Binghamton University
State University of New York

EECE 301
Signals & Systems
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Note Set #4

- System Modeling and Some Examples

System Model View

Physical View:

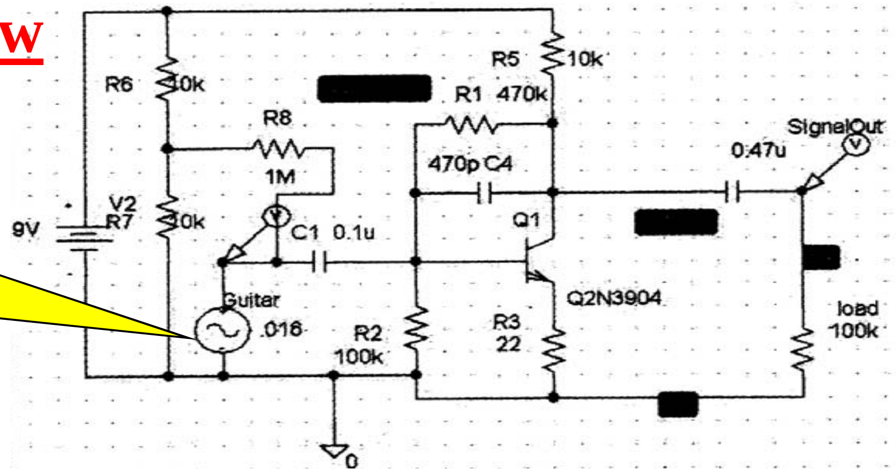
Apply input signal here as a voltage (or a current)

Get output signal here as a voltage (or a current)

Image from **llg.cubic.org/tools/sonyrm/**

Schematic View

Apply guitar signal here as a voltage



Output signal is the voltage across here

From *Pedal Power Column* by Robert Keeley, in Musician's Hotline Magazine

System View

Math Function for Input

$x(t)$ → Math Model of System → $y(t)$

Math Function for Output

Math Model… <u>quantitatively</u> relates input signal's math to the output signal's math… Allows us to understand and <u>predict</u> how the system <u>will</u> work!

Math Models for Systems

- Many physical systems are modeled w/ **Differential Eqs**
  - Because physics shows that electrical (& mechanical!) components often have "V-I Rules" that depend on derivatives

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_1 \frac{dx(t)}{dt} + b_0 x(t)$$

Given: Input $x(t)$
Find: Ouput $y(t)$

This is what it means to "solve" a differential equation!!

- However, engineers use **Other Math Models** to help solve and analyze differential eqs
  - The concept of **"Frequency Response"** and the related concept of **"Transfer Function"** are the most widely used such math models
    - > **"Fourier Transform"** is the math tool underlying Frequency Response
  - Another helpful math model is called **"Convolution"**

System Modeling

To do engineering design, we must be able to accurately predict the quantitative behavior of a circuit or other system.

This requires math models:

Circuits

Device Rules

R: $v(t)=Ri(t)$

L: $v(t)=L(di(t)/di)$

C: $dv(t)/dt=1/Ci(t)$

Circuit Rules

-KVL

-KCL

-Voltage Divider

-etc.

Differential Equation

Mechanical

Device Rules

Mass: $M(d^2p(t)/dt^2)$

Spring: $k_x p(t)$

Damping: $k_d(dp(t)/dt)$

System Rules

-Sum of forces

-etc.

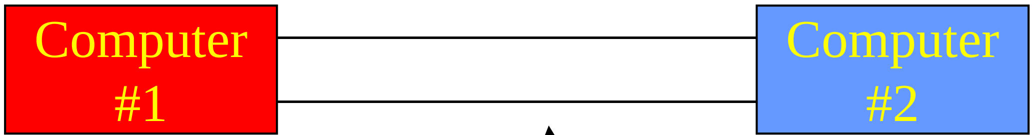
Differential Equation

Similar ideas hold for hydraulic, chemical, etc. systems…

“differential equations rule the world”

Simple Circuit Example:

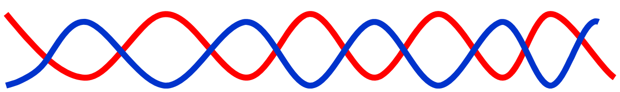
Sending info over a wire cable between two computers



Two conductors separated by an insulator
⇒ capacitance

Two practical examples of the cable

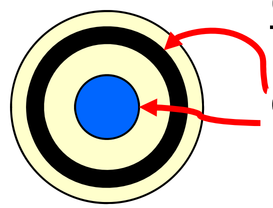
"Twisted Pair" of Insulated Wires



Typical values: 100 Ω/km

50 nF/km

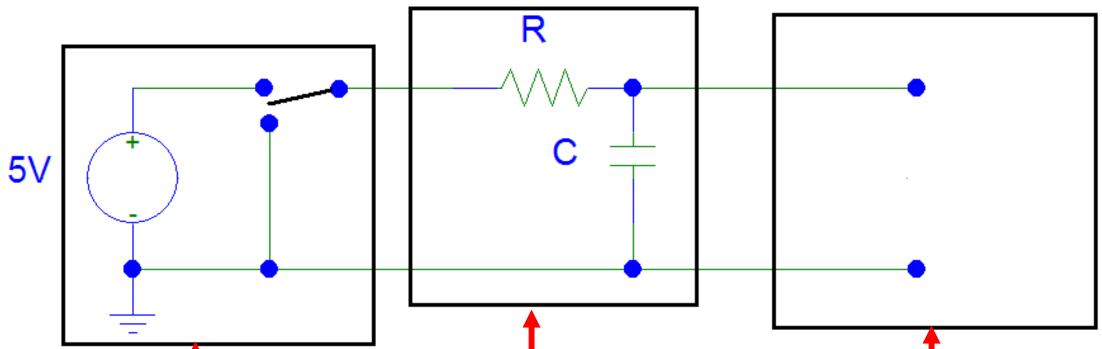
coaxial cable



conductors separated by insulator

Recall: resistance increases with wire length

Simple Model:



Driver's Thevenin Equivalent Circuit (Computer #1)

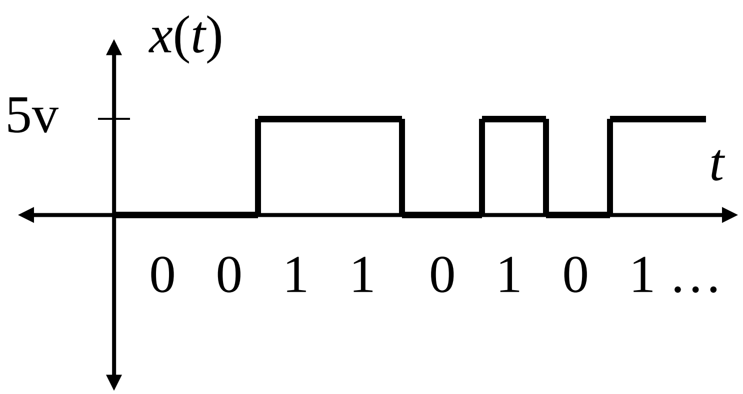
Cable Model

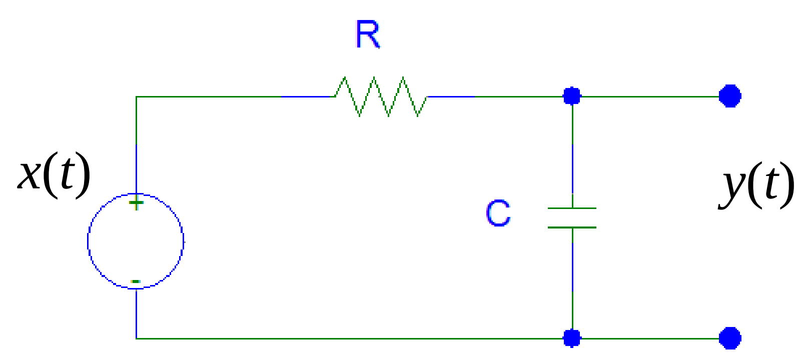
Receiver's Thevenin Equivalent Circuit (Computer #2)

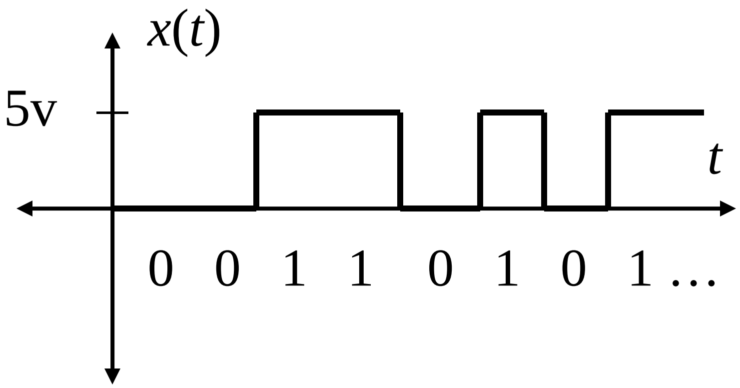
Zero Output Resistance (Ideal)

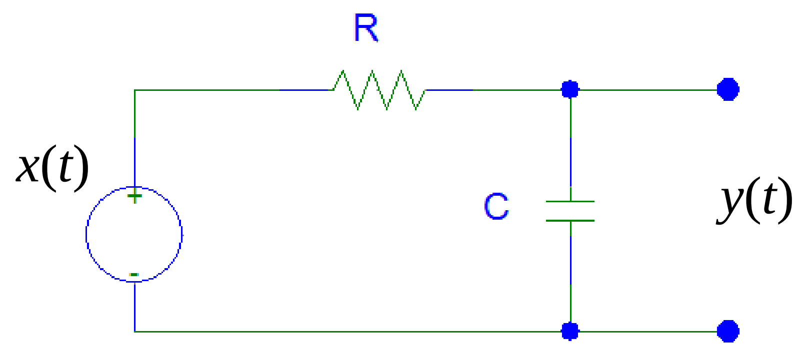
Infinite Input Resistance (Ideal)

Effective Operation:









Use Loop Equation & Device Rules:

$$x(t) = v_R(t) + y(t)$$

$$v_R(t) = Ri(t)$$

$$i(t) = C\frac{dy(t)}{dt}$$

$$\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

This is the Differential Equation to be "Solved":

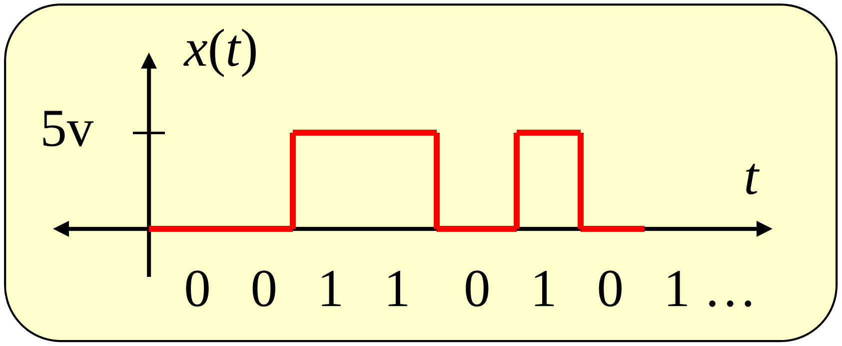
Given: Input $x(t)$ Find: Solution $y(t)$

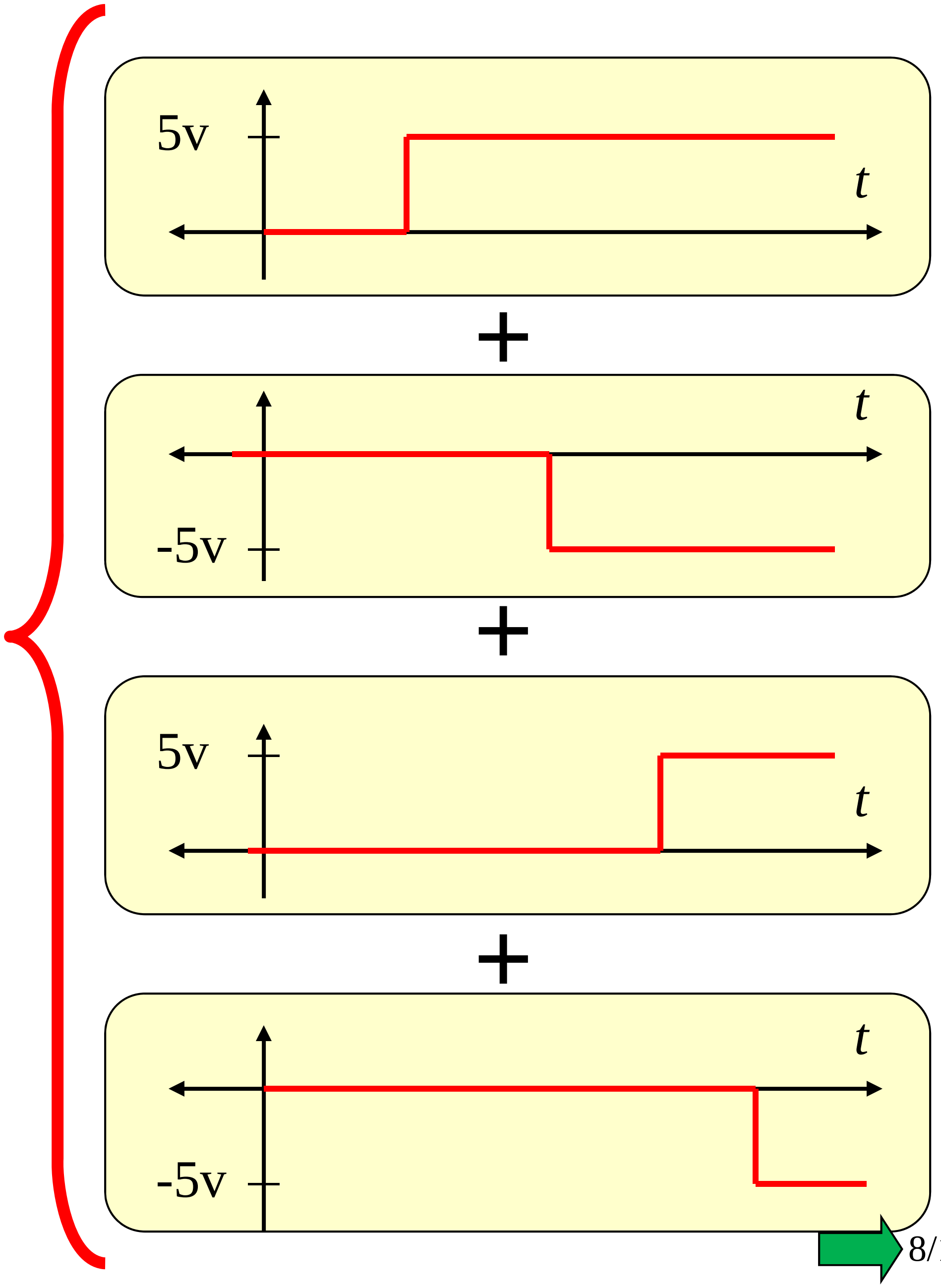
Recall: A "Solution" of the D.E. means… The function that when put into the left side causes it to reduce to the right side

Differential Equation & System … the solution is the output

Now… because this is a **linear** system (it only has *R*, *L*, *C* components!) we can analyze it by **superposition**.

Decompose the input…

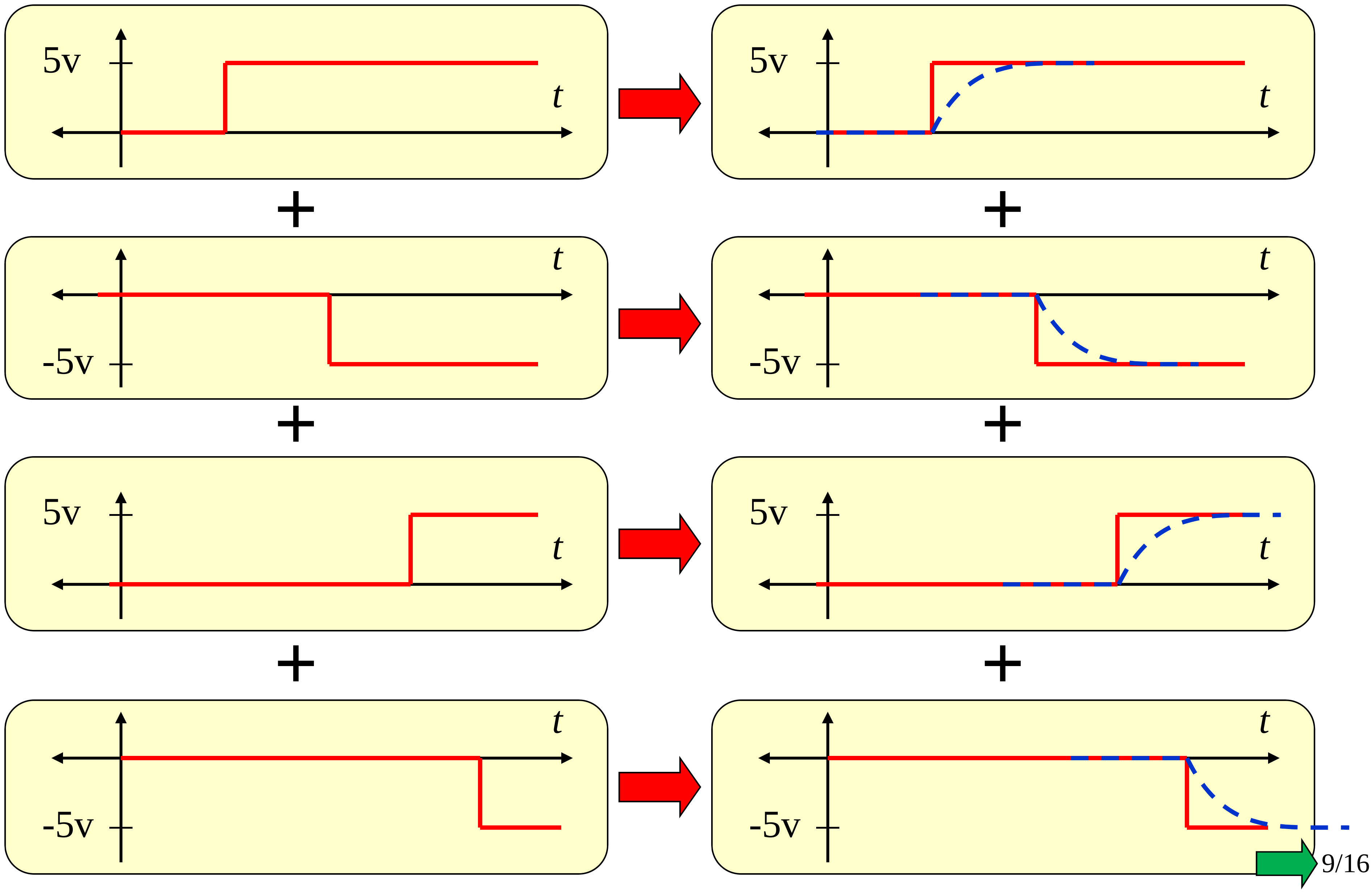




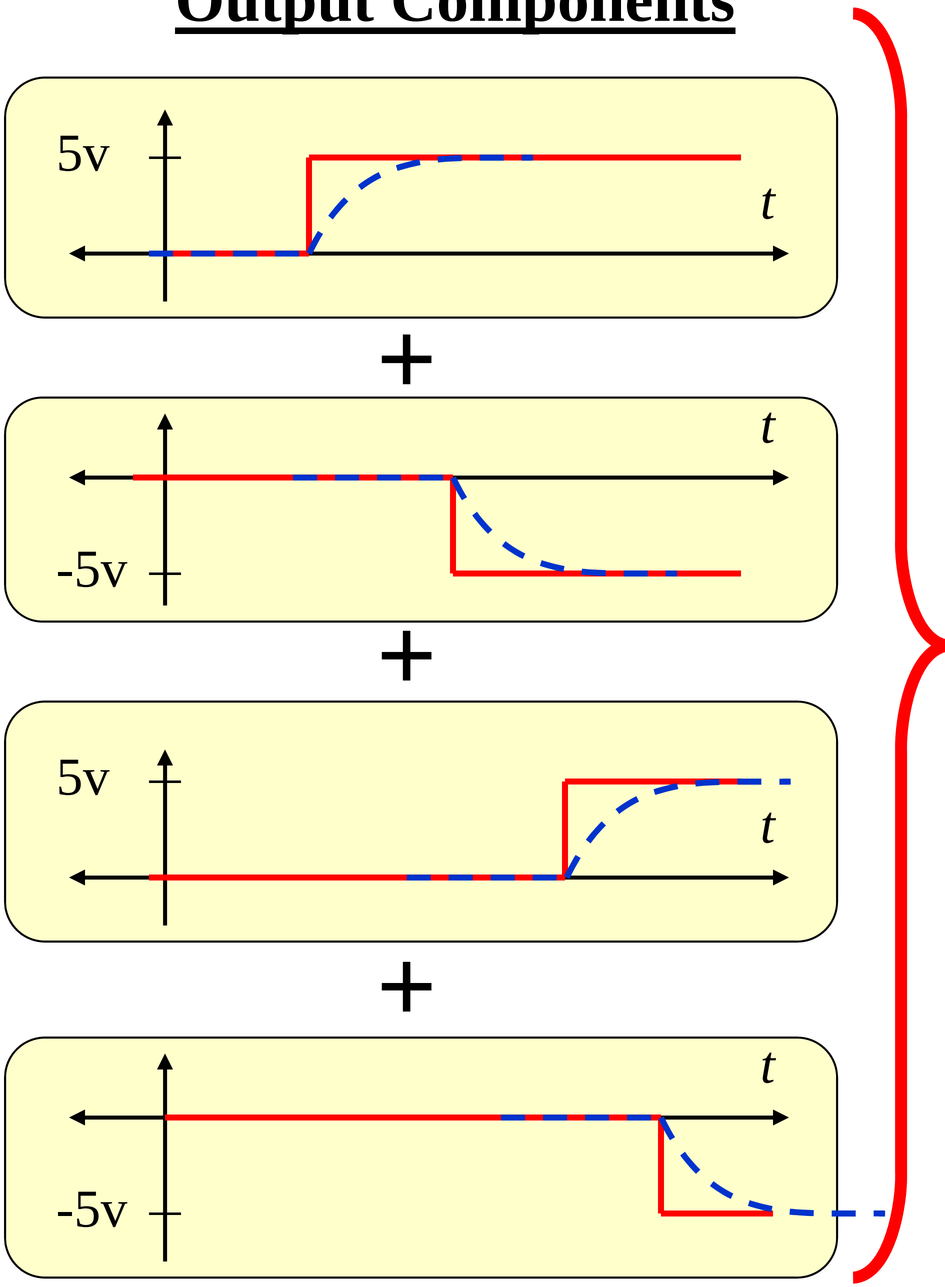
Input Components

Output Components (Blue)

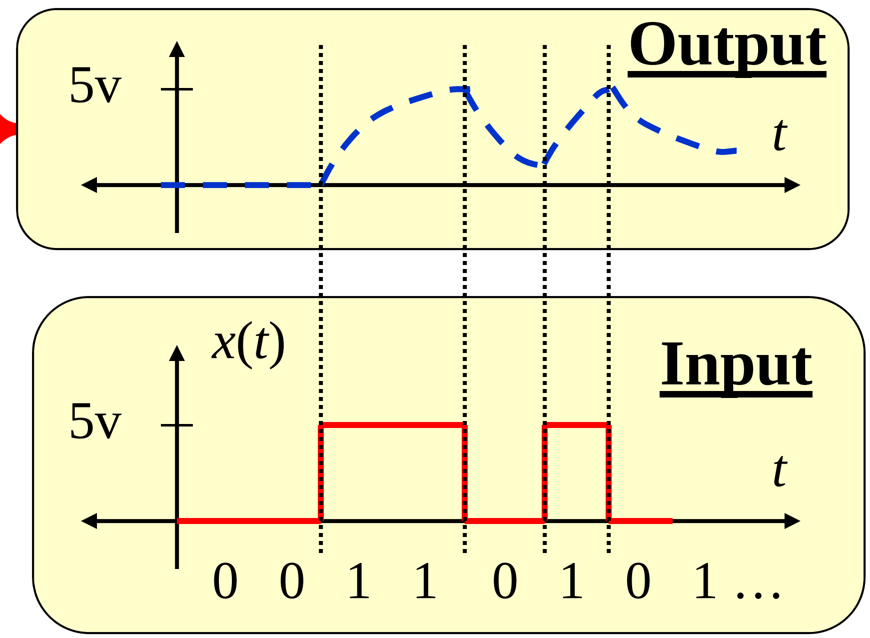
Standard Exponential Response
Learned in “Circuits”:



Output Components

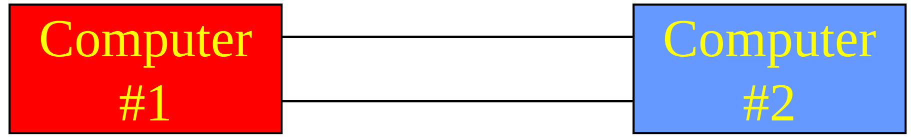


Output is a "smoothed" version of the input… it is harder to distinguish "ones" and "zeros"… it will be even harder if there is noise added onto the signal!

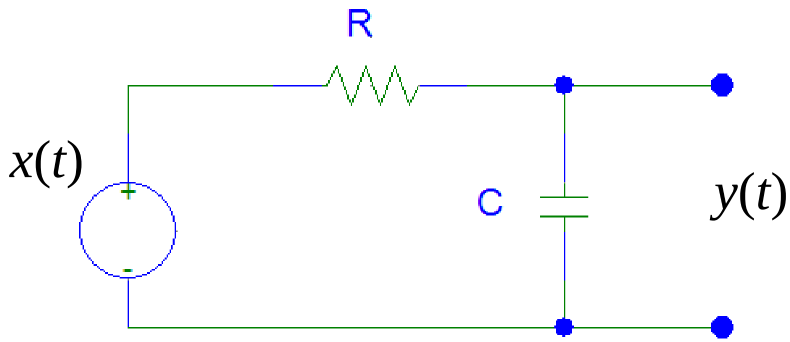


Progression of Ideas an Engineer Might Use for this Problem

Physical System:



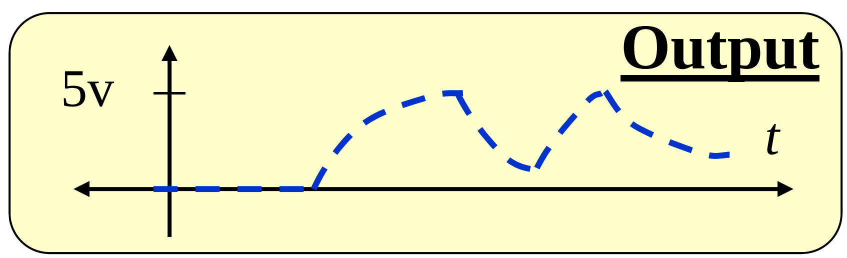
Schematic System:



Mathematical System:

$$\frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{RC} x(t)$$

Mathematical Solution:



<u>Big picture for CT Systems:</u>

Nature is filled with "Derivative Rules"

- **Capacitor and Inductor i-v Relationships**
- **Force, Mass and Acceleration Relationships**
- **Etc.**

Thus C-T Systems are mathematically modeled by <u>Differential Equations</u>

⇒There are a lot of practical C-T systems that can be <u>modeled</u> by differential equations.

In particular, we will be interested in…
Linear, Constant-Coefficient, Ordinary Diff Eqs!

D-T System Example

Recall: We are mostly interested in D-T systems that arise in computer processing of signals collected by sensors.

We illustrate with a simple automotive example: A sensor provides a measure of the "instantaneous MPG" for a car. Suppose the sensor gives this every 10 seconds. We want to keep track of and display the average MPG since "time zero".

Let $x[n]$, $n = 1, 2, 3, \ldots$ be a sequence of MPG measurements

Input

D-T signal because you are not continuously measuring!

Let $y[n]$ be the average MPG after the n^{th} measurement.

Output

Now, one way to do this is to store ALL the measurements and each time you get a new one just average them…

$$y[n] = \frac{1}{n+1}\left(x[0] + x[1] + \cdots + x[n]\right)$$

But… how much memory should we implement? Who knows how long this will run???

So we need a better way. Write $y[n]$ in terms of $y[n-1]$:

$$y[n] = \frac{1}{n+1}\left(n \underbrace{\left[\frac{1}{n}\left(x[0] + x[1] + \cdots + x[n-1]\right)\right]}_{= y[n-1]} + x[n] \right)$$

$$\Rightarrow \quad y[n] = \frac{n}{n+1} y[n-1] + \frac{1}{n+1} x[n]$$

This is a math model for this DT system

This kind of math model is called a "Difference Equation"

This system can easily be computed in software…

$$y[n] = \frac{n}{n+1} y[n-1] + \frac{1}{n+1} x[n]$$

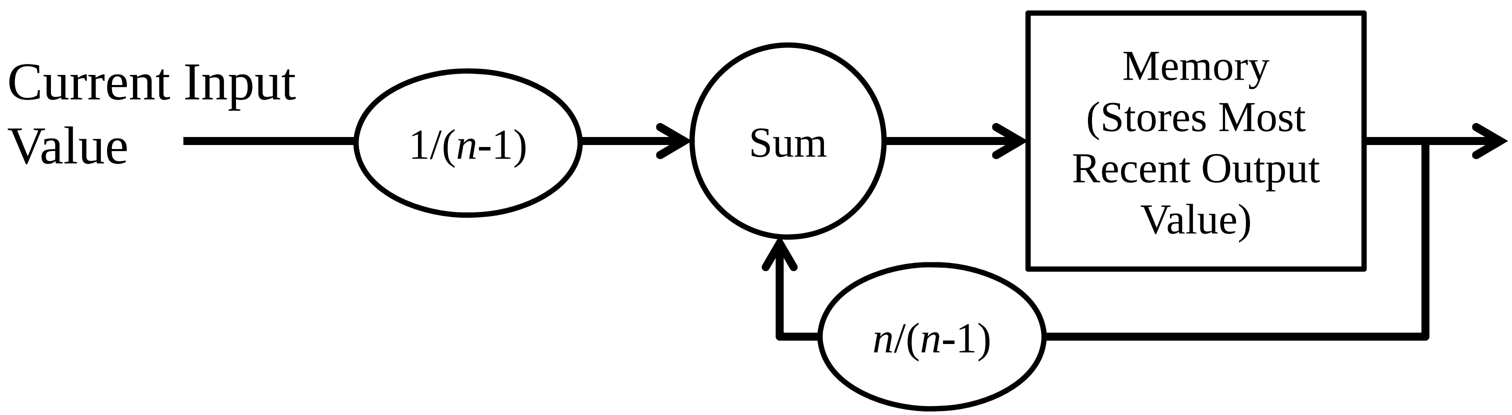
Initial Condition

n	***x*[*n*]**	***y*[*n*]**
-1	-	0
0	35	y[0]=(0/1)y[-1] +(1/1)x[0] = 35
1	39	y[1]=(1/2)y[0] +(1/2)x[0] = (35+39)/2
2	43	y[2]=(2/3)y[1] +(1/3)x[1]
3	36	Etc.
Etc.	Etc.	

```
y[0] = 0
for n = 1 to ???
    y(n) = (n/(n+1))*y(n-1) + (1/(n+1))*x(n)
end
```

All Components are "Clocked"

This system can also be computed in hardware…



BIG PICTURE

- **Physical (nature!) systems are C-T systems modeled by <u>differential equations</u>… e.g., RLC Circuits, Electric Motors, etc.**

- **D-T systems are modeled by <u>difference equations</u>… these are generally implemented using computer HW/SW**

- **Both C-T & D-T systems (at least a large subset) have:**
  - **Zero-Input part of response (due to Initial Conditions)**
    - **e.g., Homogeneous solution of CT Diff Eq**
  - **Zero-State part of Response (due to Input)**

Our Focus will be <u>mostly</u> on the Zero-State Response