EEO 401
Digital Signal Processing

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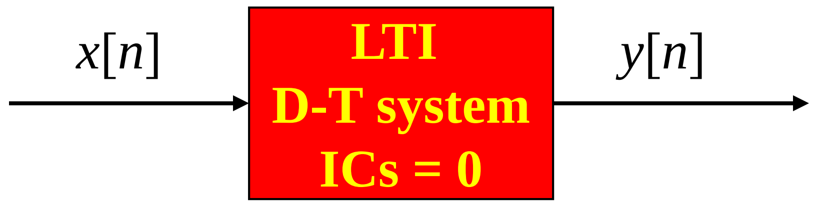
<u>Note Set #3</u>

- Convolution & Impulse Response – Review
- Reading Assignment: Sect. 2.3 of Proakis & Manolakis

Convolution for LTI D-T systems

We are trying to find $y(t)$… when all ICs = 0

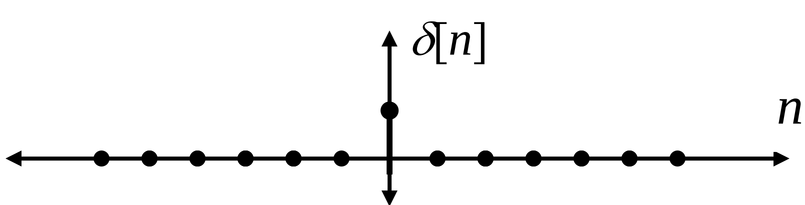
i.e. no stored "energy"

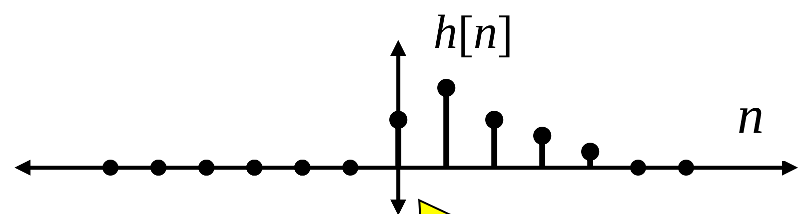


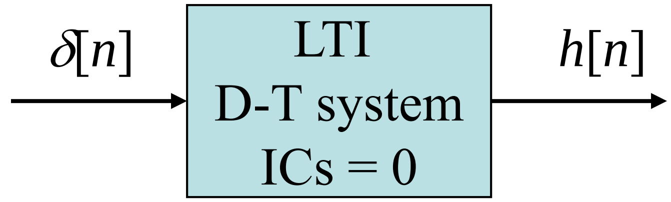
Before we can find the outptut… we need something first:

Impulse Response

The impulse response $h[n]$ is what "comes out" when $\delta[n]$ "goes in" w/ ICs=0



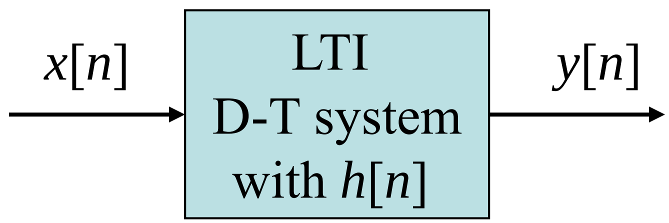




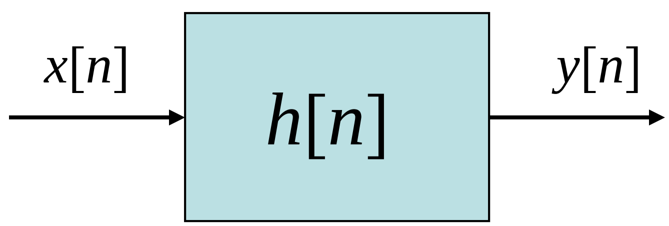
Note: If system is causal, then $h[n] = 0$ for $n < 0$

The impulse response $h[n]$ uniquely describes the system… so we can identify the system by specifying its impulse response $h[n]$.

Thus, we often show the system using a block diagram with the system's impulse response $h[n]$ inside the box representing the system:



Because impulse response $h[n]$ is only defined for LTI systems, if you see a box with the symbol $h[n]$ inside it you can assume that the system is an LTI system.

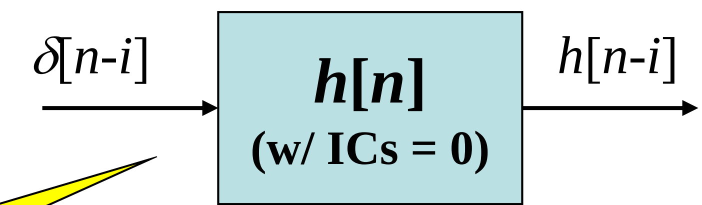


Q: How do we use *h*[*n*] to find the Zero-State Response?

A: "Convolution" We'll go through three analysis steps that will derive "The General Answer" that convolution is what we need to do to find the zero-state response

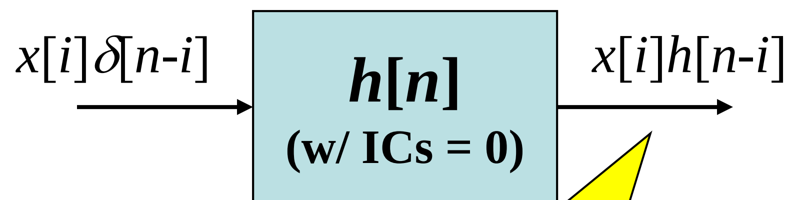
After that… we won't need to re-do these steps… **we'll just "Do Convolution"**

Step 1: Using time-invariance we know:



Shifted input gives shifted output

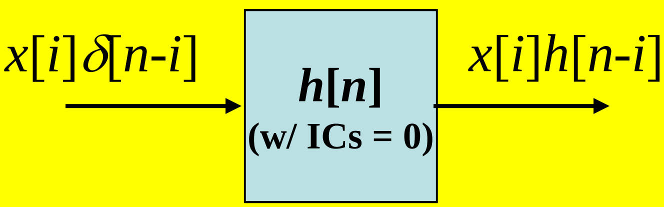
Step 2: Use "homogeneity" part of linearity:

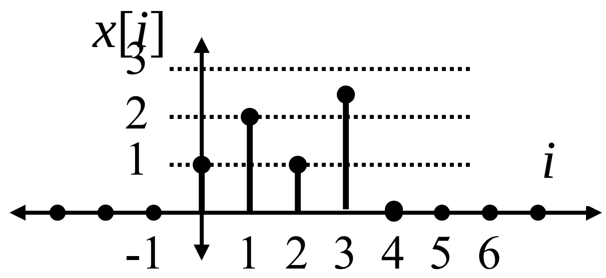


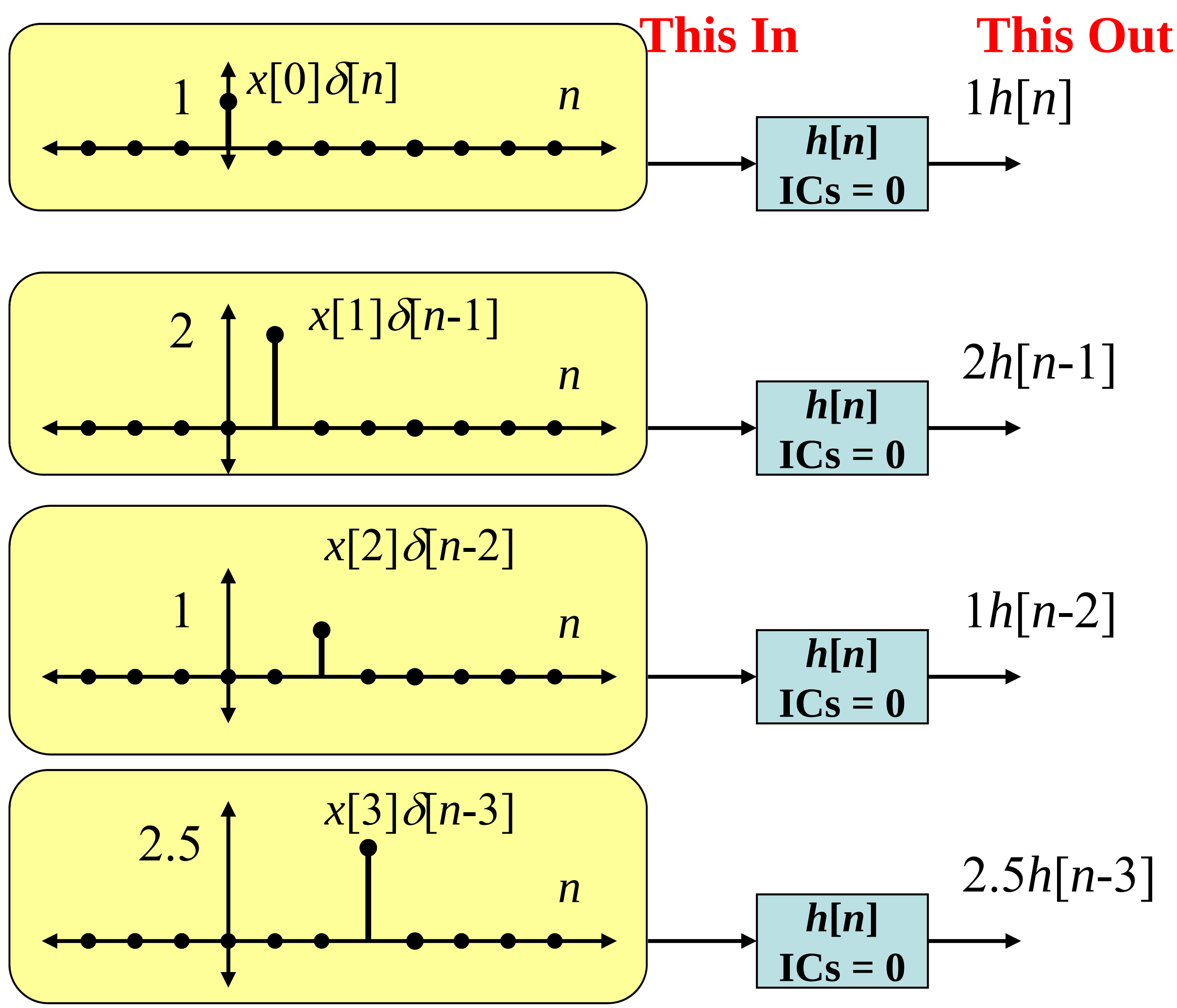
The input is a function of n so we view $x[i]$ as a fixed number for a given i

So… we scale the output by the same fixed number

Let's <u>see</u> step 2…
for a specific input:

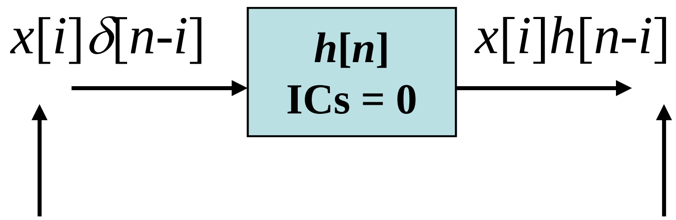






<u>Step 3:</u> Use "additivity" part of linearity

In Step 2 we looked at inputs like this:



For each i, a different input $\Rightarrow$ For each i, a different response

Now we use the additivity part of linearity:

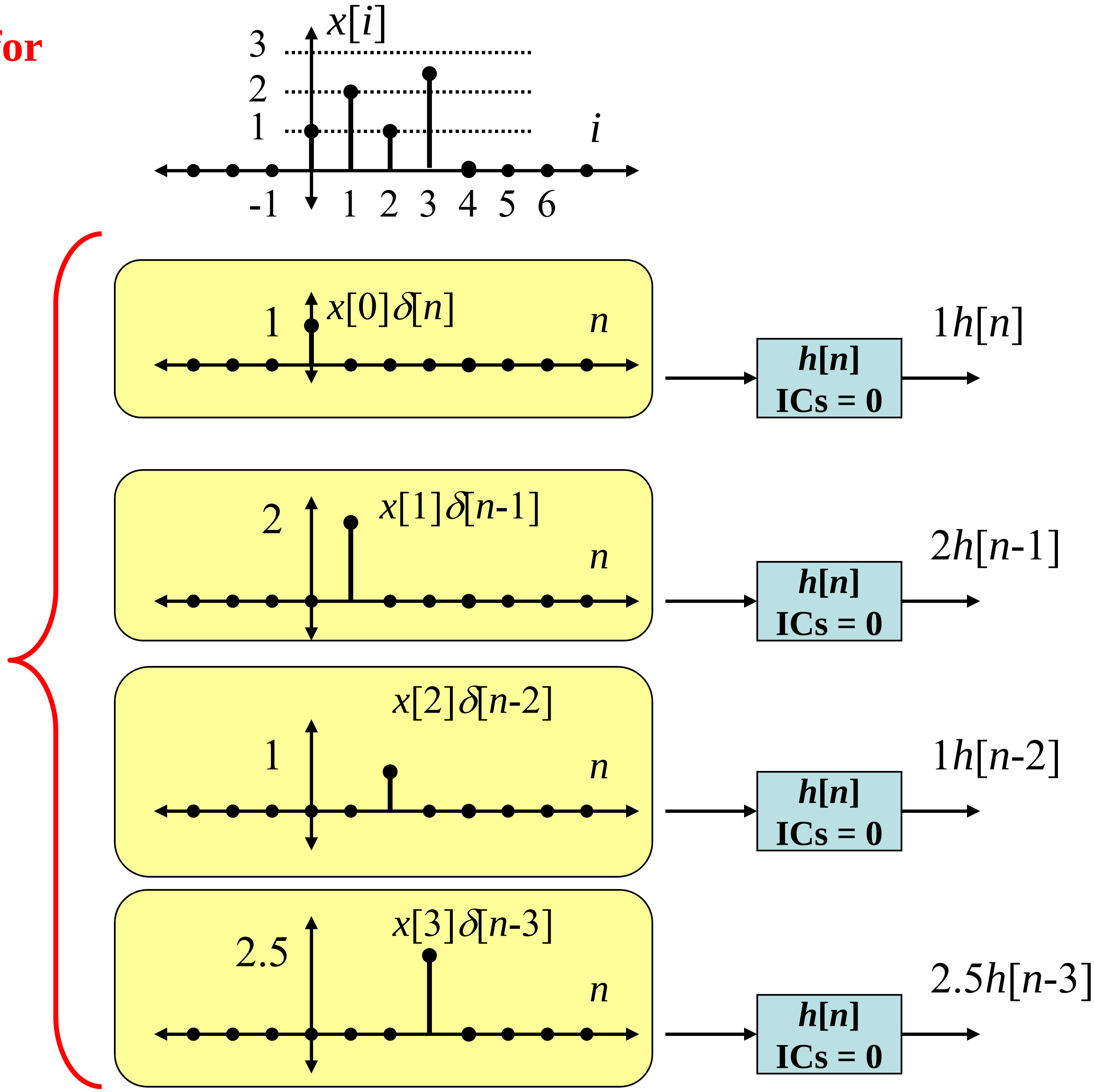
Put the <u>Sum of Those Inputs</u> In $\Rightarrow$ Get the <u>Sum of Their Responses</u> Out

Input: $\sum_{i=-\infty}^{\infty} x[i]\delta[n-i]$ → **Output:** $\sum_{i=-\infty}^{\infty} x[i]h[n-i]$

But… what is this??
On the next slide we show that it is the desired input signal $x[n]$!

Let's <u>see</u> step 3 for a specific input:

$$\sum_{i=-\infty}^{\infty} x[i]\delta[n-i]$$



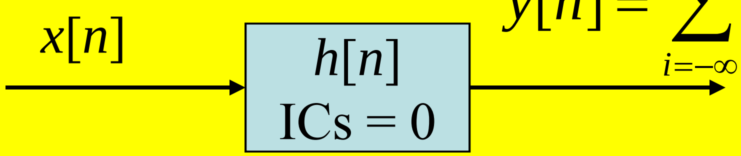
<u>Note:</u> The Sum of these "x-weighted" impulses gives $x[n]$!!

So… what we've seen is this:

Input: $\sum_{i=-\infty}^{\infty} x[i]\delta[n-i]$ → **Output:** $\sum_{i=-\infty}^{\infty} x[i]h[n-i]$

$$\underbrace{\sum_{i=-\infty}^{\infty} x[i]\delta[n-i]}_{= x[n]}$$

Or in other words… we've derived an expression that tells what comes out of a D-T LTI system with input *x*[*n*]:



$$y[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$$

CONVOLUTION!

$$y[n] = x[n] * h[n]$$

Notation for Convolution

So… now that we have derived this result we don't have to do these three steps… we "just" use this equation to find the zero-state output:

$$y[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i]$$

CONVOLUTION!

Note: In your Signals & Systems course you should have learned how to *do* convolution…. You should review that!

Big Picture

For a LTI D-T system in zero state characterized by impulse response $h[n]$, we can analytically find the output when the input is $x[n]$ by performing the convolution between $x[n]$ and $h[n]$.

$$y[n] = \sum_{i=-\infty}^{\infty} x[i]h[n-i] = \sum_{i=-\infty}^{\infty} h[i]x[n-i]$$

What if the LTI system is causal? ➡ $h[n] = 0 \; \forall n < 0$

$$y[n] = \sum_{i=-\infty}^{n} x[i]h[n-i] = \sum_{i=0}^{\infty} h[i]x[n-i]$$

What if the input "starts" @ $n = 0$? ➡ $x[n] = 0 \; \forall n < 0$

$$y[n] = \sum_{i=0}^{\infty} x[i]h[n-i] = \sum_{i=-\infty}^{n} h[i]x[n-i]$$

What if the LTI system is causal and input "starts" @ $n = 0$?

$$y[n] = \sum_{i=0}^{n} x[i]h[n-i] = \sum_{i=0}^{n} h[i]x[n-i]$$

Convolution Properties

These are things you can exploit to make it easier to solve problems

1. Commutativity $x[n]*h[n] = h[n]*x[n]$

$\Rightarrow$ You can choose which signal to "flip"

2. Associativity $x[n]*(v[n]*w[n]) = (x[n]*v[n])*w[n]$

$\Rightarrow$ Can change order $\rightarrow$ sometimes one order is easier than another

3. Distributivity $x[n]*(h_1[n]+h_2[n]) = x[n]*h_1[n]+x[n]*h_2[n]$

$\Rightarrow$ may be easier to split complicated system $h[n]$ into sum of simple ones

OR.. $\Rightarrow$ we can split complicated input into sum of simple ones

(nothing more than "linearity")

4. Convolution with impulses $x[n]*\delta[n-q] = x[n-q]$

This one is VERY easy to see using the graphical convolution steps. TRY IT!!

Checking for Stability via the Impulse Response

An LTI system is BIBO stable if and only if its impulse response is "absolutely summable":

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

Systems w/ Infinite-Duration & Finite-Duration Impulse Resp.

For simplicity of notation we focus on causal systems here.

$$y[n] = \sum_{i=0}^{\infty} h[i]x[n-i]$$

In general, the impulse response has infinite duration

A system for which $h[n]$ has infinitely many non-zero values is said to be an "infinite-duration impulse response (IIR) system.

A system for which $h[n]$ has finitely many non-zero values is said to be an "finite-duration impulse response (FIR) system. Suppose $h[n] = 0$ for $n < 0$ and for $n \geq M$ then the convolution sum becomes

$$y[n] = \sum_{i=0}^{M-1} h[i]x[n-i]$$

This is said to be an <u>order *M*</u> FIR system