus will be on systems (DT & CT) that are:

1. Linear
2. Time-Invariant

Such systems will be called LTI systems.

We will mostly focus on causal systems since those are the ones that can occur in the real world… however, it will sometimes be necessary to deal with non-causal systems in theory.