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EEO 401
Digital Signal Processing

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Note Set #30

- Linear Phase FIR Design – Frequency Sampling Method
- Reading: Sect. 10.2.3 of Proakis & Manolakis

Basic Idea

Suppose you have a desired frequency response for which you'd like to design a realizable FIR filter: $H_d^{f}(\omega)$

Note: Changing to 0 to 2π has no effect!

The corresponding impulse response is: $h_d[n] = \dfrac{1}{2\pi}\int_0^{2\pi} H_d^{f}(\omega)e^{j\omega n}d\omega$

Of course, doing this IDTFT does not get us an FIR filter!

So… recalling that the DFT

- is a sort-of stand-in for the DTFT
- And it acts on a finite-duration signal

… we wonder if we could use the IDFT on frequency-domain samples of $H_d^{f}(\omega)$

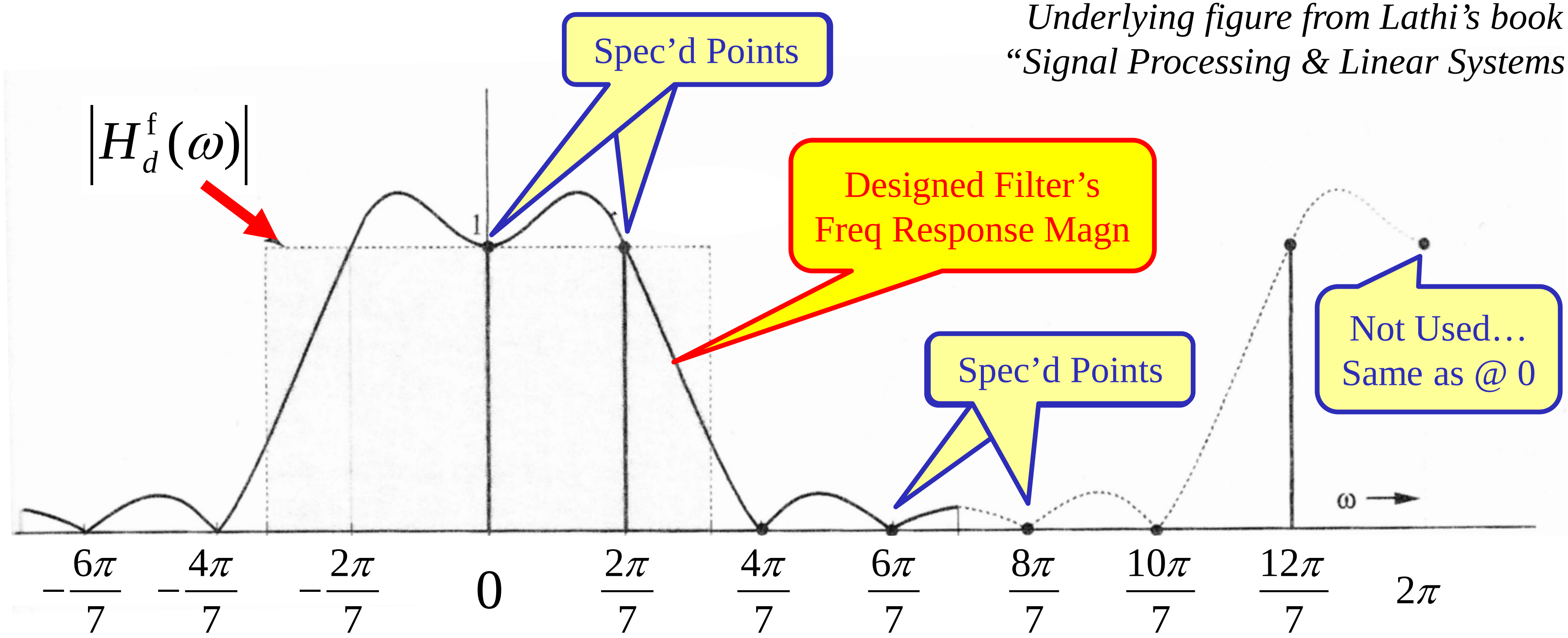
Define: $H[k] = H_d^{f}\left(k\dfrac{2\pi}{M}\right), \quad k = 0,1,2,\ldots,M-1$

Designed FIR Impulse Response

Then an IDFT gives : $h[n] = \dfrac{1}{M}\sum_{k=0}^{M-1} H[k]e^{j\frac{2\pi kn}{M}}, \quad n = 0,1,2,\ldots,M-1$

Then a DTFT of that gives the designed FR: $H^{f}(\omega) = \sum_{k=0}^{M-1} h[n]e^{j\omega n}, \quad -\pi \le \omega \le \pi$

Underlying figure from Lathi's book "Signal Processing & Linear Systems"



Increasing the number of "Spec'd Points" improves the design!

Ensuring Linear Phase Design

The designer specs the magnitude and then a linear phase is imparted:

$$\left|H[k]\right| = \mathit{specified}, \quad k = 0,1,2,\ldots,M-1$$

$$h[n] = \frac{1}{M}\sum_{k=0}^{M-1}\left[\left|H[k]\right| e^{-jk\frac{2\pi}{M}(M-1)/2}\right] e^{j\frac{2\pi kn}{M}}, \quad n = 0,1,2,\ldots,M-1$$

Expanded Idea – Using Window

The idea here is to use a finer frequency sampling grid than is set by the desired filter length.

Basic Form: M = FIR Length

$$|H[k]| = \left| H_d^{\mathrm{f}}\left(k\frac{2\pi}{M}\right)\right|, \quad k = 0,1,2,\ldots,M-1$$

$$h[n] = \frac{1}{M}\sum_{k=0}^{M-1}\left[|H[k]|e^{-jk\frac{2\pi}{M}(M-1)/2}\right]e^{j\frac{2\pi kn}{M}}, \quad n = 0,1,2,\ldots,M-1$$

Extended Form: N >> M = FIR Length

$$|H[k]| = \left| H_d^{\mathrm{f}}\left(k\frac{2\pi}{N}\right)\right|, \quad k = 0,1,2,\ldots,N-1$$

Use Finer Frequency Grid

$$\tilde{h}[n] = \frac{1}{N}\sum_{k=0}^{N-1}\left[|H[k]|e^{-jk\frac{2\pi}{N}(M-1)/2}\right]e^{j\frac{2\pi kn}{N}}, \quad n = 0,1,2,\ldots,N-1$$

$$h[n] = \tilde{h}[n]w_M[n], \quad n = 0,1,2,\ldots,M-1$$

$w_M[n]$ is length M window

MATLAB fir2

% FIR2 FIR arbitrary shape filter design using the frequency sampling method.
% B = FIR2(N,F,A) designs an Nth **order** linear phase FIR digital filter
% with the frequency response specified by vectors F and A and returns
% the filter coefficients in length N+1 vector B.
%
% The vectors F and A specify the frequency and magnitude breakpoints for
% the desired frequency response. The frequencies in F must be given in
% increasing order with 0.0 < F < 1.0 and 1.0 corresponding to half the
% sample rate. The first and last elements of F must equal 0 and 1
% respectively.

Default Window: Hamming

```
f = [0 0.6 0.6 1];      % Frequency breakpoints
m = [1 1 0 0];          % Magnitude breakpoints
b = fir2(30,f,m);       % Frequency sampling-based FIR filter design
```

Uses 2 x NPT frequency points

Default: NPT = 512

% B = FIR2(N,F,A,NPT) specifies the number of points, NPT, for the grid
% onto which FIR2 linearly interpolates the frequency response. NPT must
% be greater than 1/2 the filter order (NPT > N/2). If desired, you can
% interpolate F and A before passing them to FIR2.

MATLAB fir1 vs. fir2

From the MATLAB website documentation:

- "**Use fir1** for window-based standard lowpass, bandpass, highpass, bandstop, and multiband configurations."
- **Use fir2** for windowed filters with arbitrary frequency response.

More details and examples of using fir1 & fir2 are available at:

- http://www.mathworks.com/help/signal/ref/fir1.html
- http://www.mathworks.com/help/signal/ref/fir2.html