Binghamton University
State University of New York

EECE 301
Signals & Systems
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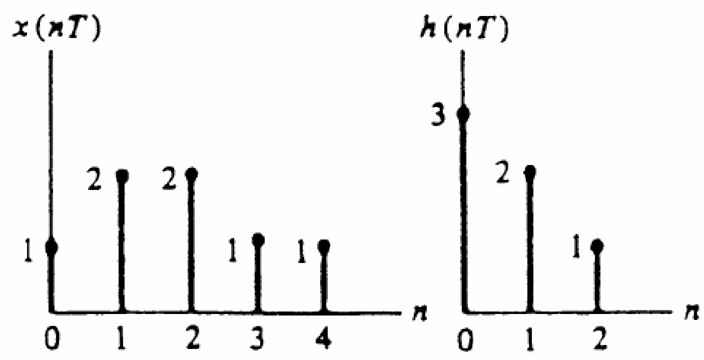
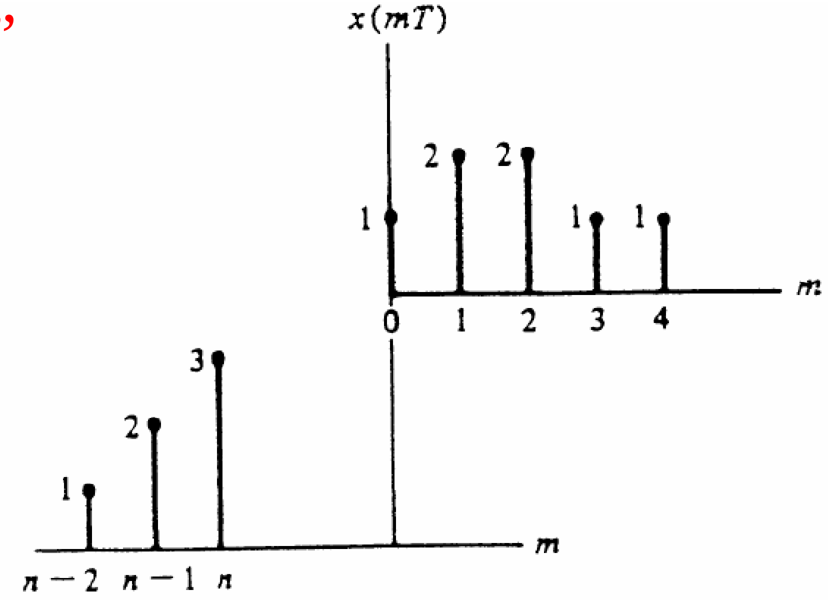
<u>Discussion #3b</u>

- DT Convolution Examples

Convolution Example “Table view”



(a) Input and unit pulse response



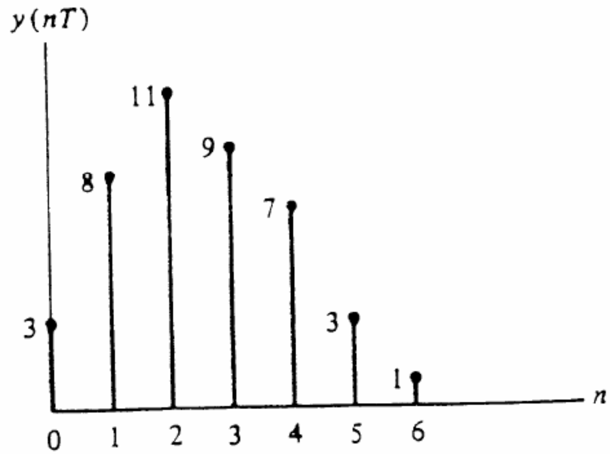
(b) Functions for computing convolution sum

			Samples $x(mT)$					
	0	0	1	2	2	1	1	0
$n = 0$	1	2	3	0	0	0	0	0
$n = 1$	0	1	2	3	0	0	0	0
$n = 2$	0	0	1	2	3	0	0	0
$n = 3$	0	0	0	1	2	3	0	0
$n = 4$	0	0	0	0	1	2	3	0
$n = 5$	0	0	0	0	0	1	2	3
$n = 6$	0	0	0	0	0	0	1	2

(c) Table for evaluating summation

$h(-m)$

$h(1-m)$



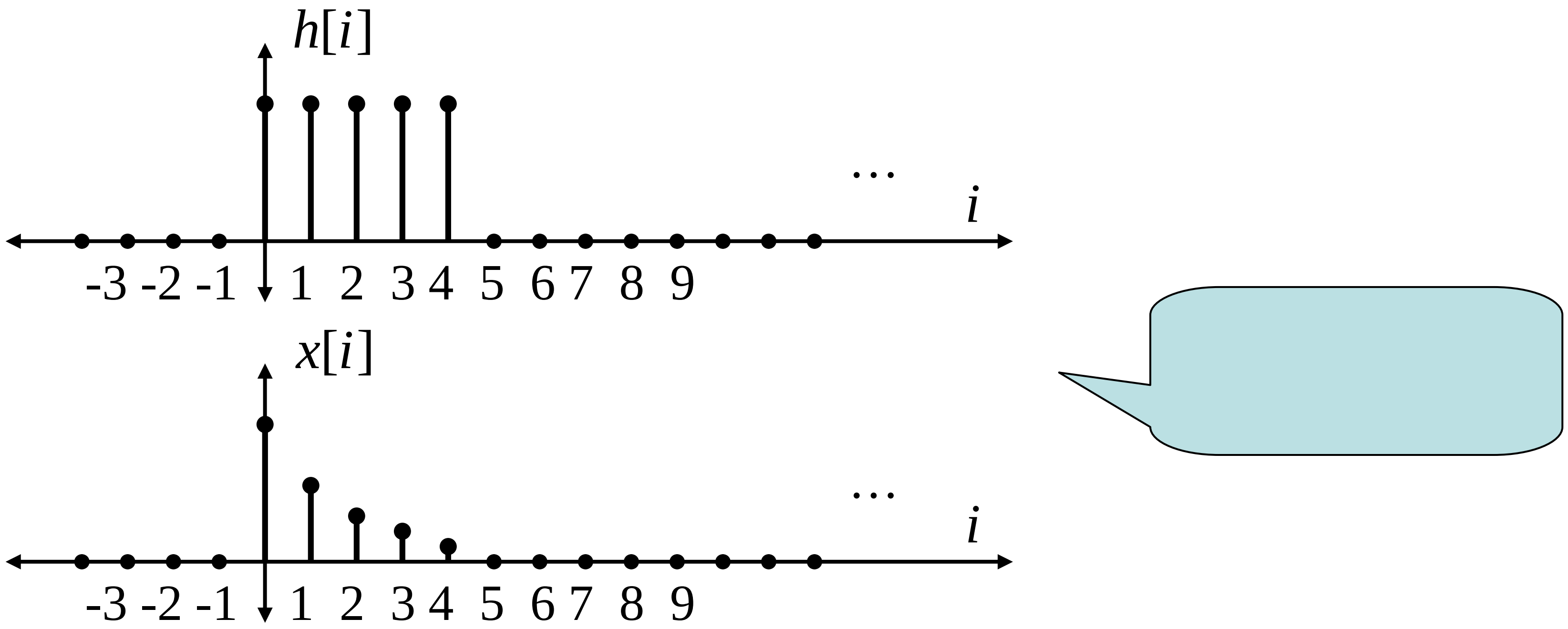
(d) Output

Discrete-Time Convolution Example: "Sliding Tape View"

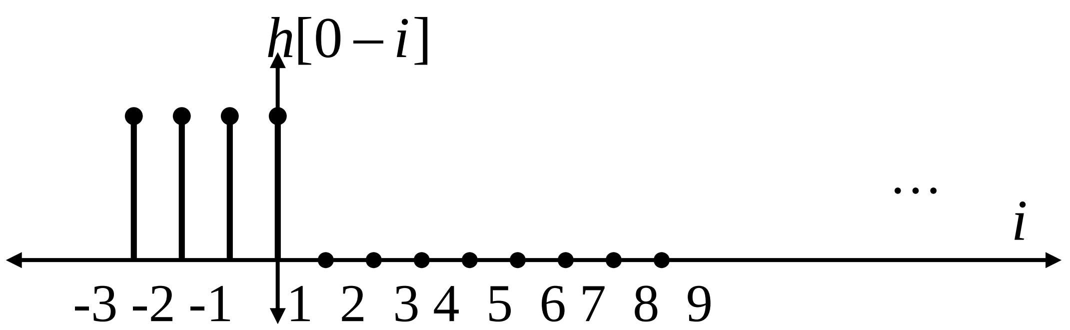
		k=-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	Rewrite signals in k
	x(k)	0	0	0	0	0	0	1	2	2	1	1	0	0	0	0	0	
	x(-k)	0	0	1	1	2	2	1	0	0	0	0	0	0	0	0	0	Flip x(k)
n=-1	x(-1-k)	0	1	1	2	2	1	0	0	0	0	0	0	0	0	0	0	<--- Shifted x(-k) by -1
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(-1-k)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Sum ----> y(-1) = 0
n=0	x(-k)	0	0	1	1	2	2	1	0	0	0	0	0	0	0	0	0	<--- Shifted x(-k) by 0
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(-k)	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	Sum ----> y(0) = 3
n=1	x(1-k)	0	0	0	1	1	2	2	1	0	0	0	0	0	0	0	0	<--- Shifted x(-k) by 1
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(1-k)	0	0	0	0	0	0	6	2	0	0	0	0	0	0	0	0	Sum ----> y(1) = 8
n=2	x(2-k)	0	0	0	0	1	1	2	2	1	0	0	0	0	0	0	0	<--- Shifted x(-k) by 2
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(2-k)	0	0	0	0	0	0	6	4	1	0	0	0	0	0	0	0	Sum ----> y(2) = 11
n=3	x(3-k)	0	0	0	0	0	1	1	2	2	1	0	0	0	0	0	0	<--- Shifted x(-k) by 3
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(3-k)	0	0	0	0	0	0	3	4	2	0	0	0	0	0	0	0	Sum ----> y(3) = 9
n=4	x(4-k)	0	0	0	0	0	0	1	1	2	2	1	0	0	0	0	0	<--- Shifted x(-k) by 4
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(4-k)	0	0	0	0	0	0	3	2	2	0	0	0	0	0	0	0	Sum ----> y(4) = 7
n=5	x(5-k)	0	0	0	0	0	0	0	1	1	2	2	1	0	0	0	0	<--- Shifted x(-k) by 5
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(5-k)	0	0	0	0	0	0	0	2	1	0	0	0	0	0	0	0	Sum ----> y(5) = 3
n=6	x(6-k)	0	0	0	0	0	0	0	0	1	1	2	2	1	0	0	0	<--- Shifted x(-k) by 6
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(6-k)	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	Sum ----> y(6) = 1
n=7	x(7-k)	0	0	0	0	0	0	0	0	0	1	1	2	2	1	0	0	<--- Shifted x(-k) by 7
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(7-k)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Sum ----> y(7) = 0
n=8	x(8-k)	0	0	0	0	0	0	0	0	0	0	1	1	2	2	1	0	<--- Shifted x(-k) by 8
	h(k)	0	0	0	0	0	0	3	2	1	0	0	0	0	0	0	0	
	h(k)x(8-k)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Sum ----> y(8) = 0

D-T Convolution Examples

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n] \qquad h[n] = u[n] - u[n-4]$$



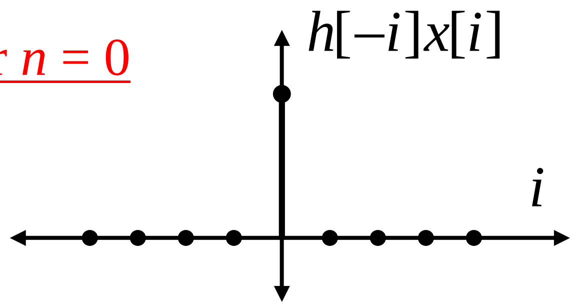
Choose to flip and slide $h[n]$ This shows $h[n-i]$ for $n = 0$



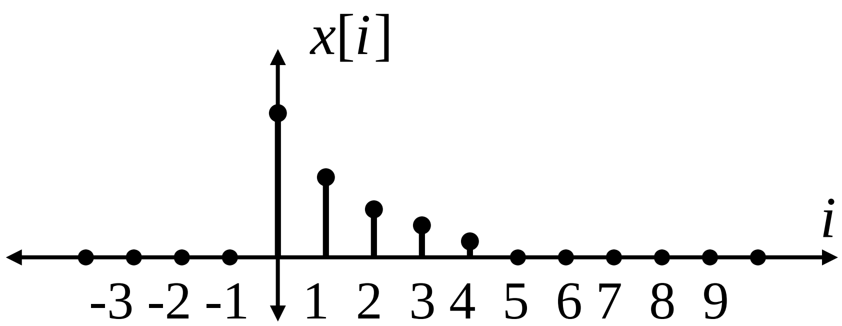
For $n < 0$ $\quad h[n-i]x(i) = 0 \quad \forall i$

$$\Rightarrow \boxed{y[n] = 0 \quad for \quad n < 0}$$

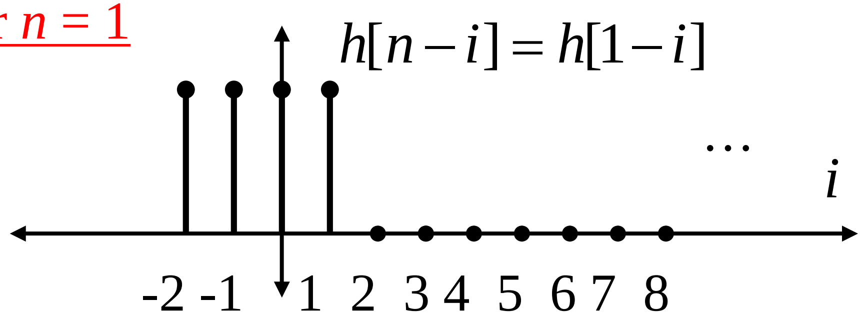
For $n = 0$



$\Rightarrow y[0] = 1$



For $n = 1$



$$\boxed{\Rightarrow y(1) = 1 + \tfrac{1}{2} = 3/2}$$

Notice that for $n = 0$, $n = 1$, …, $n = 3$

The general result is:

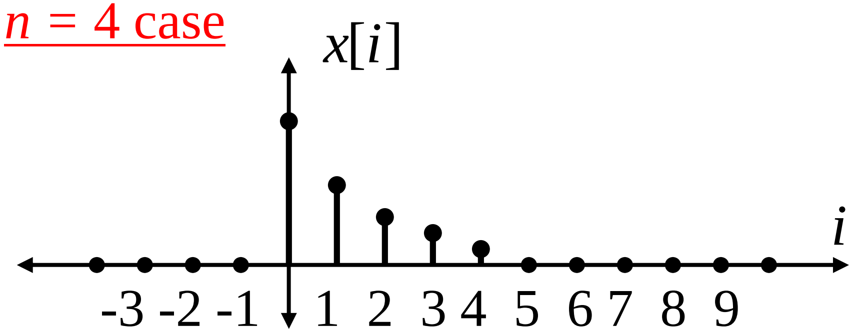
$$y[n] = \sum_{i=0}^{n} \left(\tfrac{1}{2}\right)^{i} \qquad for \quad n = 0,1,2,3$$

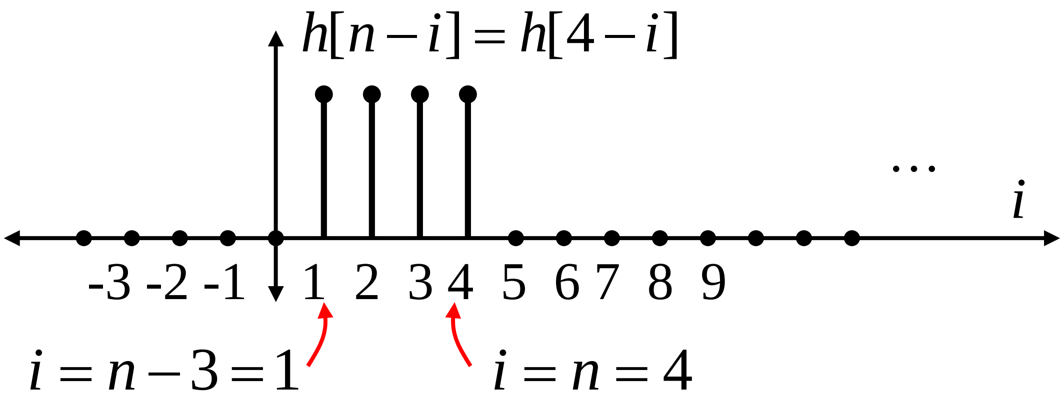
$$= \frac{1 - \left(\tfrac{1}{2}\right)^{n+1}}{1 - \tfrac{1}{2}} \qquad (Geometric \quad Sum)$$

$$\boxed{y[n] = 2\left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right] \qquad for \quad n = 0,1,2,3}$$

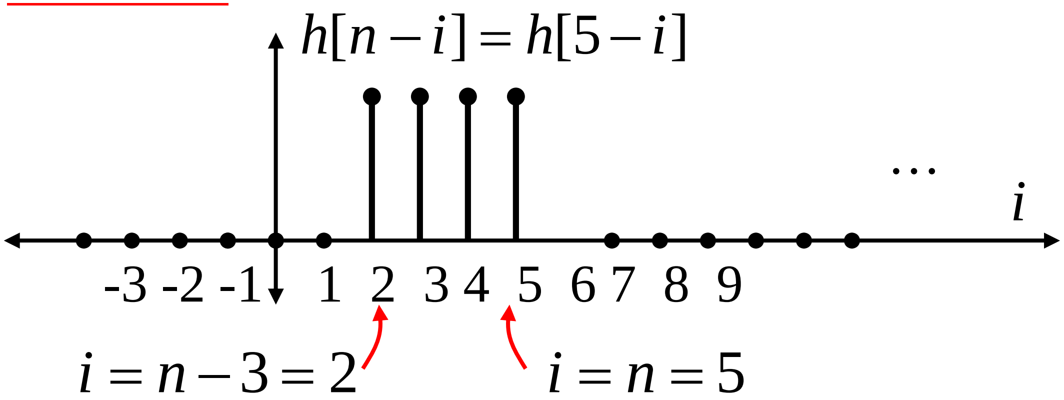
Now for n = 4, n = 5, …

n = 4 case





n = 5 case



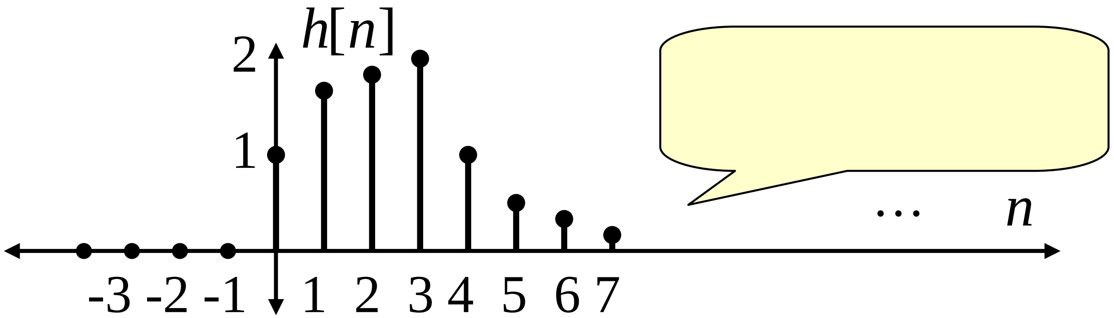
Notice that: for n = 4, 5, 6, …

$$y[n]=\sum_{i=n-3}^{n}\left(\tfrac{1}{2}\right)^{i}\quad for\;\; n=4,5,6,\ldots$$

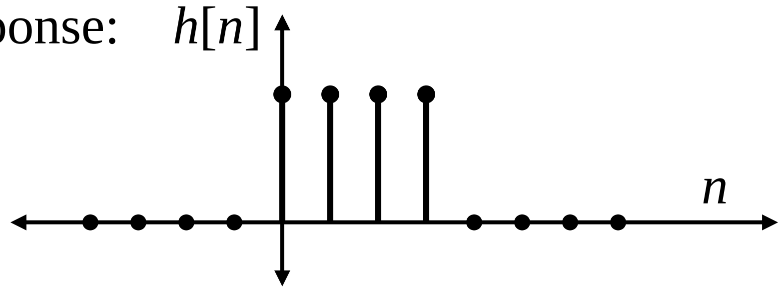
$$=\frac{\left(\tfrac{1}{2}\right)^{n-3}-\left(\tfrac{1}{2}\right)^{n+1}}{1-\tfrac{1}{2}}\qquad then\;\; simplify!$$

Then we can write out the solution as:

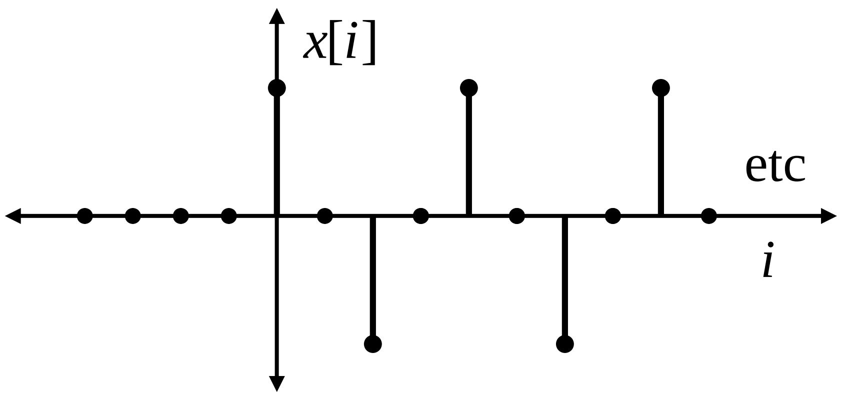
$$y[n]=\begin{cases}0, & n<0\\ 2\left[1-(1/2)^{n+1}\right], & n=0,1,2,3\\ 2\left[(1/2)^{n-3}-(1/2)^{n+1}\right], & n=4,5,6,\ldots\end{cases}$$

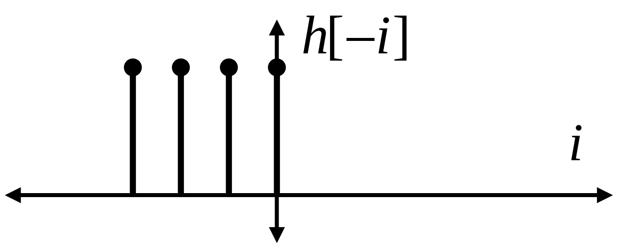


2. Same Impulse Response:



$$x[n] = \cos\left(\pi/2\, n\right) u[n]$$





Again $y[n] = 0$ for $n < 0$:

$$y[0] = 1$$

$$y[1] = 1 + 0 = 1$$

$$y[2] = 1 + 0 - 1 = 0$$

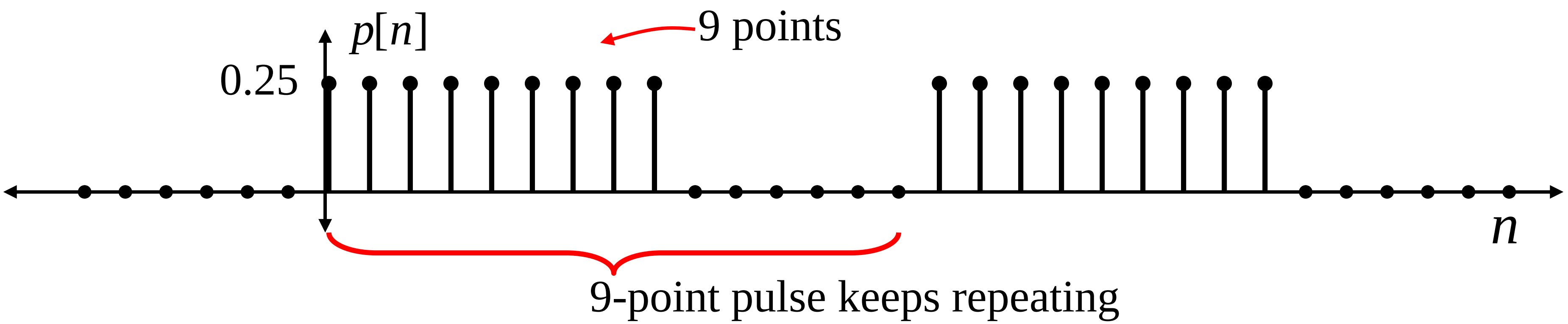
$$y[3] = 1 + 0 - 1 + 0 = 0$$

$$y[4] = 0 - 1 + 0 + 1 = 0$$

$$y[5] = -1 + 0 + 1 + 0 = 0$$

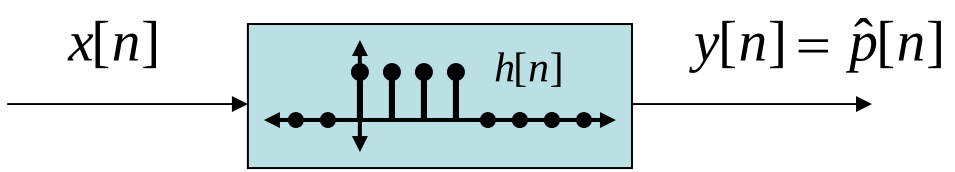
Notice: $y[n] = 0 \quad \forall\, n = 2, 3, 4, 5, \ldots$!

So suppose we had a <u>desired</u> part of our signal as:



But say we "receive" our desired pulse signal with an "interfering" sinusoid:

$$x[n] = p[n] + \cos\left(\pi/2\, n\right)u[n]$$



From above we know that system "zeros out" (or suppresses) the sinusoid…

We also know that the system will "pass" the pulses, although their edges will be smoothed.

Matlab Explorations

disc_03_DT_conv.m

```
%%% Matlab exploration for Pulses with Interfering Sinusoid

p=[ones(1,9) zeros(1,6)];   %%% Create one pulse and zeros
p=[p p p p p];   %%% stack 5 of them together
p=0.25*p;   %%% adjust its amplitude to be 0.25
subplot(3,1,1)
stem(0:74,p)   %%% look at the sequence of pulses
xlabel('Sample Index, n')
ylabel('Pulsed Signal p[n]')
x=p+cos((pi/2)*(0:74));  % add in an interfering sinusoid
subplot(3,1,2)
stem(0:74,x)
xlabel('Sample Index, n')
ylabel('x[n] Input = pulse + sinusoid')
y=conv(x,ones(1,4));   %% filter out sinusoid with DT Conv.
subplot(3,1,3)
stem(0:77,y)
xlabel('Sample Index, n')
ylabel('y[n] = Output')
%%%  Note that pulses are free of sinusoidal interference but have been "smoothed"
```