BINGHAMTON UNIVERSITY

State University of New York

EEO 401
Digital Signal Processing

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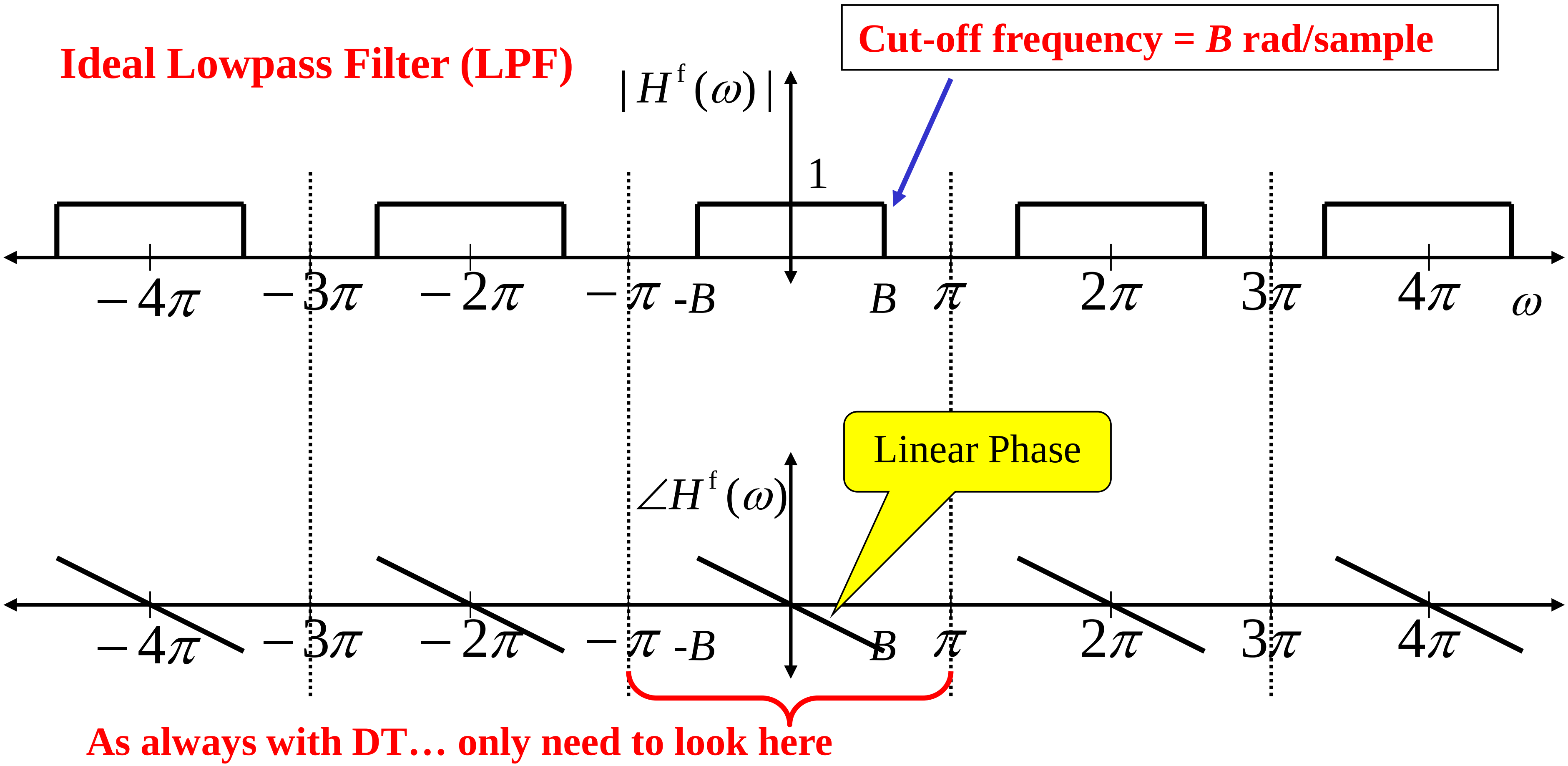
Note Set #27

- General Issues for Design of Digital Filters
- Reading: Sect. 10.1 & 10.2.1 of Proakis & Manolakis

Ideal D-T Filters

Here is the so-called "ideal filter" frequency response for a Lowpass Filter



There are also highpass, bandpass, and bandstop filters; and there are other more specialized types

Why can't an ideal filter exist in practice?? Same as for CT…. It is non-causal

For the ideal LPF $H^{f}(\omega) = p_{2B}(\omega)e^{-j\omega n_d} \quad \omega \in [-\pi, \pi)$ repeats elsewhere

Now consider applying a delta function as its input: $x[n] = \delta[n] \leftrightarrow X^{f}(\omega) = 1$

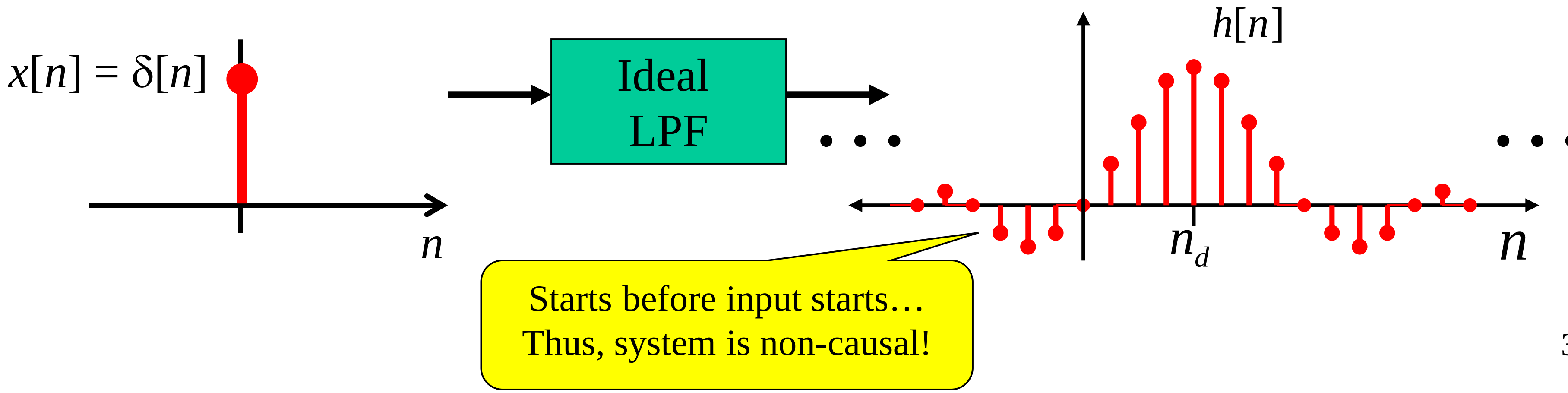
Then the output has DTFT:

$$Y^{f}(\omega) = X^{f}(\omega)H^{f}(\omega) = p_{2B}(\omega)e^{-j\omega n_d} \quad \omega \in [-\pi, \pi) \quad \text{repeats elsewhere}$$

Linear Phase Imparts Delay

From the DTFT Table: $\frac{B}{\pi}\text{sinc}\left[\frac{B}{\pi}n\right] \leftrightarrow \sum_{k=-\infty}^{\infty} p_{2B}(\omega + 2\pi k)$

So the response to a delta (applied at $n = 0$) is: $h[n] = (B/\pi)\,\text{sinc}\left[(B/\pi)(n - n_d)\right]$



Paley-Wiener Theorem

So we've seen that a causal filter can't give that perfect rectangular frequency response of the ideal filter.

But…. What ***can*** it give??? The Paley-Wiener Theorem answers that…

If $h[n]$ has finite energy and $h[n] = 0 \ \forall n < 0$ then

$$\int_{-\pi}^{\pi} \left| \ln\left(\left| H^{\mathrm{f}}(\omega) \right| \right) \right| d\omega < \infty$$

Conversely, if $|H^{\mathrm{f}}(\omega)|$ is square integrable and satisfies this… then there exists a phase $\theta(\omega)$ so that

$$H^{\mathrm{f}}(\omega) = \left| H^{\mathrm{f}}(\omega) \right| e^{j\theta(\omega)}$$

has an IDTFT $h[n]$ that is causal.

So what does the Paley-Wiener Theorem actually say about what shapes we can have for frequency response of a causal filter??

$$\int_{-\pi}^{\pi} \left| \ln\left(\left| H^{\mathrm{f}}(\omega) \right| \right) \right| d\omega < \infty$$

Clearly if $|H^{\mathrm{f}}(\omega)| = 0$ on any interval then $|\ln(|H^{\mathrm{f}}(\omega)|)|$ is infinite on that interval

➔ So can't have a perfect stop band

But… it is possible for $|H^{\mathrm{f}}(\omega)| = 0$ at finitely many points… just not over intervals.

Rational Transfer Function

Just because the P-W Theorem says that there is a causal filter that exists for some desired $|H(\omega)|$ does not mean it has a rational transfer function.

Recall that our focus is on DT systems with rational transfer functions:

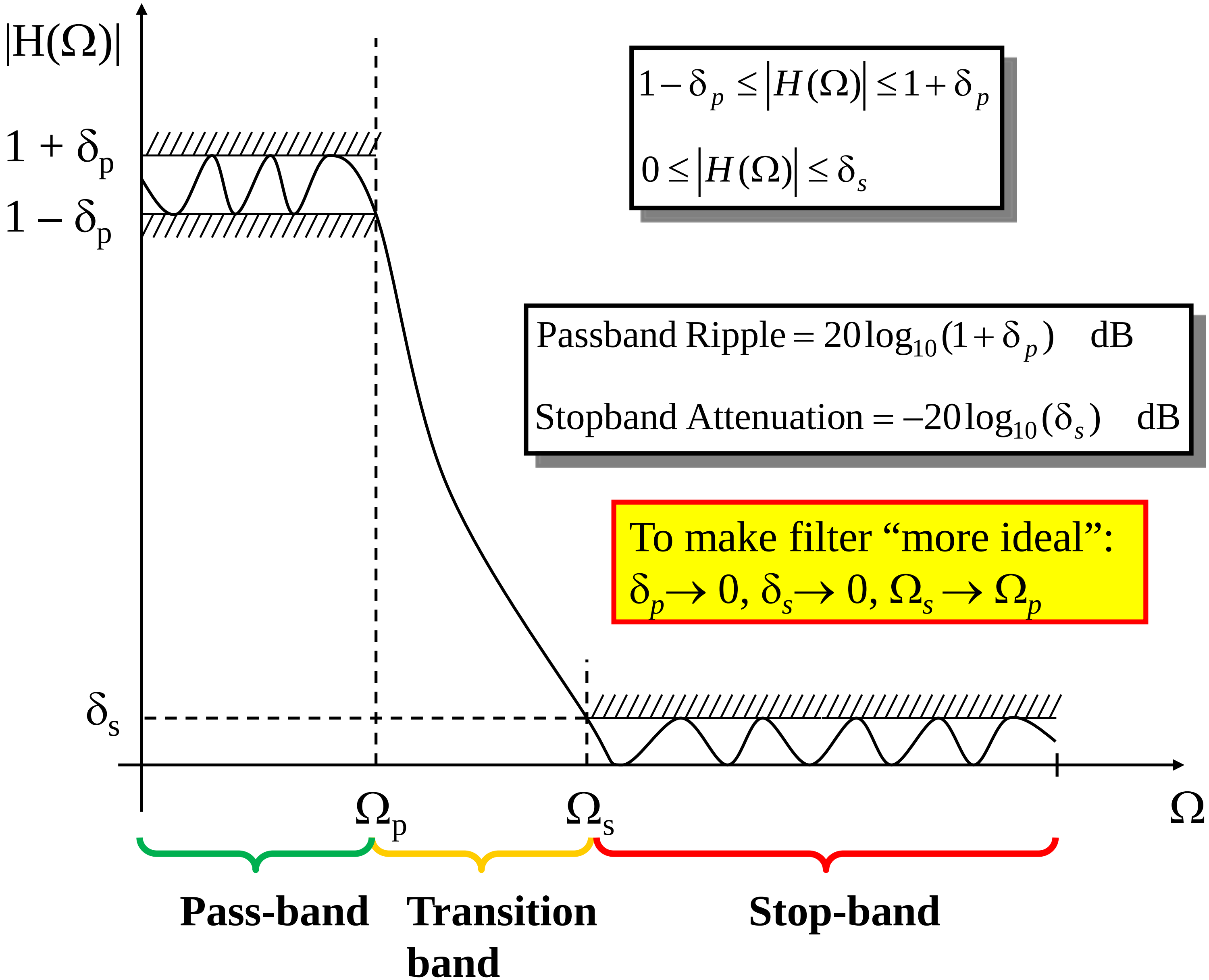
➔ Standard-Form Difference Eq
➔ Poles & Zeros
➔ Block Diagram

$$H^{\mathrm{f}}(\omega) = \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{1 + \sum_{k=1}^{N} a_k e^{-j\omega k}} \qquad H^{\mathrm{z}}(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

Our Goal: Find rational approximate to desired $H^{\mathrm{f}}_d(\omega)$

Practical Filter Specification

Lowpass Filter Specification



$$1-\delta_p \le |H(\Omega)| \le 1+\delta_p$$

$$0 \le |H(\Omega)| \le \delta_s$$

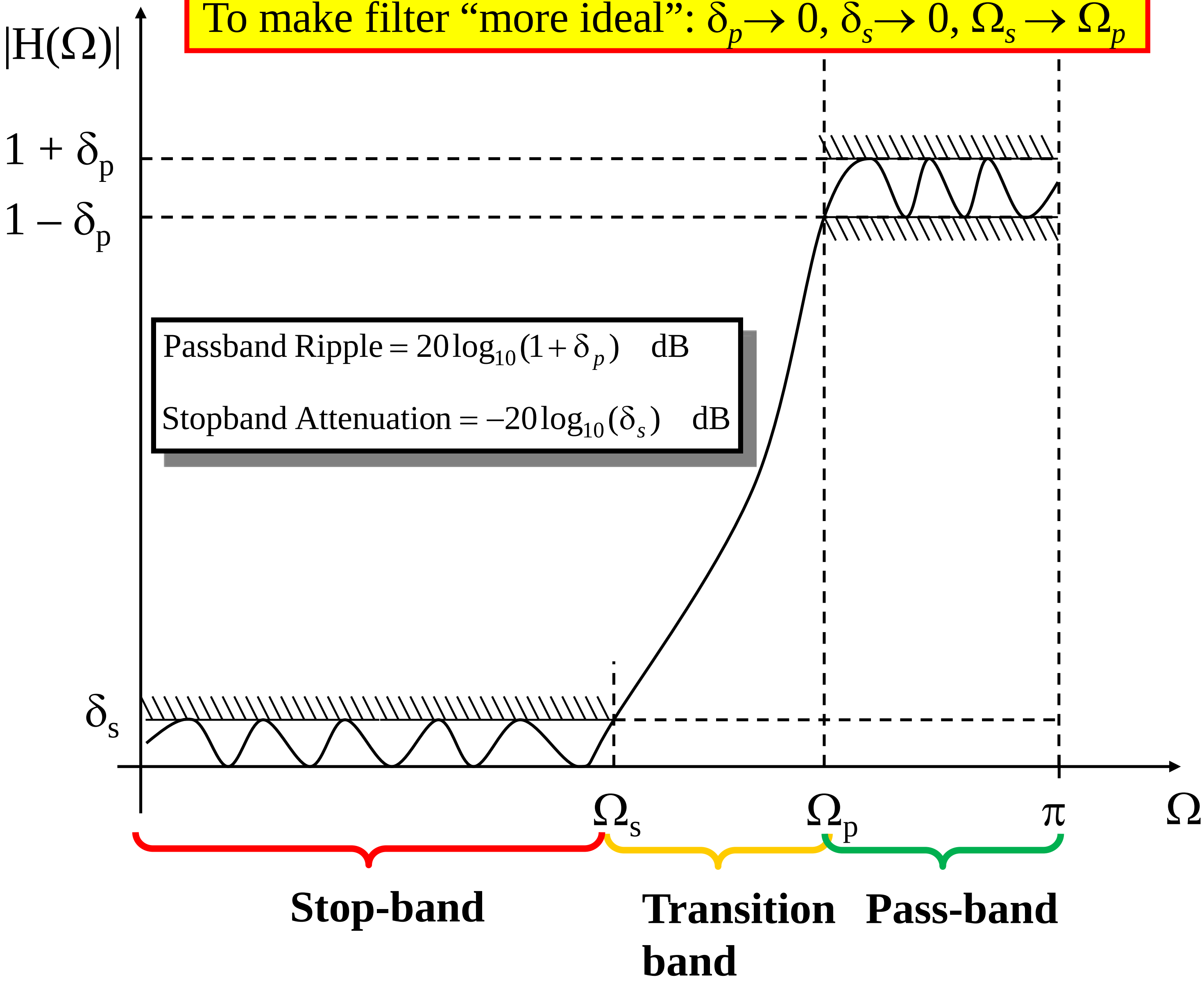
$$\text{Passband Ripple} = 20\log_{10}(1+\delta_p) \quad \text{dB}$$

$$\text{Stopband Attenuation} = -20\log_{10}(\delta_s) \quad \text{dB}$$

To make filter "more ideal":
$\delta_p \rightarrow 0$, $\delta_s \rightarrow 0$, $\Omega_s \rightarrow \Omega_p$

Highpass Filter Specification

To make filter "more ideal": $\delta_p \rightarrow 0$, $\delta_s \rightarrow 0$, $\Omega_s \rightarrow \Omega_p$

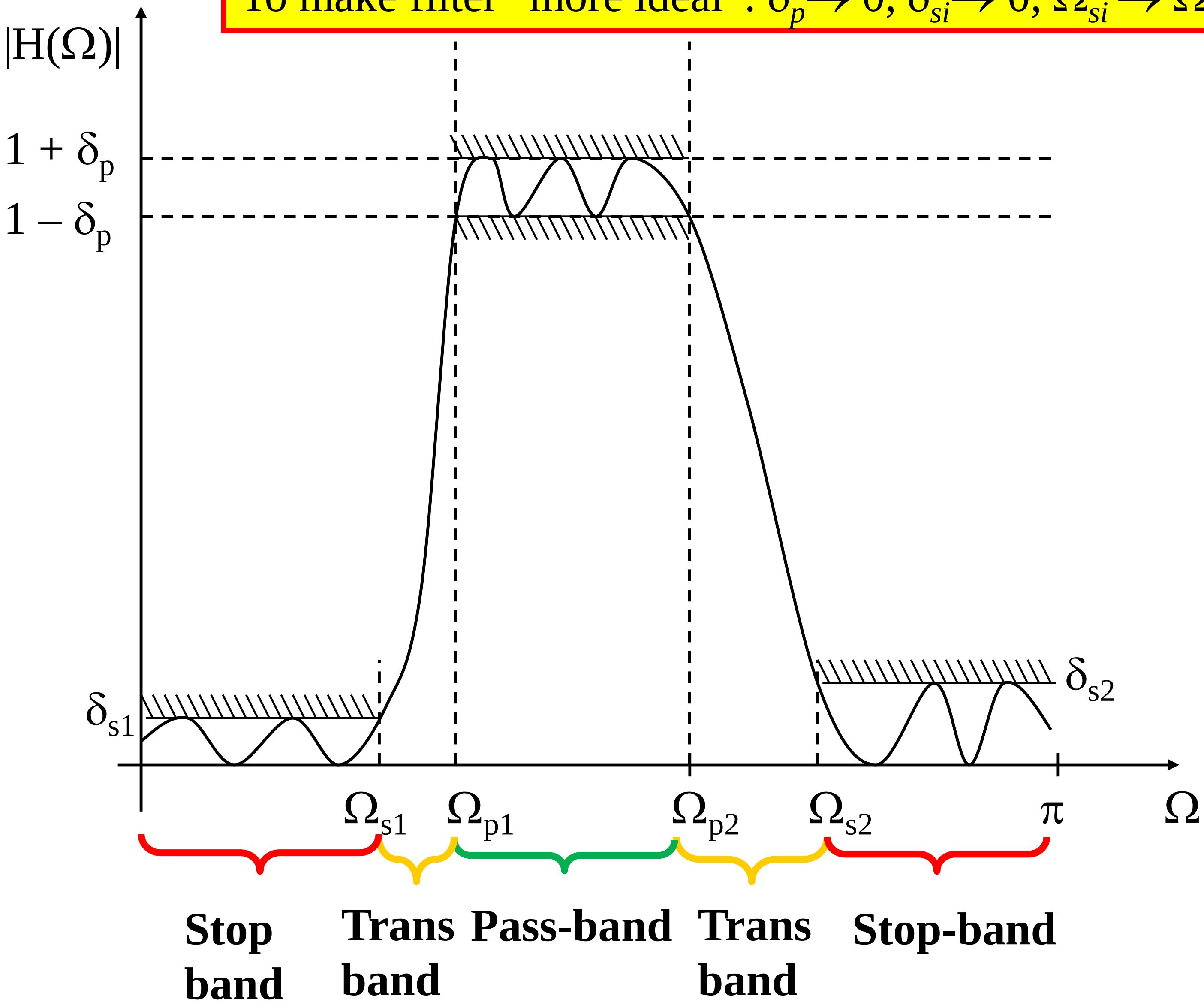


$$\text{Passband Ripple} = 20\log_{10}(1+\delta_p) \quad \text{dB}$$

$$\text{Stopband Attenuation} = -20\log_{10}(\delta_s) \quad \text{dB}$$

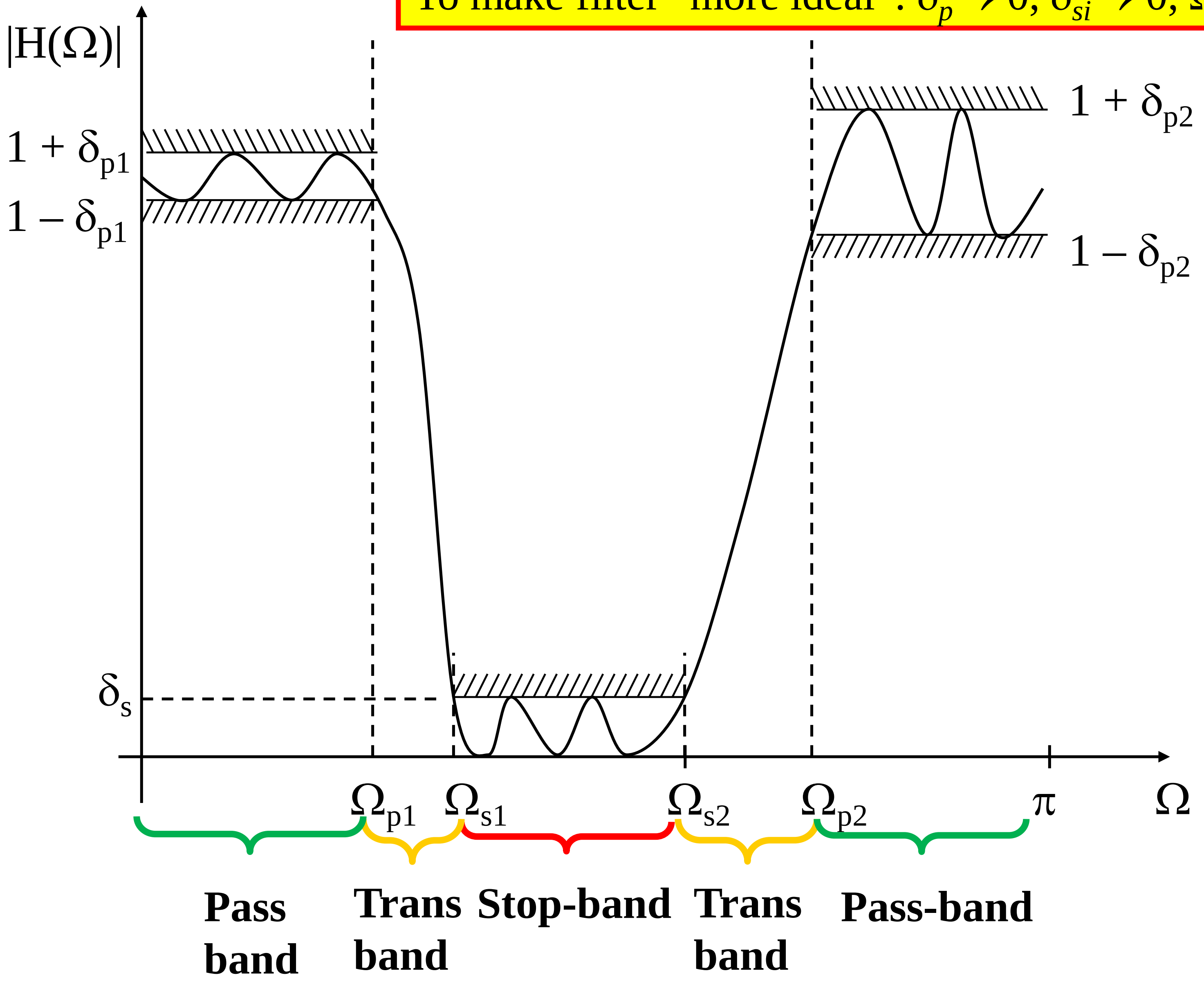
Bandpass Filter Specification

To make filter "more ideal": $\delta_p \to 0$, $\delta_{si} \to 0$, $\Omega_{si} \to \Omega_{pi}$



Bandstop Filter Specification

To make filter "more ideal": $\delta_p \rightarrow 0$, $\delta_{si} \rightarrow 0$, $\Omega_{si} \rightarrow \Omega_{pi}$



Our Goal: Find rational approximate to desired $H^{f}_{d}(\omega)$

<u>Some Issues to Consider</u>:

- Achieve the specs (band edges and ripples) with the least complexity
  - Complexity does have links to the “delay” through the filter
- FIR vs IIR filters
  - FIR can achieve the desirable linear phase response
  - IIR can achieve magnitude specs with lower complexity
- Recursive vs. Non-Recursive
  - Connections to FIR vs IIR
  - Impact on complexity
- Stability of filter
  - FIR filters are inherently stable
  - IIR filter design needs attention to stability