BINGHAMTON UNIVERSITY
State University of New York

EEO 401
Digital Signal Processing

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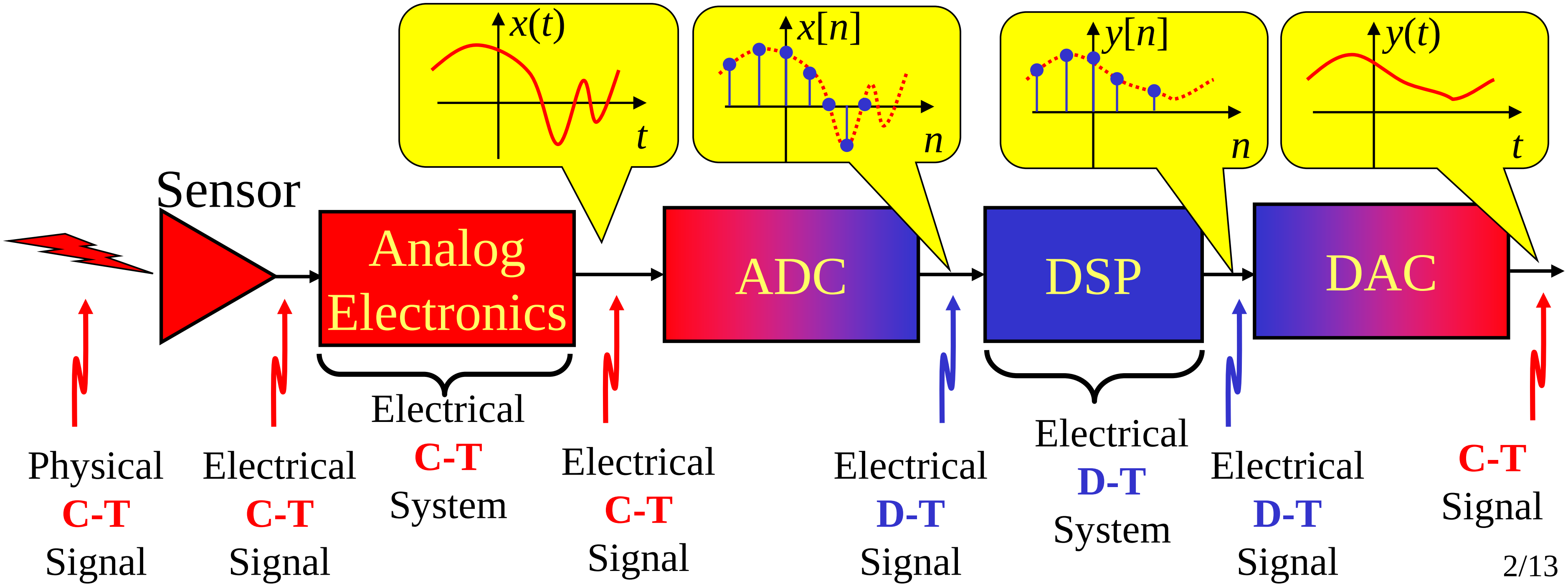
Note Set #1

- Introduction
- Reading Assignment: Ch. 1 of Proakis & Manolakis

DSP Scenario

- Modern systems generally…
  - get a **continuous-time signal** from a sensor
  - a **cont.-time system** modifies the signal
  - an "analog-to-digital converter" (ADC or A-to-D) sample the signal to create a **discrete-time signal** … a "stream of numbers"
  - A **discrete-time system** to do the processing
  - and then (if desired) convert back to analog (not shown here)



- Our focus will be on
  - Sampling theory
  - Frequency-Domain Models for DT Signals
  - Frequency-Domain Models for DT Systems
  - Processing structures for implementing DSP systems
  - Methods for designing DSP systems

Ensures that samples are equivalent to CT signal

Provides math to understand ***HOW*** the DSP works

Gives practical ways to ***MAKE*** DSP work

- Section 1.2 Classification of Signals
  - Multichannel vs. **Single Channel**
  - CT vs **DT**
  - Discrete-Valued vs **Continuous-Valued**
  - Random vs **Deterministic**

Transforms & Notation

Proakis & Manolakis don't use this superscript Notation. I borrowed it from Porat's DSP Book

Fourier Transform for CT Signals

$$X^F(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$

$$x(t) = \int_{-\infty}^{\infty} X^F(F) e^{j2\pi Ft} dF$$

Fourier Transform for DT Signals

$$X^{\mathrm{f}}(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

Discrete
Time
Fourier
Transform

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X^{\mathrm{f}}(\omega) e^{j\omega n} d\theta$$

Set $z = e^{j\omega}$

Z Transform for DT Signals

$$X^{\mathrm{z}}(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Inverse ZT done using partial fractions & a ZT table

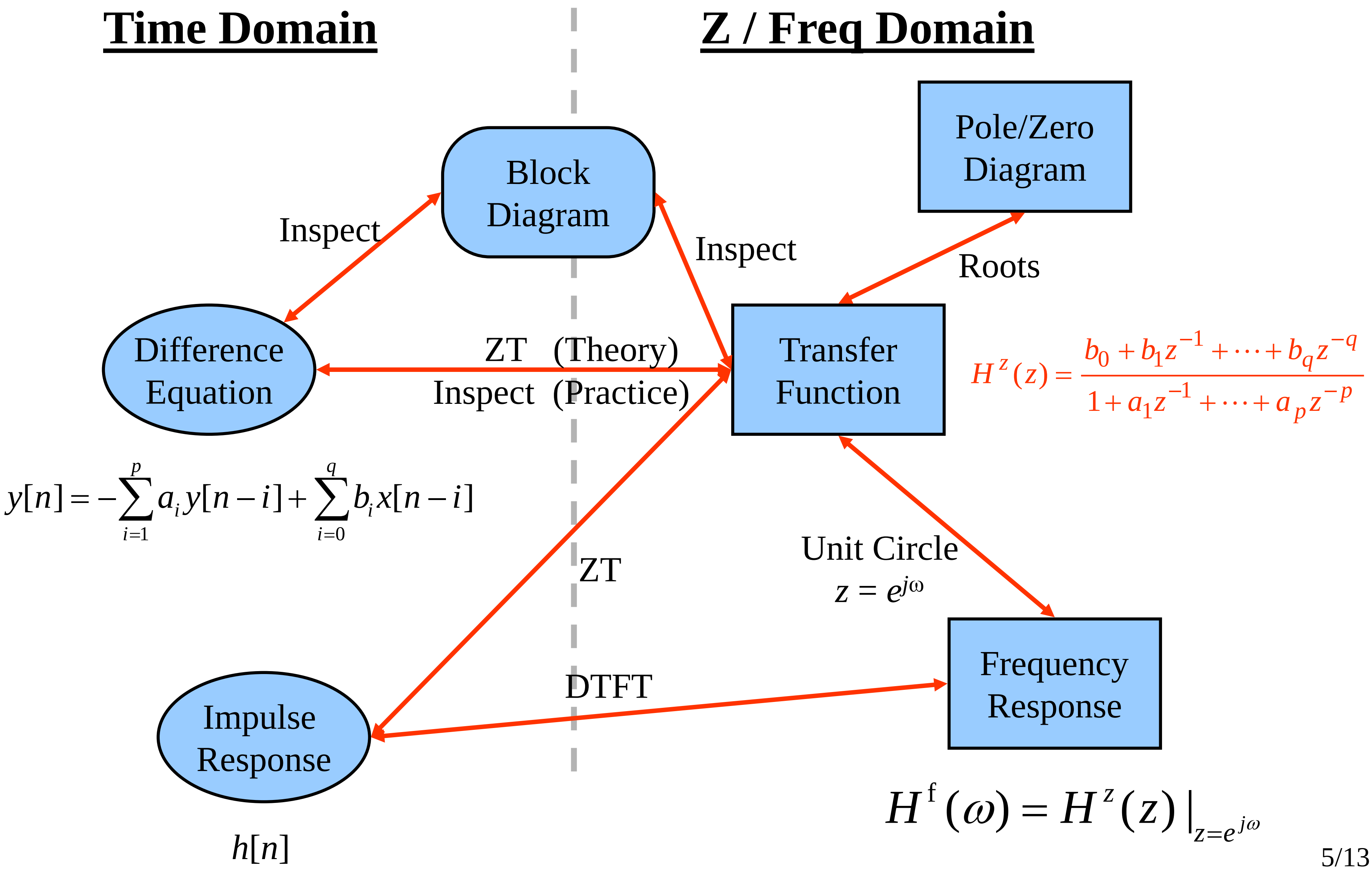
Discrete Fourier Transform for DT Signals

Discrete **F**ourier **T**ransform

$$X^{\mathrm{d}}[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \quad k = 0, 1, 2, \ldots, N-1$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X^{\mathrm{d}}[k] e^{j2\pi kn/N} \quad n = 0, 1, 2, \ldots, N-1$$

Discrete-Time System Relationships



Sinusoidal Time Function

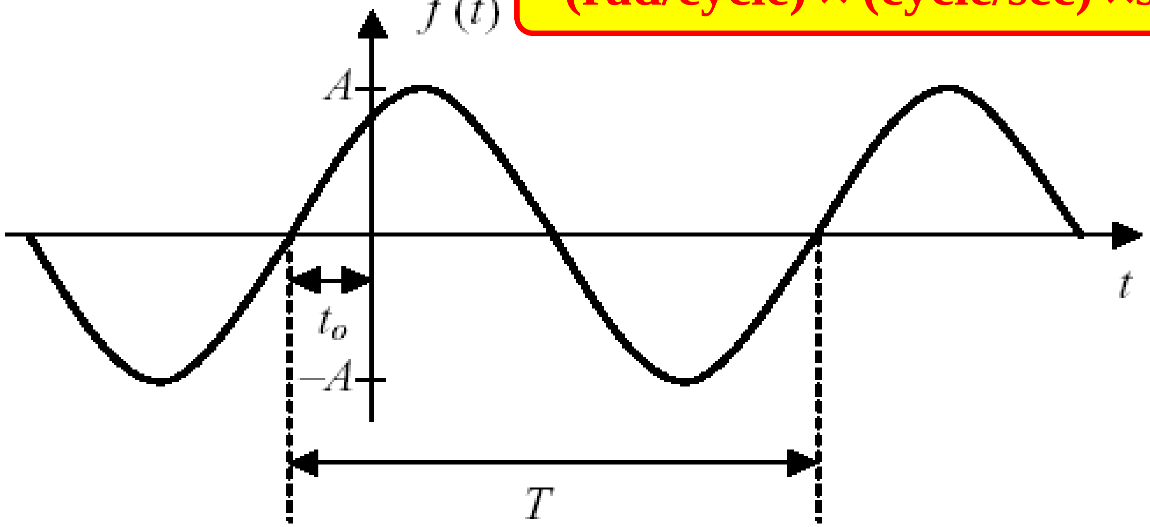
A sinusoid is completely defined by its three parameters:

- **Amplitude** A (for us typically in volts or amps but could be other unit)
- **Frequency** F_o in Hz
- **Phase** ϕ in rad (not degrees!)

$$x(t) = A\sin(2\pi F_o t + \phi)$$

(rad/cycle) × (cycle/sec) ×sec + rad = rad

(Similar for cosine)



T is the Period in seconds (actually seconds/cycle) of the sinusoid… $T = 1/F_o$

t_o is a time shift… it is related the phase ϕ

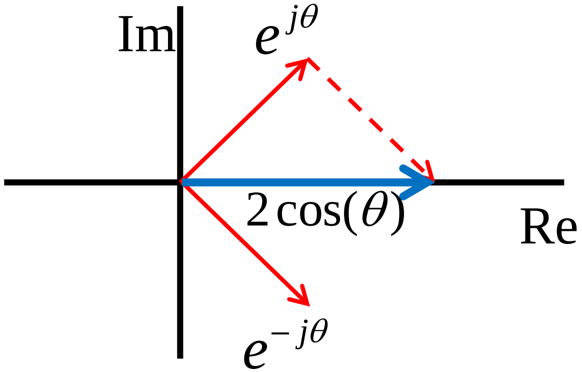
Complex Sinusoidal Time Function

In many cases it is desirable to write a real-valued sinusoid in terms of "complex-valued sinusoids". This is a math trick that – believe it or not! – makes things ***easier*** to work with!!!

$$x(t) = A\cos(2\pi F_o t + \phi) = \frac{A}{2}\left[e^{j(2\pi F_o t + \phi)} + e^{-j(2\pi F_o t + \phi)} \right]$$

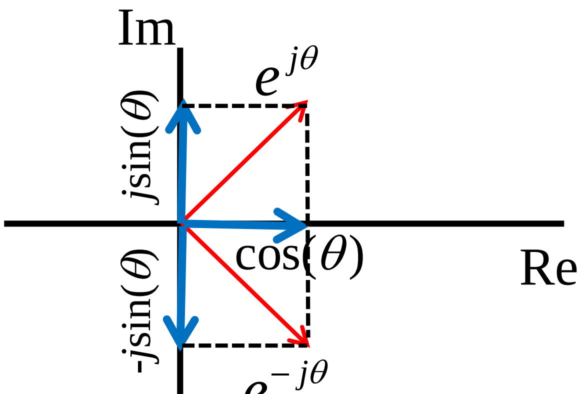
This comes from Euler's Formula:

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$



Another form of Euler's Formula:

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$
$$e^{-j\theta} = \cos(\theta) - j\sin(\theta)$$

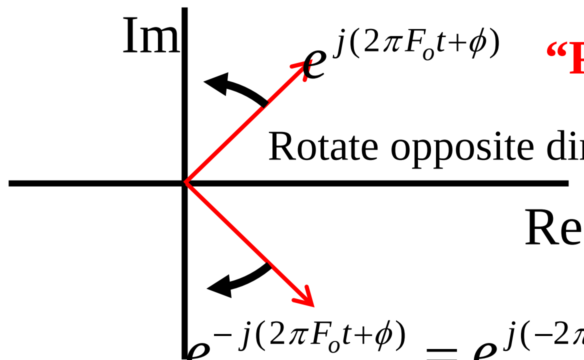


Exploring the Complex Sinusoidal Terms

$$x(t) = A\cos(2\pi F_o t + \phi) = \frac{A}{2}\left[e^{j(2\pi F_o t + \phi)} + e^{-j(2\pi F_o t + \phi)} \right]$$

Two complex values with opposite angles

Imaginary part always cancels!



$e^{j(2\pi F_o t + \phi)}$ **"Positive Frequency" Term**

Rotate opposite directions… due to negative sign

$e^{-j(2\pi F_o t + \phi)} = e^{j(-2\pi F_o t - \phi)}$ **"Negative Frequency" Term**

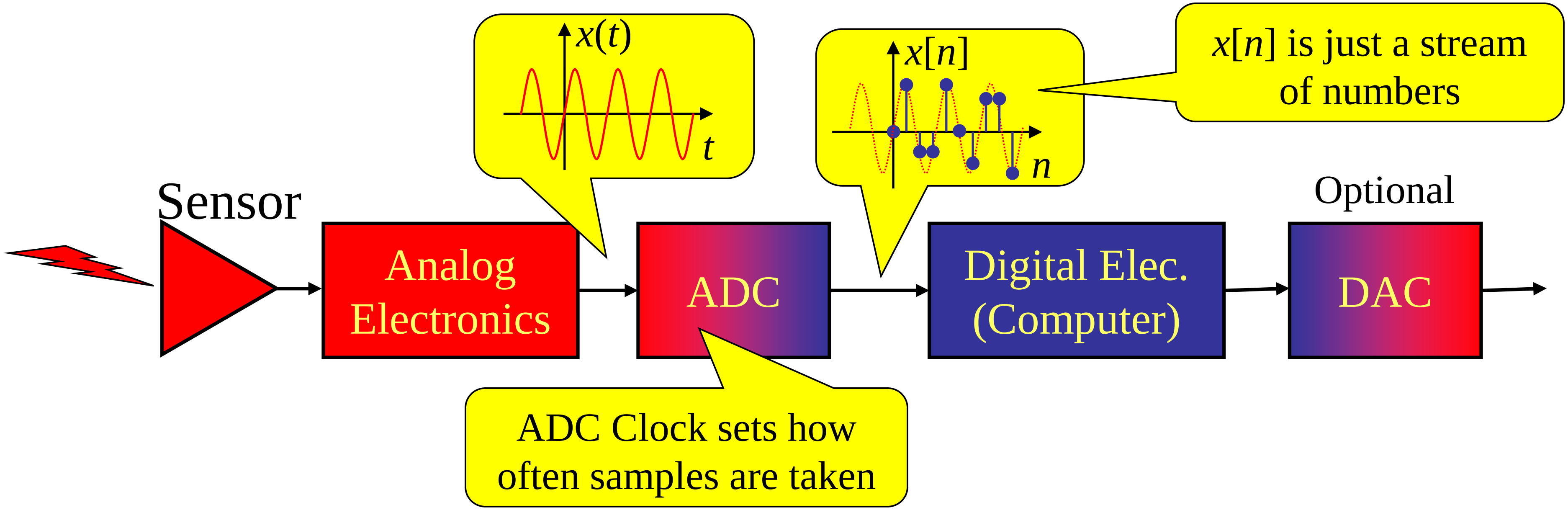
Here is a link to a Quicktime movie of these rotating…

http://www.cic.unb.br/~mylene/PSMM/DSPFIRST/chapters/2sines/demos/phasors/graphics/phasorsn.mov

Link to another Web Demo of this…

1. Open the web page
2. Click on the box at the top labeled Two

Sampling Sinusoids… DT Sinusoids



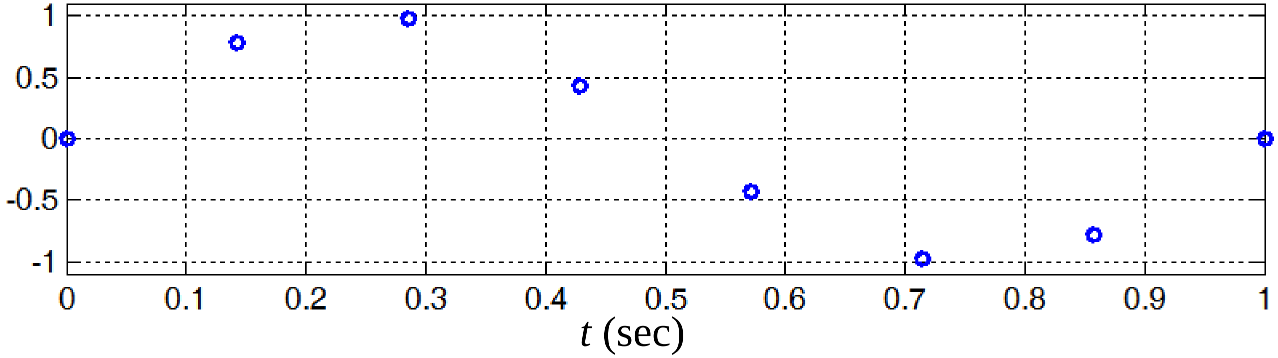
How closely should the samples be spaced??

At first thought we might think we need to have the samples still "look like" the original sinusoid… But that turns out to be excessive, as our theory will show eventually show.

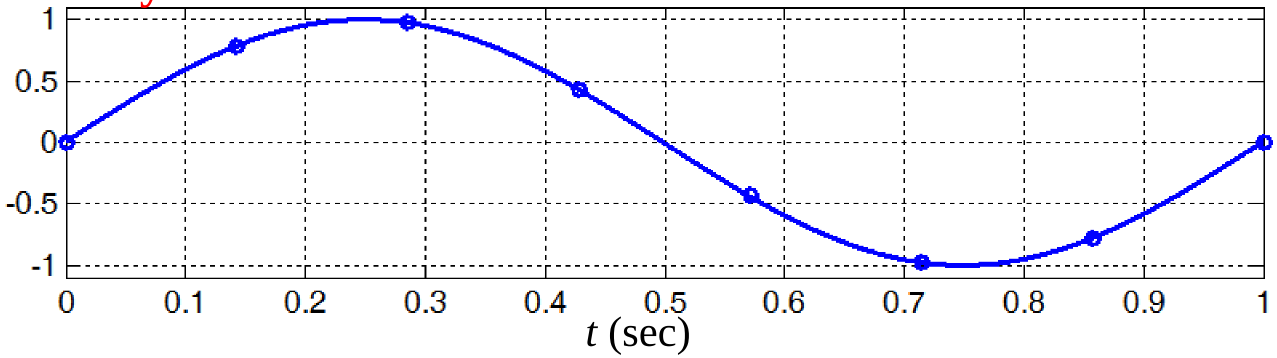
Looking at the samples $x[n]$ above they don't quite really look like a sinusoid… yet they are taken at a rate suitable for most applications!

So… how do we determine how fast we need to sample???

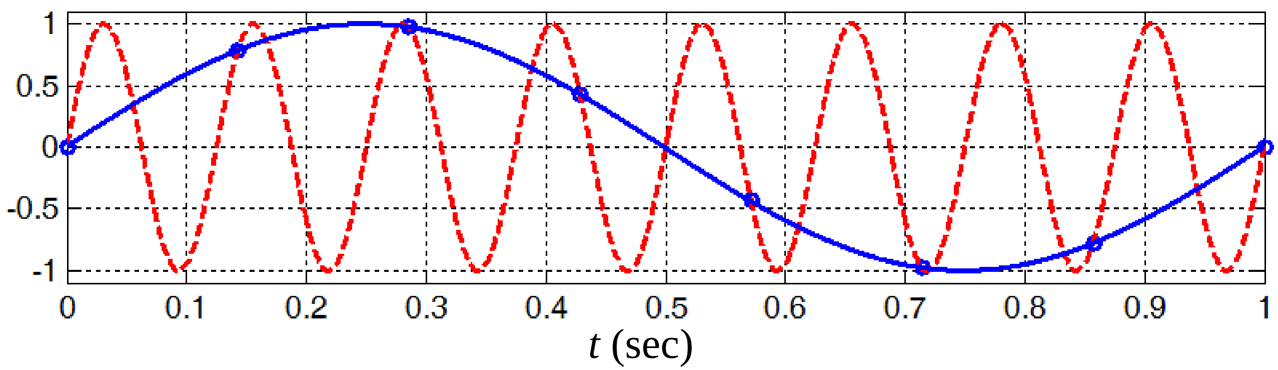
DT Samples…. What CT Sinusoid did they come from????



They could have come from this blue one…



But…They could have come from this RED one!!!



Thus… if we want to be able to tell these two apart we need to sample faster!!

Let T_s be the time spacing between samples… Then $F_s = 1/T_s$ as the "sampling frequency" in samples/sec.

Then if we have a CT sinusoid $x(t) = \cos(2\pi f_o t)$ that is sampled we have

$$x(t) = \cos(2\pi F_o t) \quad \longrightarrow \quad x[n] = x(nT_s) = \cos(\underbrace{2\pi F_o T_s}_{\triangleq \omega_o} n)$$

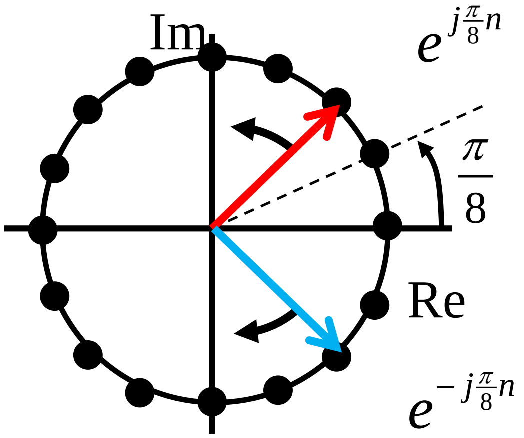
Discrete-Time Sinusoid

$$x[n] = \cos(\omega_o n) \qquad \omega_o \triangleq 2\pi \frac{F_o}{F_s}$$

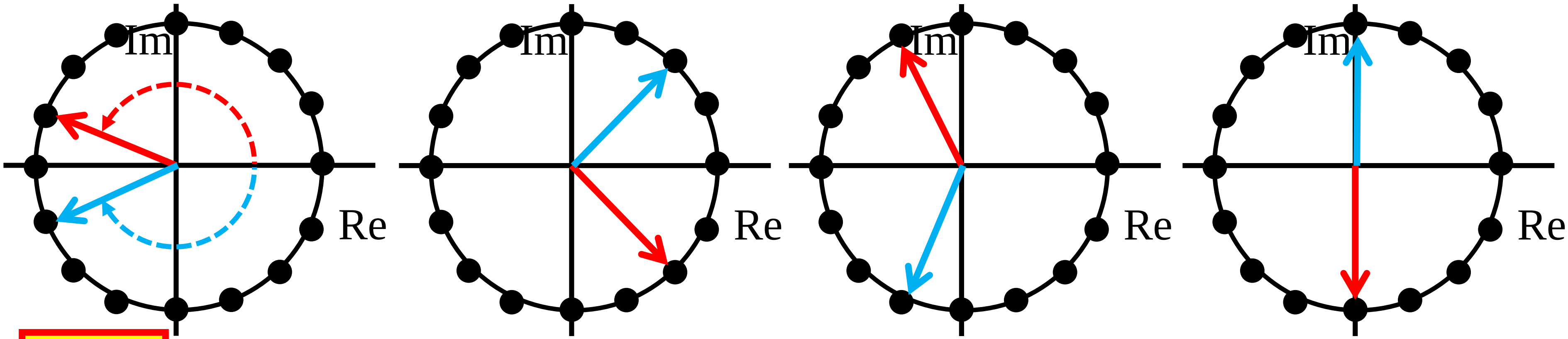
Units are "rad/sample"

So… to help visualize this:

$$\cos(\omega_o n) = \tfrac{1}{2}\left[e^{j\omega_o n} + e^{-j\omega_o n} \right]$$



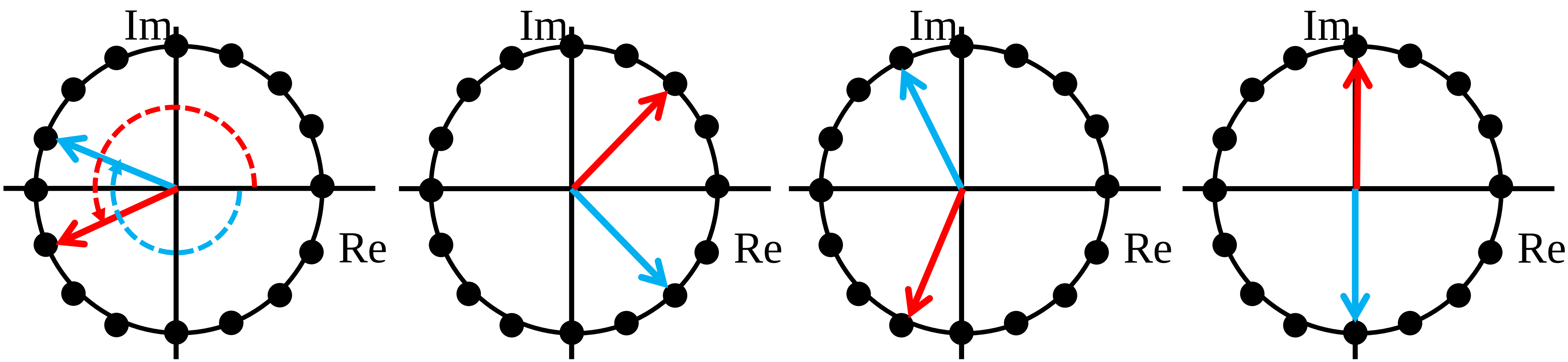
$n = 1$ **$n = 2$** **$n = 3$** **$n = 4$**



$$\omega_o = \frac{7\pi}{8}$$

$$\omega_o = \frac{9\pi}{8}$$

So… a DT frequency > π rad/sample looks exactly like some other frequency < π. <u>This is called "Aliasing"</u>.



$n = 1$ **$n = 2$** **$n = 3$** **$n = 4$**

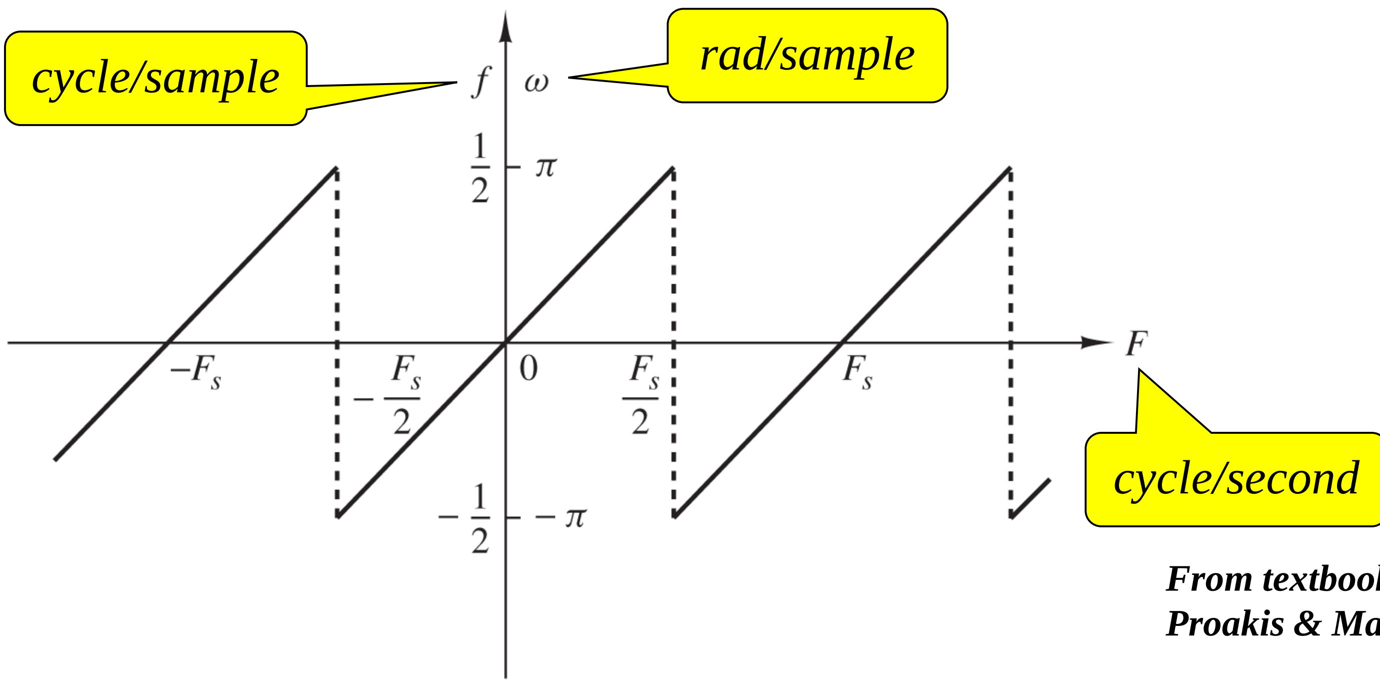
So to avoid this "aliasing" when sampling a CT sinusoid to make a DT sinusoid we must require that:

$$F_o < \frac{F_s}{2} \quad \Rightarrow \quad \omega_o < 2\pi \frac{F_s/2}{F_s} = \pi$$

Thus… for "proper sampling" we need to choose our <u>sampling rate</u> to be more than <u>double</u> the <u>highest frequency we expect</u>!!!

Aside: This is consistent with some real-world facts you may know about:

- High-Fidelity Audio contains frequencies up to only about 20 kHz
- CD digital audio has a sampling frequency of F_s = 44.k Hz > 2x20kHz



From textbook by Proakis & Manolakis

Figure 1.4.4 Relationship between the continuous-time and discrete-time frequency variables in the case of periodic sampling.